Towards systematic error estimates in asymptotically safe gravity-matter systems

HEP Seminar, National Centre for Nuclear Research

December 16, 2025

Marc Schiffer, Radboud University Nijmegen

Based on

M. Riabokon, MS, F. Wagner: **Phys.Rev.D 112 (2025) 10, 106003**

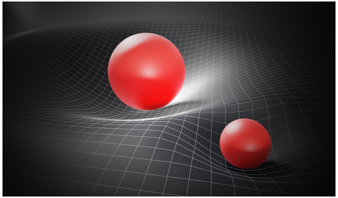
G. de Brito, M. Reichert, MS: **arXiv: 2510.09572**

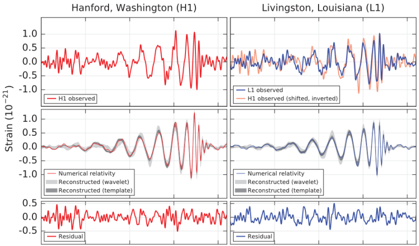
Radboud Universiteit

NWO

Why Quantum Gravity with matter?

General Relativity





LIGO, 2016

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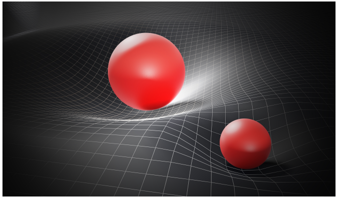

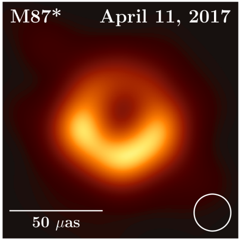

EHT, 2019

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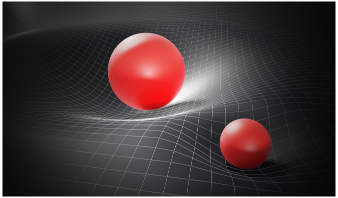

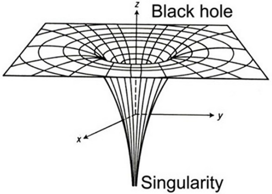

Northern Arizona University

Scale of QG: $M_{\mathrm{Pl}} = \sqrt{\frac{\hbar c}{g}}$

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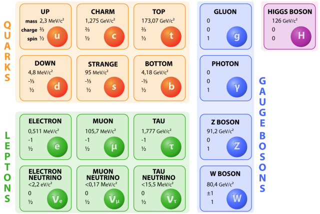
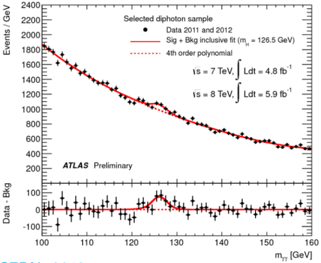
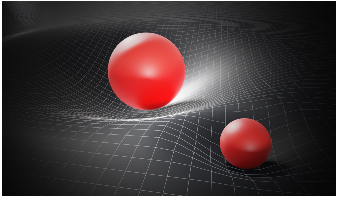
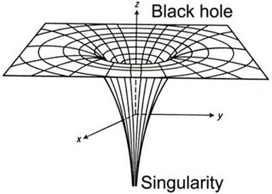




CERN, 2012

General Relativity





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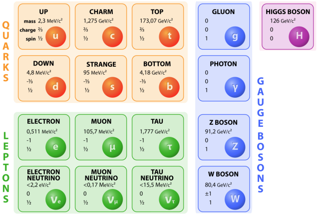

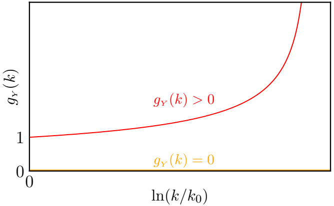

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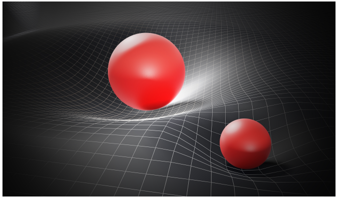

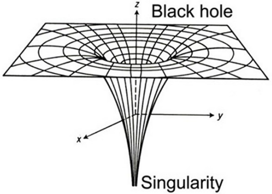

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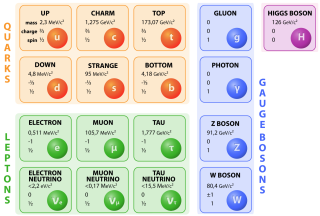

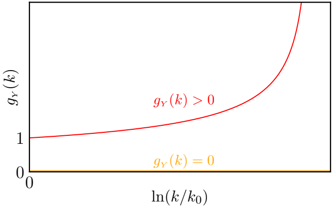

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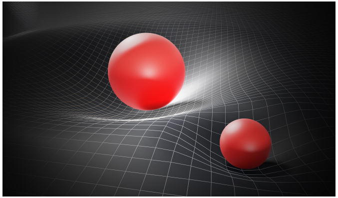

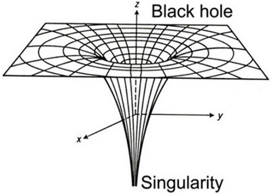

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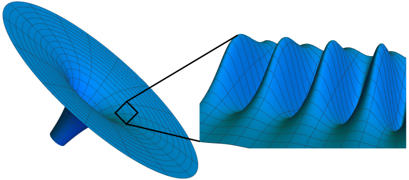
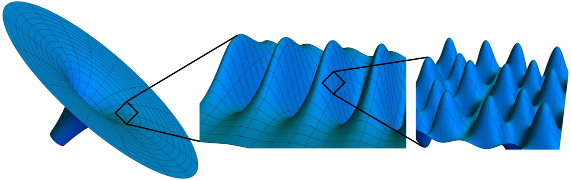
Quantum nature of spacetime: might provide UV-completion for GR and SM!

Asymptotically Safe Quantum Gravity





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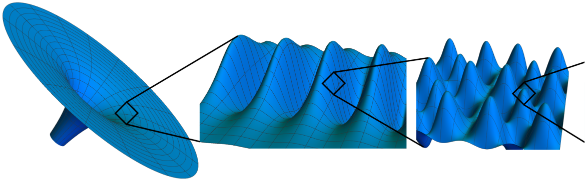
Asymptotically Safe Quantum Gravity



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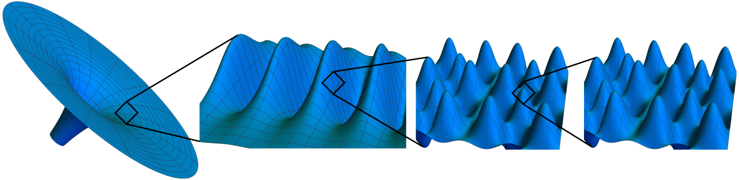

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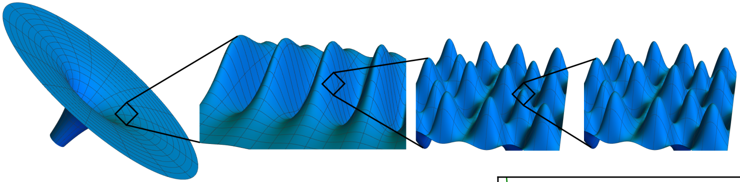

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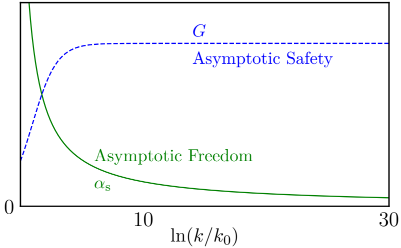

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free fixed point, $\alpha_{s,*} = 0$
Asymptotic safety:
interacting fixed point, $G_* \neq 0$

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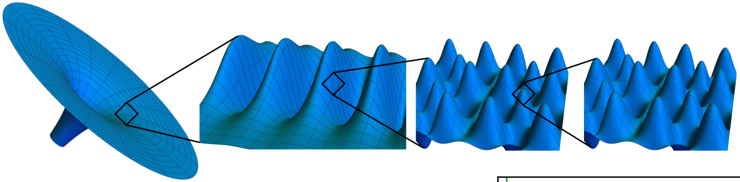

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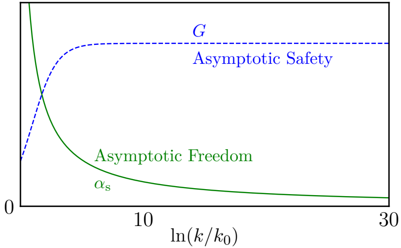
- Key idea of asymptotic safety:
  Quantum realization of
  scale symmetry
  - imposes infinitely many conditions on theory
  - Examples for AS:
    gauge-Yukawa systems (perturbative)
    [Litim, Sannino; 2014]

    Gross-Neveu, etc.
    [Braun, Gies, Scherer; 2010], see also review [Eichhorn, 2019]



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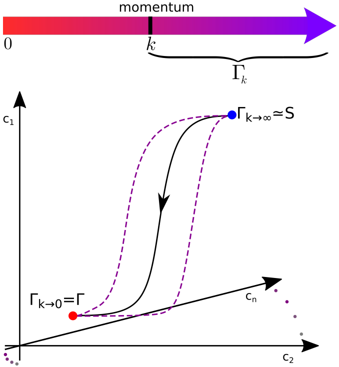
Use different methods in concerted sfashion to extract evidence for and physical features of asymptotically safe quantum gravity.

see also [Ambjorn, Gizbert-Studnicki, Goerlich, Nemeth; 2024]

Tool: Functional Renormalization Group

Main idea:

include quantum fluctuations step by step



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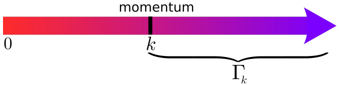
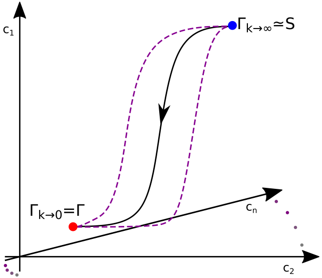
$$e^{-\Gamma_k[\phi]} \sim \int \mathcal{D}\varphi \; e^{-S[\varphi]-\frac{1}{2}\int_p \varphi(-p) R_k(p^2) \varphi(p)}\,, \qquad R_k(p^2) \begin{cases} > 0 \text{ if } p^2 < k^2 \text{ (supression)} \\ = 0 \text{ if } p^2 > k^2 \text{ (no supression)} \end{cases}$$





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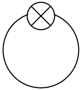
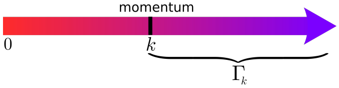
Flow Equation

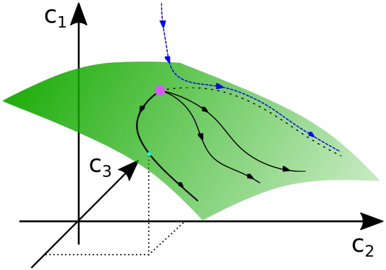
[Wetterich, 1993], [Ellwanger, 1993], [Morris, 1994], [Reuter, 1996]

$$k\,\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left(\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\,\partial_k R_k\right)$$

$$= \frac{1}{2}$$







→ search for fixed points $k\,\partial_k g_i = 0$

→ describe RG-flow in theory-space

Γ_k : requires truncation ⇒ not exact

Predictivity in asymptotic safety

scale invariance

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our universe at low energies

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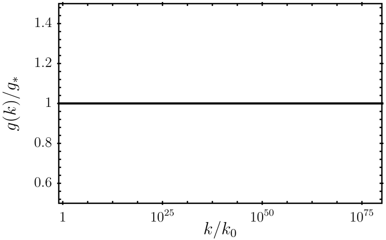

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our universe at low energies

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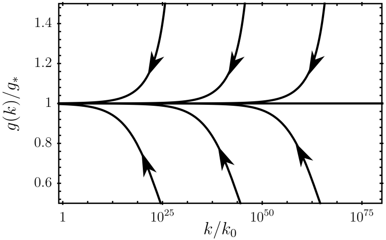

Irrelevant direction:
prediction in IR

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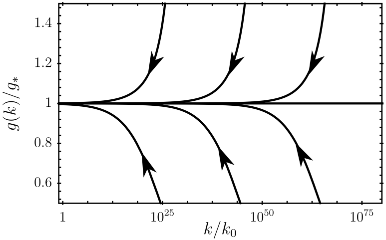

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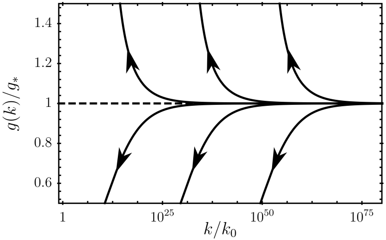

Relevant direction:
free parameter

Predictivity in asymptotic safety

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- More technically: predictivity encoded in critical exponents Θ_I:

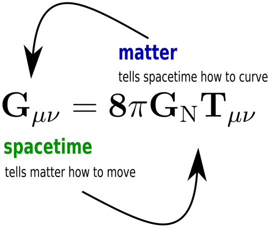
$$g_j(k) = g_j^* + \sum_I c_I V_j^I \left(\frac{k}{k_0}\right)^{-\Theta_I} \quad \text{with} \quad \Theta_I = -\text{eig}\left(\left(\frac{\partial \beta_{g_j}}{\partial g_i}\right)\Big|_{\mathbf{g}=\mathbf{g}^*}\right)$$

Irrelevant direction:
prediction in IR

$$\Theta_I < 0$$

Relevant direction:
free parameter

$$\Theta_I > 0$$



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- Key questions for gravity-matter systems:

AS in gravity-matter systems

matter

tells spacetime how to curve

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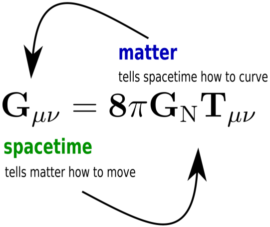

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  - Is there a viable phenomenology?
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  - Robustness of phenomenological implications?
    [Kotlarski, Kowalska, Rizzo, Sessolo; 2023], [Riabokon, MS, Wagner; 2025], [de Brito, Reichert, MS; 2025]

The $U(1)$ sector of the Standard Model

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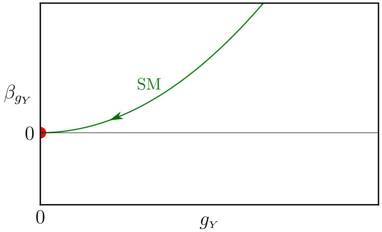
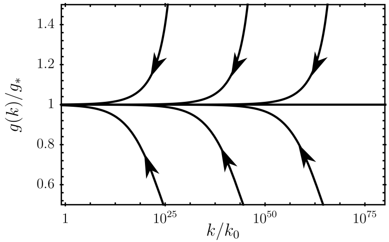
$$\beta_{g_Y} = \frac{1}{16\pi^2}\frac{41}{6}g_Y^3 + \mathcal{O}(g_Y^4)$$





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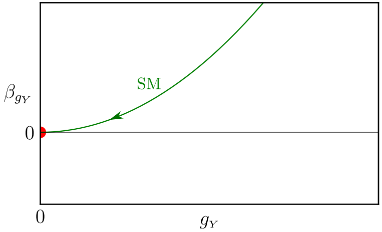

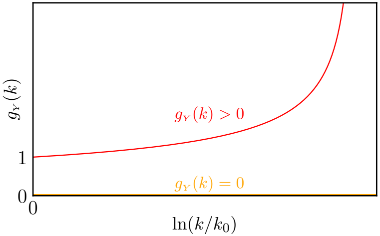

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$$f_g \begin{cases} = \text{const.} \geq 0\,, & \text{for } k > M_{\text{Pl}} \\ \approx 0\,, & \text{for } k < M_{\text{Pl}} \end{cases}$$

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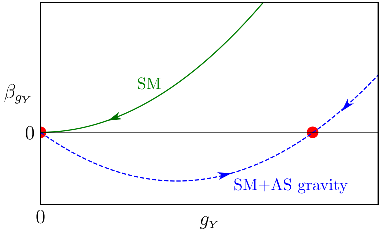

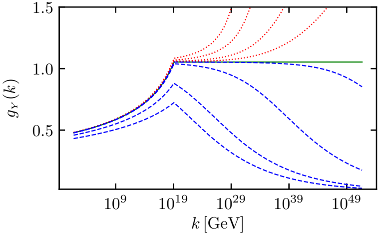

$$\beta_{g_Y} = -f_g\, g_Y + \frac{1}{16\pi^2}\frac{41}{6} g_Y^3 + \mathcal{O}(g_Y^4)$$

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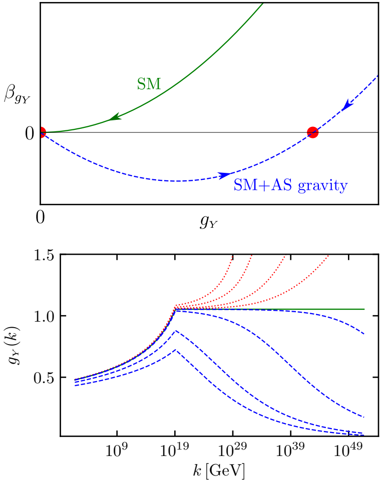
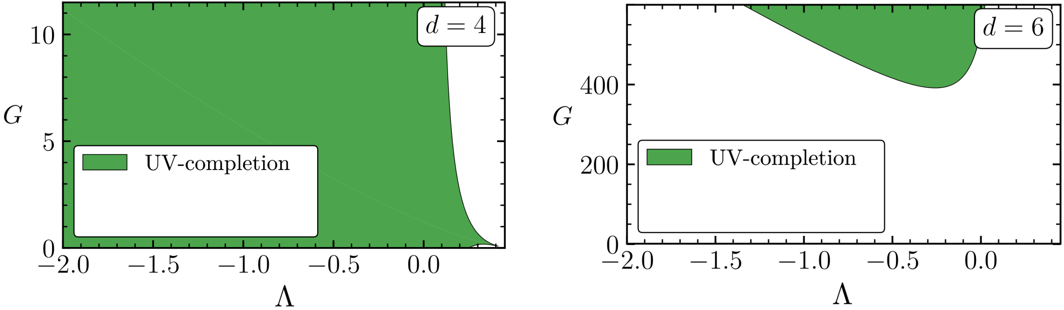


Metric fluctuations might induce a UV completion of the $U(1)$-sector.
$\Rightarrow$ Upper bound on $g_Y(k)$ (i.e., constraints on gravity) [Eichhorn, Versteegen; 2017]

Example: The $U(1)$ sector in $d > 4$

- $\beta_{g_Y} = g_Y \left(\frac{d-4}{2} - f_g(d)\right) + \mathcal{O}(g_Y^3)$
  $\Rightarrow$ UV-completion shifts into more strongly coupled regime



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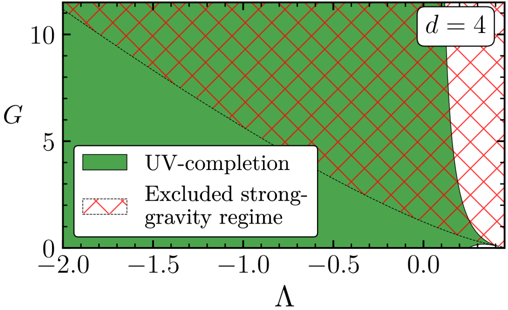

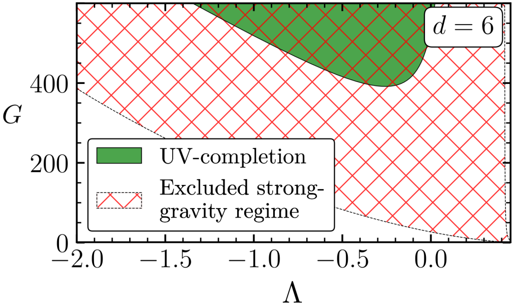

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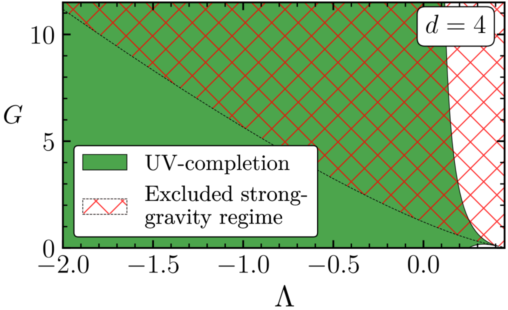

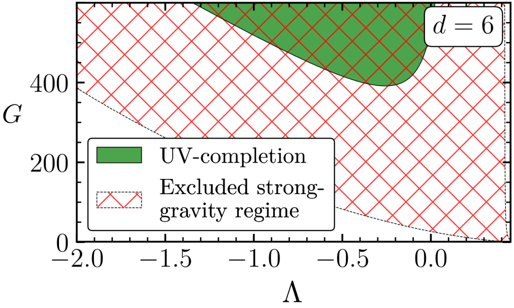

$U(1)$ gauge sector might remain UV-incomplete in $d \geq 6$, even in the presence of gravity.
[Eichhorn, MS; 2019]

Robustness of f_g: Litim-type shapefunction

- Litim-type shapefunction: $r_k(z) = a_{\mathrm{L}}(1-z)\Theta(1-z)$

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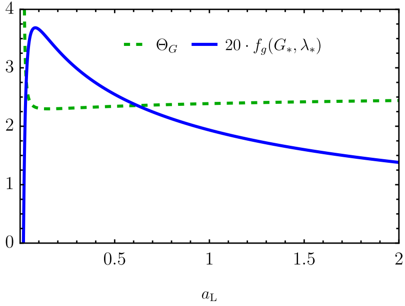

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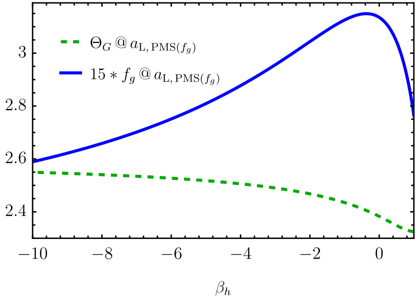
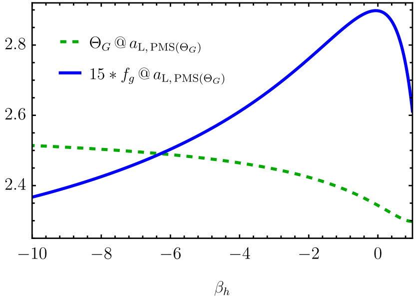
- Track each as function of β_h





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- Litim-type shapefunction: $r_k(z) = a_{\mathrm{L}}(1-z)\Theta(1-z)$
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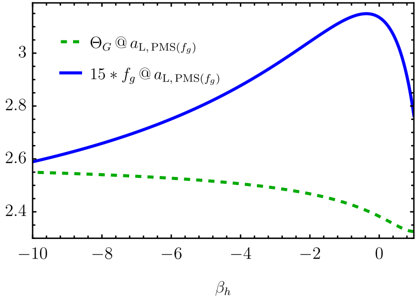
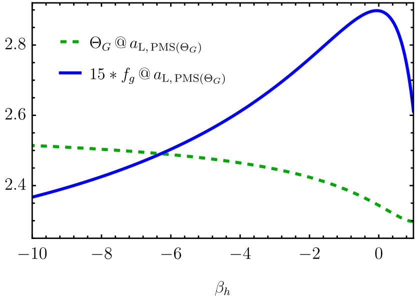
- Track each as function of β_h
- Can additionally minimize β_h-dependence.





Robustness of f_g: Adding matter

- Start from previous picture, and add scalars, fermions gauge fields
- In short: scalars and gauge fields move curves to the right, fermions to the left.
- SM matter content: fermions dominate, hence no PMS point
- Caveat: background approximation likely to break down for SM matter content

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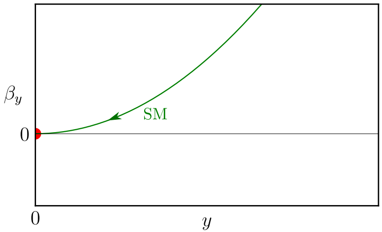
Summary

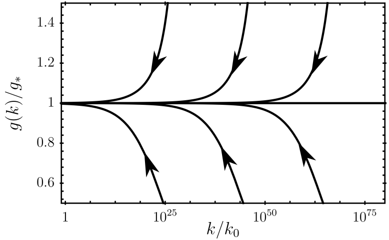
- For minimal matter content: PMS point of f_g in a_i and β_h
- This point is global maximum of f_g; similarities between shape-functions

Towards the Yukawa sector of the Standard Model

Single Yukawa coupling y:

$$\beta_y = \qquad \#_{\text{SM}}\, y^3 + \mathcal{O}(y^4)$$





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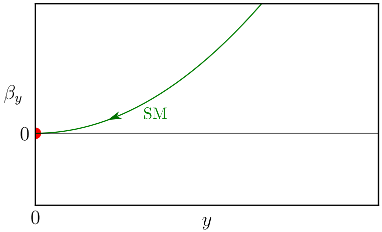

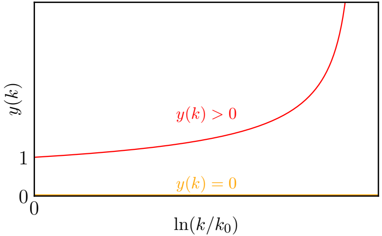

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$$\beta_y = -f_y\, y + \#_{\mathrm{SM}}\, y^3 + \mathcal{O}(y^4)$$

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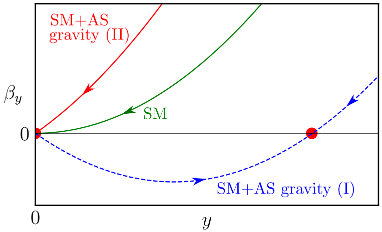

[Oda, Yamada; 2015], [Eichhorn, Held, Pawlowski; 2016], [Eichhorn, Held; 2017]

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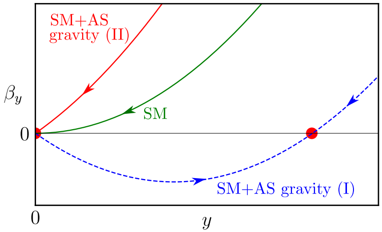

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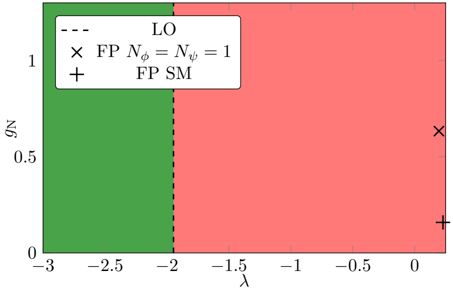
UV completion of the simple Yukawa system: constraints on gravitational dynamics
Additionally: top mass might be retro-dicted [Eichhorn, Held, 2017]

Simple Yukawa system: state of the art (LO)



- Simplest approximation: $\lambda_{\text{crit}} \approx -2$

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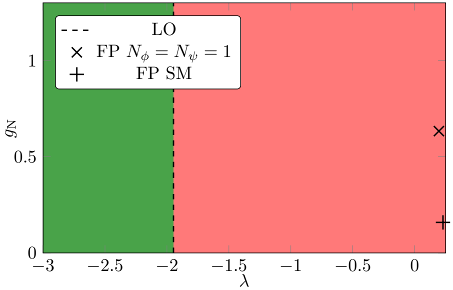

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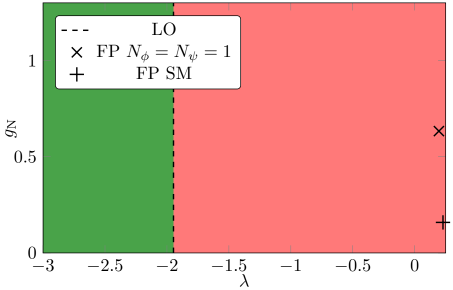

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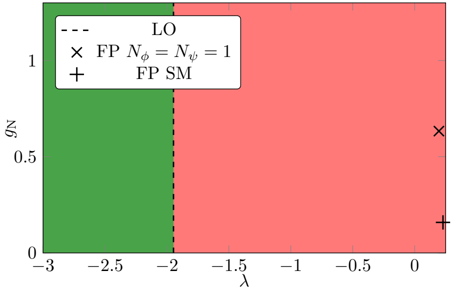

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Leading order: $\Theta_{y,\text{MM}} = -0.58$ and $\Theta_{y,\text{SM}} = -0.22$
This would result in vanishing Yukawa couplings in IR
$\Rightarrow$ no-go theorem for fundamental Yukawa interactions in ASQG!?

Contributions beyond LO

$$\theta_I = -\mathsf{eig}\left(M_{ij}\right) \,, \quad \text{with} \quad M_{ij} = \left(\frac{\partial \beta_{g_j}}{\partial g_i}\right)\Bigg|_{\mathbf{g}=\mathbf{g}^*}$$

- At LO: $\theta_y = f_y g$;
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- At LO: $\theta_y = f_y g$;
- NLO: g^2 contributions to Θ_y;
- Possible sources:
  - direct contributions to M_{11}
    - contribution $\sim y\, g_{\text{ind}}$ to β_y with $g_{\text{ind},*} \neq 0$
  - off-diagonal contributions in M_{ij} - contribution to β_y can be independent of y itself

Direct contributions: induced operators

- Operators share symmetries of kinetic terms
- Feature Gaussian fixed point for $g = 0$ and *shifted* Gaussian fixed point for $g \neq 0$
- Existence of fixed point for induced operators at g_*: non-trivial test for ASQG (stability of truncations)
  [Eichhorn, Gies; 2011], [Eichhorn; 2021], [Christiansen, Eichhorn; 2017], [Eichhorn, Held; 2017], [Eichhorn, MS; 2019], ...
- Assume induced coupling b

$$\beta_b = d_{\mathcal{O}}\, b + c_1\, b + \mathcal{O}(g^2)\,, \quad \Rightarrow \quad b_* = -g_* \frac{c_1}{d_{\mathcal{O}}} + \mathcal{O}(g^2)\,.$$

- Contribution to θ_y:

$$\theta_y\Big|_{\text{ind}} = -c_2\, g_*\, b_* = -g_*^2 \frac{c_1\, c_2}{d_{\mathcal{O}}} + \mathcal{O}(g^3)$$

Off-diagonal contributions

- Break symmetries of kinetic terms
- Always feature Gaussian fixed point;
  interacting fixed point possible, but not needed!
- Consider coupling κ: contributes linearly to β_y, and y contributes linearly to β_κ

$$M = \begin{bmatrix} a_{11}\, g & a_{12}\, g \\ a_{21}\, g & \mathrm{d}_{\mathcal{O}} + a_{22}\, g \end{bmatrix} .$$

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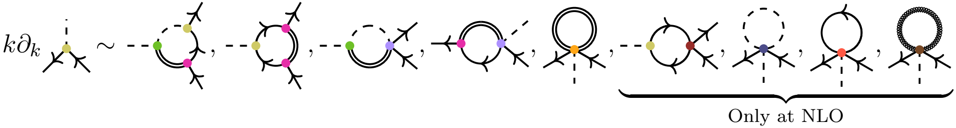
$$M = \begin{bmatrix} a_{11}\, g & a_{12}\, g \\ a_{21}\, g & \mathrm{d}_{\mathcal{O}} + a_{22}\, g \end{bmatrix}.$$

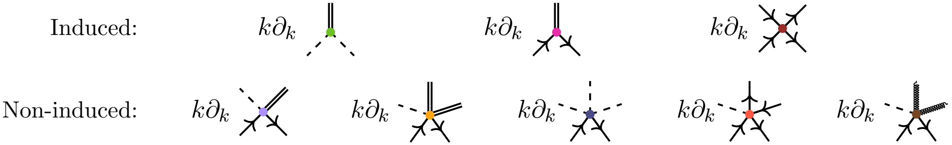
- This yields

$$\theta_y\Big|_{\mathrm{non-induced}} = -g_*^2 \frac{a_{12}a_{21}}{\mathrm{d}_{\mathcal{O}}} + \mathcal{O}(g^3)$$

Yukawa flow equation



Resummed vertices



Simple Yukawa system: Summary of NLO Operators

- **Induced**
- contributions to β_y with $g_* \neq 0$

Vertex	Operator	Coupl.
$h_{\mu\nu}\bar{\psi}\psi$	$R^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu \psi$	σ_{Ric}
	$R\,\bar{\psi}\,\not{D}\psi$	σ_{R}
$h_{\mu\nu}\phi^2$	$R^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi$	ρ_{Ric}
	$R\,\partial_\mu\phi\,\partial^\mu\phi$	ρ_{R}
$(\bar{\psi}\psi)^2$	$(\bar{\psi}\gamma_\mu\psi)^2 + (i\,\bar{\psi}\gamma_\mu\gamma_5\psi)^2$	λ_+
$\phi^2\bar{\psi}\psi$	$\partial_\mu\phi\,\partial^\mu\phi\,\bar{\psi}\,\not{D}\psi$	$\chi_{1/2}$
ϕ^4	$(\partial_\mu\phi\partial^\mu\phi)^2$	K_2
$h^2_{\mu\nu}\bar{\psi}\psi$	$R^2\,\bar{\psi}\,\not{D}\psi$	$\sigma_{R^2,i}$
$h^2_{\mu\nu}\phi^2$	$R^2\,\partial_\mu\phi\,\partial^\mu\phi$	$\rho_{R^2,i}$

see [Eichhorn, MS; 2022] for references

contribute via η_i only $\Rightarrow$ neglect

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contribute via η_i only $\Rightarrow$ neglect

- **Non induced**
- contributions to β_y with $\hat{g}_{i,*} = 0$

Vertex	Operator	Coupl.
$\phi\bar{\psi}\psi$	$\Box\phi\bar{\psi}\psi$	$y_{\Box,1}$
	$\phi\;\bar{\psi}\Box\psi$	$y_{\Box,2}$
$h_{\mu\nu}\phi\bar{\psi}\psi$	$R\,\phi\bar{\psi}\psi$	y_{R}
$h^2_{\mu\nu}\phi\bar{\psi}\psi$	$R^2\,\phi\bar{\psi}\psi$	y_{R^2}
	$C^2_{\mu\nu\rho\sigma}\,\phi\bar{\psi}\psi$	y_{C^2}
$\phi^3\bar{\psi}\psi$	$(\partial_\mu\phi)^2\,\phi\bar{\psi}\psi$	y_{ϕ^2}
$\phi(\bar{\psi}\psi)^2$	$(\bar{\psi}\not{D}\psi)\,\phi\bar{\psi}\psi$	$y_{\bar{\psi}\psi}$
$A^2\phi\bar{\psi}\psi$	$(F_{\mu\nu}F^{\mu\nu})\,\phi\bar{\psi}\psi$	y_{A^2}

Simple Yukawa system: Summary of NLO Operators

- **Induced**
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$A^2\phi\bar{\psi}\psi$	$(F_{\mu\nu}F^{\mu\nu})\,\phi\bar{\psi}\psi$	y_{A^2}

contribute via η_i only $\Rightarrow$ neglect

Induced operators shift FP, but admit lower-triangular stability sub-matrix

Simple Yukawa system: Summary of NLO Operators

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- contributions to β_y with $g_* \neq 0$

Vertex	Operator	Coupl.
$h_{\mu\nu}\bar{\psi}\psi$	$R^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi$	σ_{Ric}
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$h^2_{\mu\nu}\phi^2$	$R^2\,\partial_\mu\phi\,\partial^\mu\phi$	$\rho_{R^2,i}$

see [Eichhorn, MS; 2022] for references

- **Non induced**
- contributions to β_y with $\hat{g}_{i,*} = 0$

Vertex	Operator	Coupl.
$\phi\bar{\psi}\psi$	$\Box\phi\bar{\psi}\psi$	$y_{\Box,1}$
	$\phi\,\bar{\psi}\Box\psi$	$y_{\Box,2}$
$h_{\mu\nu}\phi\bar{\psi}\psi$	$R\,\phi\bar{\psi}\psi$	y_{R}
$h^2_{\mu\nu}\phi\bar{\psi}\psi$	$R^2\,\phi\bar{\psi}\psi$	y_{R^2}
	$C^2_{\mu\nu\rho\sigma}\,\phi\bar{\psi}\psi$	y_{C^2}
$\phi^3\bar{\psi}\psi$	$(\partial_\mu\phi)^2\,\phi\bar{\psi}\psi$	y_{ϕ^2}
$\phi(\bar{\psi}\psi)^2$	$(\bar{\psi}\not{D}\psi)\,\phi\bar{\psi}\psi$	$y_{\bar{\psi}\psi}$
$A^2\phi\bar{\psi}\psi$	$(F_{\mu\nu}F^{\mu\nu})\,\phi\bar{\psi}\psi$	y_{A^2}

contribute via η_i only $\Rightarrow$ neglect

Induced operators shift FP, but admit lower-triangular stability sub-matrix

Modulo momentum-dependences: No further g^2 contribution to θ_y

Yukawa system: Main results

- Fixed point exists for minimal matter and SM matter
- Critical exponents of higher-order operators: close to canonical mass dimension
- Yukawa couplings are relevant at asymptotically safe fixed point!

$$\theta_{y,\mathrm{MM}} = 3.1^{+1.8}_{-1.1}\,, \qquad \theta_{y,\mathrm{SM}} = 2.2^{+1.3}_{-1.0}$$

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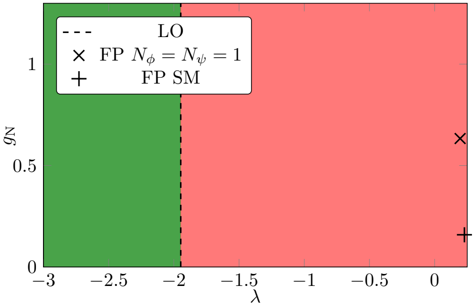
- Contributions from various couplings:

$$\begin{aligned}\theta_{y,\mathrm{SM}} &= -1.3g + g^2(\underbrace{-13}_{y_R} + \underbrace{0.81}_{y_\Box} + \underbrace{32}_{y_{R^2}} \underbrace{-17}_{y_{C^2}} + \underbrace{40}_{y_{\bar{\psi}\psi}} + \underbrace{1.8}_{y_{A^2}} \\ &\qquad \underbrace{-1.6}_{\rho_{\mathsf{Ric}}} + \underbrace{1.0}_{\rho_{\mathsf{R}}} \underbrace{-0.40}_{\sigma_{\mathsf{Ric}}} + \underbrace{0.53}_{\sigma_{\mathsf{R}}} + \underbrace{0.13}_{\lambda_+}) \\ &= -1.3g + 44g^2\,.\end{aligned}$$

Yukawa coupling at NLO



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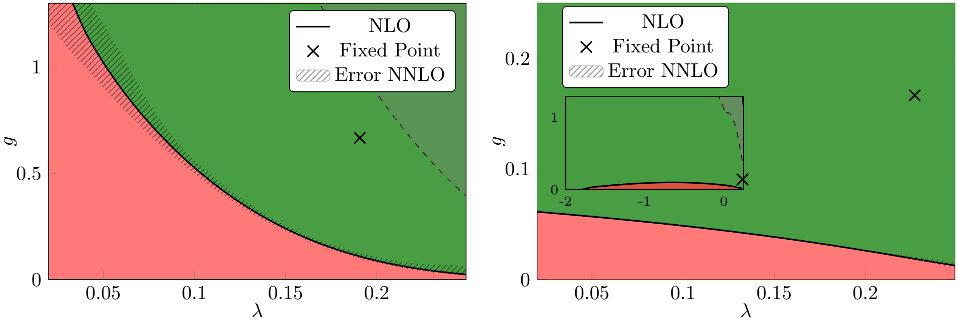

- NLO contributions to stability matrix: generate new regime where $\theta_y > 0$
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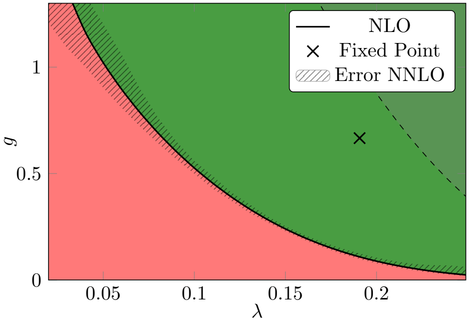

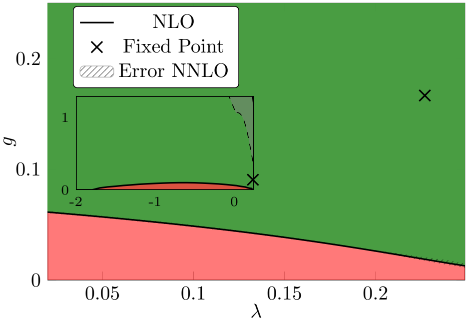

- NLO contributions to stability matrix: generate new regime where $\theta_y > 0$
- Fixed point lies inside that new regime!

- LO: $\theta_y < 0$, NLO: $\theta_y > 0$; What about NNLO (i.e., g^3-contributions)?

Estimating NNLO effects ('error bars')

- previously observed: $R\,\phi\,\bar{\psi}\psi$, $R^2\,\phi\,\bar{\psi}\psi$ and $C^2\,\phi\,\bar{\psi}\psi$ dominate
- estimate effect of $R^3\,\phi\,\bar{\psi}\psi$-type operator

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$$M_{\text{NLO}} = \begin{pmatrix} 1.3\,g & 1.1\,g & 0.54\,g & 1.3\,g & -2.2\,g & -0.14\,g & -0.048\,g \\ -23\,g & 2-0.73\,g & -5.5\,g & 0.46\,g & 8.5\,g & {\color{red}-0.014\,g} & -0.0093\,g \\ 3.0\,g & 1.6\,g & 2+1.8\,g & 0 & 0 & 0 & 0 \\ 95\,g & 36\,g & 25\,g & 4+2.4\,g & -43\,g & 0 & 0 \\ 31\,g & 13\,g & 5.7\,g & 34\,g & 4-5.9\,g & 0 & 0 \\ {\color{red}-1100}\,g & -680\,g & -370\,g & 630\,g & 410\,g & 4-0.39\,g & 0 \\ -150\,g & -46\,g & -23\,g & 5.5\,g & 18\,g & 0 & 4-1.4\,g \end{pmatrix}$$

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$$\tilde{M}_{\mathrm{NLO}} = \begin{pmatrix} 1.3\,g & 2.3\,g & -1.5\,g & -4.0\,g & -8.7\,g & -21\,g & -0.62\,g \\ -11\,g & 2-0.75\,g & 6.9\,g & -14\,g & 5.6\,g & -1.0\,g & -0.056\,g \\ -1.1\,g & -1.3\,g & 2+1.8\,g & 0 & 0 & 0 & 0 \\ -16\,g & -13\,g & 10\,g & 4+3.0\,g & 39\,g & 0 & 0 \\ -14\,g & -11\,g & -11\,g & -37\,g & 4-6.4\,g & 0 & 0 \\ -7.4\,g & -9.5\,g & -6.5\,g & -20\,g & -12\,g & 4-0.38\,g & 0 \\ -12\,g & -7.5\,g & 4.8\,g & -5.9\,g & 0.62\,g & 0 & 4-1.4\,g \end{pmatrix}$$

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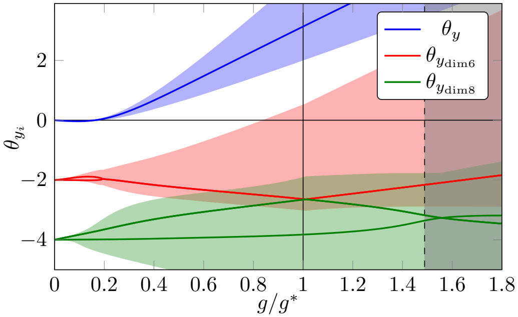
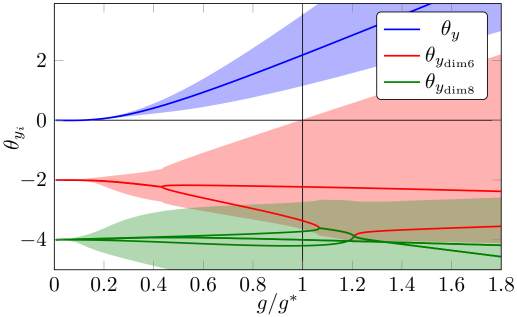
- Simulate effect of NNLO operator on θ_y (pick appropriate range for $\#_r$)

$$M = \left(\begin{array}{cccccccc} & & & & & & & 0 \\ & & & & & & & \#g \\ & & & & & & & 0 \\ & & & \tilde{M}_{\text{NLO}} & & & & \#g \\ & & & & & & & \#g \\ & & & & & & & \#g \\ & & & & & & & \#g \\ \#g & \#g & \#g & \#g & \#g & \#g & \#g & 6+\#g \end{array}\right)$$

Results of (N)NLO simulations





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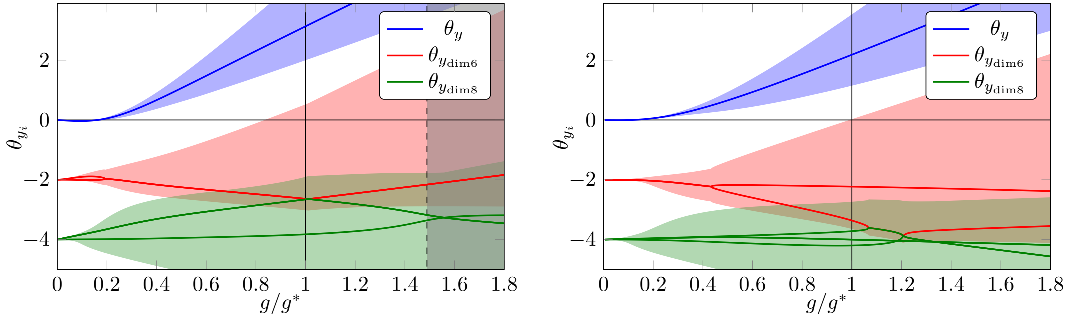

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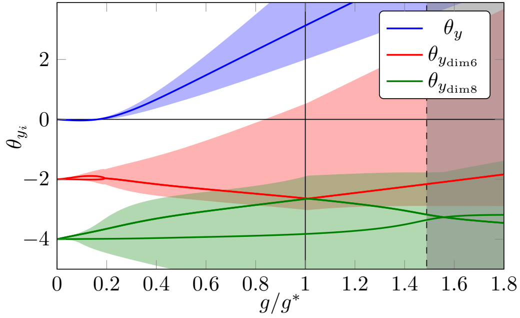

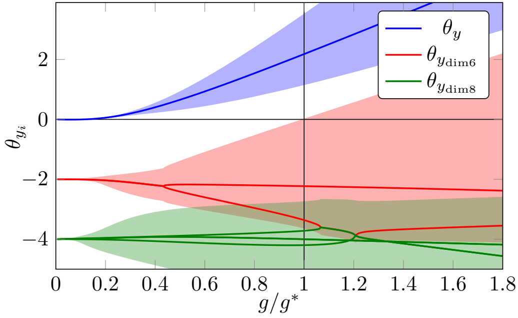

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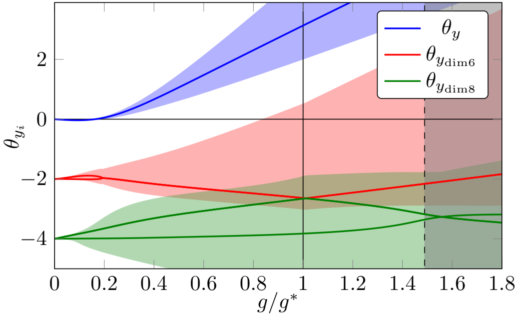

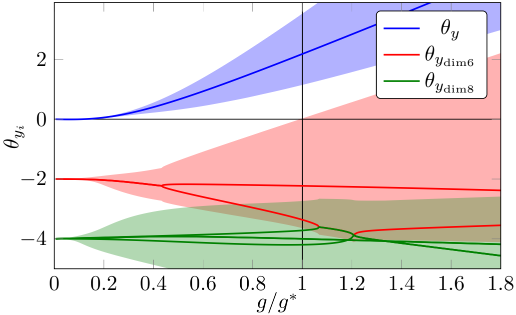

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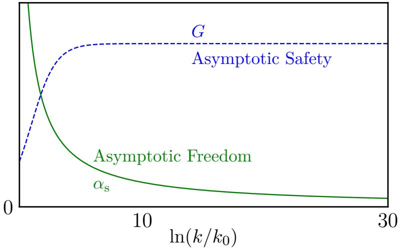
- (N)NLO simulations: insights into robustness
- large(ish) error-bars on θ_y
- small error-bars on region where $\theta_y \approx 0$

Summary

- Asymptotic safety: quantum realization of scale invariance



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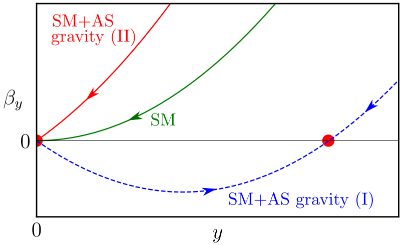

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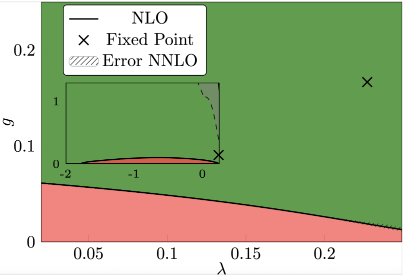

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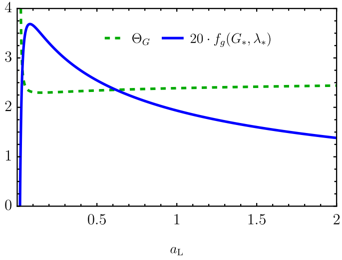

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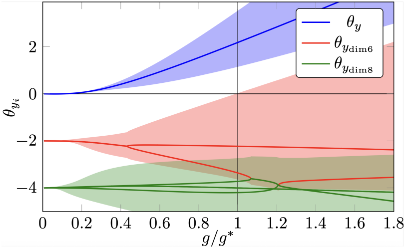

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Stay tuned!

Thank you for your attention!

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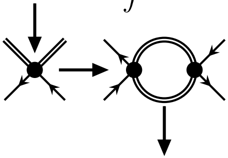

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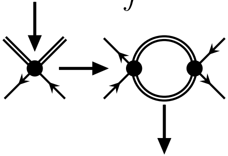

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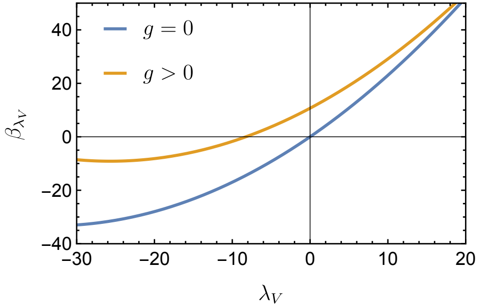

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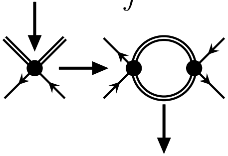

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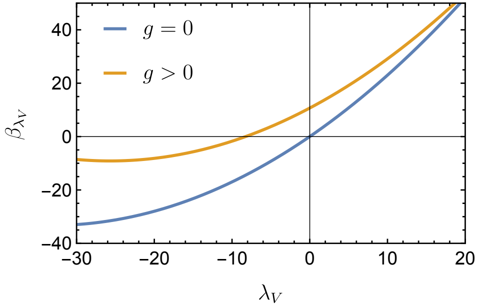

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- For $g > 0$: $\lambda_{V,*} \neq 0$

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- both cases: well-behaved extensions of the Reuter FP!

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- Plug FP-values into non-induced operators, and compute critical exponents at

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