# Excited bound states and their role in dark matter production

Stefan Lederer

Technical University of Munich

in collaboration with:

M. Beneke, M. Garny, T. Binder, J. Heisig, K. Urban, L. De Ros

based on:

arXiv:2308.01336 arXiv:2411.08737

### Outline

- 1 <u>Introduction</u>: Thermal Production and Freeze-out
- 2 <u>Theory background</u>: bound states in PNREFT
- 3 <u>Bound state formation</u>: semi-classical picture
- 4 <u>Intermezzo</u>: Perturbative Unitarity Violation in BSF
- 5 <u>Toy model</u>: Bound States in dark gauge sectors
- 6 <u>Realistic model</u>: colored + charged t-channel mediator

## Dark Matter Phenomenology – crash course

#### **Dark Matter:**

#### Key ingredient in modern cosmology

(gravitational evidence from all length scales)

- Dark
- Abundant

...

- Cold (probably)
- Some strong constraints from cosmology:
  - Cosmic Microwave Background  $\Omega_{DM}h^2 = 0.120 \pm 0.001$  [Planck, 2018]
  - Baryogenesis (must not be altered)
  - Structure formation (no warm / hot DM)
- else: No evidence from SM physics! (in)direct detection / collider searches / ...

 $\rightarrow$  How is DM produced in the early Universe?





<sup>[</sup>Bullet Cluster; X-ray: NASA/CXC/CfA/]

Boltzmann equation: 
$$\frac{dY}{dx} = \frac{1}{3Hs} \frac{ds}{dx} \frac{1}{2} \langle \sigma v \rangle \langle Y^2 - Y^{eq^2} \rangle$$

"time" coordinate:  $x = \frac{m_{\chi}}{T}$ 

particle abundance:  $Y = \frac{n}{s} = \frac{\text{particle number}}{\text{entropy}}$ 



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Thermally averaged annihilation cross-section

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#### dark U(1)





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#### **Efficient BSF prevents Chemical Decoupling !**

## Theory background

### **PNREFT**

#### Potential Non-Relativistic Effective Theory

## Theory Background: long-range Potentials

For interactions with **light mediators**, perturbation theory breaks down:

- $1 \gg \alpha$  perturbative QFT
- $1 \gg v$  NREFT
- $\alpha \gtrsim v$  PNREFT



→ Resum interaction to **all orders**  $\Rightarrow$  Coulomb-potential.

"Sommerfeld Effect" = long-range potentials between heavy particles



for DM: [Hisano et al.: 2005]

## Theory background: bound states in SU(N)

$$V(r) = \frac{\alpha_{\text{eff}}}{r} \qquad \chi_1 = \frac{\chi_1}{\chi_2}$$

Attractive, potentials can also host bound states!

 $\rightarrow$  What is the potential strength?

**QED** (or U(1)):  $\alpha_{\text{eff}} = -Q_{\chi_1}Q_{\chi_2}\alpha_{\text{em}}$ 

(For DM, representations may differ from  $F \otimes F^*$ .)

 $\rightarrow$  2-body states exist in different eigenstates of the potential.

 $\Rightarrow$  SU(N) gauge interactions can yield **different** & repulsive potentials.

## Theory background: Multipole interactions

~ "Which process forms bound states ?"

Scales in PNREFT: $M_{\chi}$ mass• $p \sim Mv$ momentum• $E \sim M\alpha^2, Mv^2$ energy

"Ultra-soft" emissions,  $\omega \sim M_{\chi} \alpha^2$ , can be expanded in <u>multipole</u> orders L:



 $\mathcal{L} \supset g (\mathbf{r} \cdot \mathbf{p}_{\omega})^{L} \propto g \alpha^{L} \ll 1$ 

These provide **bound-state formation**, Bremsstrahlung & bound-to-bound transitions.

## Theory background: PNREFT Lagrangian

We are interested in **annihilation** and **bound states**.

- $\Rightarrow$  project into 2-particle space:
  - $\rightarrow$  2 separate species (*S*, *B*) for scattering & bound states.
  - $\rightarrow$  independent potentials!
  - $\rightarrow$  include only the leading multipole operator.

$$\mathcal{L}_{BSF} = \mathcal{S}^{\dagger}(R, \vec{r}) \left( i\partial^{0} + \frac{\vec{\partial}_{\vec{r}}^{2}}{M_{\chi}} - \delta M_{\mathcal{S}} + \frac{\alpha_{s}}{r} \right) \mathcal{S}(R, \vec{r}) + \mathcal{B}^{\dagger}(R, \vec{r}) \left( i\partial^{0} + \frac{\vec{\partial}_{\vec{r}}^{2}}{M_{\chi}} - \delta M_{\mathcal{B}} + \frac{\alpha_{b}}{r} \right) \mathcal{B}(R, \vec{r})$$

think ",S = scattering state". if  $\alpha_s > 0$ , also bound states S exist

think "B = bound state". also B scattering-states exist

+ ,,h.c."

 $+ g_a^{\text{eff}} p_{\phi}^a r^a P_a(\hat{p}_{\phi} \cdot \hat{r}) \, \mathcal{B}^{\dagger}(R, \vec{r}) \, \phi^{\dagger}(R) \, \mathcal{S}(R, \vec{r})$ 

+ ,,i $\delta(r)$  annihilation operators"

The equations of motion are simply the Schrödinger equation.

## radiative Bound State Formation

*"Seeing the formula" ≠ "Undestanding the physics"* 

## General **BSF** expression



The computation is basically Quantum Mechanics. General result for all {*n*, *l*, *p*, *l*', *L*,  $\alpha_s$ ,  $\alpha_b$ }: Some notation n = major quantum number l', l = partial-wave numbers  $\kappa \equiv \frac{\alpha_s}{\alpha_b}$ ,  $\zeta_n \equiv \frac{\alpha_b}{nv}$ ,  $\zeta_s \equiv \frac{\alpha_s}{v}$ 

> [Gordon: 1929] [Beneke, Binder, Garny, **SL**, De Ros: 2024]

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amplitude

$$|M|^2 \propto \Sigma_m |\langle nlm| r^L |\vec{p}, l' \rangle |^2 \propto I_A \times I_R$$

radial overlap

$$= \frac{2^{4\ell+2}\zeta_n^{2\ell+3}}{(\mu\nu)^{3+2L}\left(1+\zeta_n^2\right)^{2\ell+4}} \frac{\Gamma(\ell'+1)^2\Gamma(n+\ell+1)}{n\Gamma(2\ell+2)^2\Gamma(n-\ell)} S_{\ell'}(\zeta_s) e^{-4\zeta_s\gamma_n}$$
$$\times \left|\frac{1-e^{2i(2(n-\ell)\gamma_n-\gamma_F-\gamma_R)}}{n\kappa\zeta_n\left(\zeta_n^2-1+\frac{2}{\kappa}\right)}\right|^2 |F_+(0)|^2 |R_{\ell'-\ell}^L|^2 ,$$

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$$\begin{split} I_{R} &= \frac{2^{4\ell+2} \zeta_{n}^{2\ell+3}}{(\mu v)^{3+2L} \left(1+\zeta_{n}^{2}\right)^{2\ell+4}} \frac{\Gamma(\ell'+1)^{2} \Gamma(n+\ell+1)}{n \Gamma(2\ell+2)^{2} \Gamma(n-\ell)} \underbrace{S_{\ell'}(\zeta_{s})}_{\text{Sommerfeld factor}} \\ &\times \left| \underbrace{\frac{1-\mathrm{e}^{2\mathrm{i}(2(n-\ell)} \gamma_{n}-\gamma_{F}-\gamma_{R})}{n \kappa \zeta_{n} \left(\zeta_{n}^{2}-1+\frac{2}{\kappa}\right)}}_{\mathrm{sin}^{2}(\textit{phase})} \right|^{2} |F_{+}(0)|^{2} \left| R_{\ell'-\ell}^{L} \right|^{2}, \\ &\qquad \mathrm{a\ rational\ polynomial}}_{\mathrm{a\ single}} \end{split}$$

## radiative Bound State Formation

"Seeing the formula" ≠ "Undestanding the physics"

### Abelian scenario: $\alpha_b = \alpha_s$

<u>Attractive initial state</u>:  $V_{\text{initial}}(r) = V_{\text{final}}(r)$ 



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n=l-1

10 000.

#### <u>Repulsive initial state</u>: $V_{\text{initial}}(r) < 0$



Too low v: particles bounce off each other

10 000.

⇒ Higher n are enhanced ! BSF stronger than in the attractive case !

1/v

1000.

100.

 $\propto (\sigma v)$  [a.u.]

10.

#### <u>Repulsive initial state</u>: $V_{\text{initial}}(r) < 0$





⇒ Higher n are enhanced ! BSF stronger than in the attractive case !

Anomalous **enhancement over the "Abelian" case** of identical potentials.



#### **Classical analogy**: smooth matching of orbits $r_n = b_l$ , $p_n = p(r = b_l)$



The enhancement grows strongly with n !

## Intermezzo: perturbative Unitarity Violation

 $2 \rightarrow 2$  scattering bound from S-matrix unitarity:

$$(\sigma_{2\to 2}v)_{l'} \leq (\sigma v)_{l'}^{uni} \equiv \frac{4\pi (2l'+1)}{m^2 v} \sim \frac{1}{v}$$

## Perturbative Unitarity Violation in BSF

• <u>Proven</u>: for maximal angular momentum *l=n-1* 

$$\max_{v} \left\{ (\sigma v)_{l' \to l=n-1}^{BSF} \right\} \propto \sqrt{n} \times (\sigma v)_{l'}^{\text{uni}}$$

- <u>Proven</u>: When summed in *n*, there will always be UVi below some critical velocity.
- <u>Proven</u>: The UVi depends on the coupling <u>ratio</u>  $\kappa < 1$ .

#### For any small coupling, high n violate unitarity !



## Unitarity violation in dark SU(N)

$$\kappa = \kappa(N_C) = \frac{-1}{N_C^2 - 1}$$

 $\Rightarrow$  for every N<sub>C</sub>, at fixed  $\alpha$ , find the velocity where UVi first occurs:



Intermezzo end.

## Bound states in thermal production

## 1. dark-sector toy models

 $\chi$  = DM: charged under a *new* (dark) symmetry U(1) or SU(N)

## Quasi-steady state approximation

Every bound state = one species in the BME.  $\rightarrow$  huge system of equations!

$$\frac{dY_{\chi}}{dx} = \frac{1}{3Hs} \frac{ds}{dx} \left[ \frac{1}{2} \langle \sigma_{\bar{\chi}\chi}^{annh} v \rangle \left( Y_{\chi}^{2} - Y_{\chi}^{eq2} \right) + \sum_{i} \frac{1}{2} \langle \sigma_{BSF,i} v \rangle \left( Y_{\chi}^{2} - Y_{\chi}^{eq2} \frac{Y_{i}}{Y_{i}^{eq}} \right) \right]$$
  
coupled system
$$\frac{dY_{i}}{dx} = \frac{1}{3Hs} \frac{ds}{dx} \left[ \Gamma_{dec}^{B} \left( Y_{i} - Y_{i}^{eq} \right) - \sum_{j \neq i} \Gamma_{trans}^{j \neq i} \left( Y_{j} - Y_{j}^{eq} \frac{Y_{i}}{Y_{i}^{eq}} \right) + \Gamma_{ion} \left( Y_{i} - Y_{i}^{eq} \frac{Y_{\chi}^{2}}{Y_{\chi}^{eq2}} \right) \right]$$



[Redi et al.: 1702.01141] [Petraki et al.: 2112.00042] [Garny, Heisig: 2112.01499]

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$$0 = \frac{dY_i}{dx} = \frac{1}{3Hs} \frac{ds}{dx} \left[ \Gamma^B_{dec} \left( \mathbf{Y}_i - \mathbf{Y}^{eq}_i \right) - \sum_{j \neq i} \Gamma^{j \rightarrow i}_{trans} \left( \mathbf{Y}_j - \mathbf{Y}^{eq}_j \frac{\mathbf{Y}_i}{\mathbf{Y}^{eq}_i} \right) + \Gamma_{ion} \left( \mathbf{Y}_i - \mathbf{Y}^{eq}_i \frac{\mathbf{Y}^2_{\chi}}{\mathbf{Y}^{eq2}_{\chi}} \right) \right]$$

"quasi steady-state" approximation



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## **Effective cross-section**



", **Critical scaling**" for freeze-out:  $\langle \sigma v \rangle_{ann}^{eff} \propto \frac{1}{T} \propto x$ 

## **Effective cross-section**



## Effective cross-section: dark U(1)



Includes bound-to-bound

#### transitions allowed. transitions. dark U(1) dark SU(3) Dark QED effective cross section, $\alpha = 0.1$ Dark QCD effective cross section, $\alpha(m) = 0.025$ $10^{2}$ $10^{5}$ $10^{4}$ $n \leq 100$ - $n \le 1000$ $10^{3}$ n ≤ 100 n ≤ 10 $10^{1}$ $10^{2}$ --- n < 10 $\cdots n = 1$ $\langle \sigma \, v \rangle_{\rm eff} \times m^2$ $\left<\sigma\,v\right>_{\rm eff}\times m^2$ $\cdots n = 1$ SE only $10^{1}$ - SE only $10^{0}$ $10^{0}$ $10^{-1}$ $10^{-2}$ $10^{-3}$ $10^{-1}$ $10^{-4}$ $10^{\overline{2}}$ $10^{2}$ $10^{3}$ $10^{4}$ $10^{5}$ $10^{3}$ $10^{4}$ $10^{5}$ $10^{6}$ $10^{8}$ $10^{9}$ $10^{6}$ $10^{1}$ $10^{7}$ 10 x = m/Tx = m/T

### **Efficient BSF prevents Chemical Decoupling !**

[Binder, Garny, Heisig, SL, Urban: 2308.01336]

No bound-to-bound

## "Freeze-Out" including BSF – dark SU(3)



## "Freeze-Out" including BSF – dark SU(3)



## Bound states in thermal production

## 2. t-channel superWIMP

 $\chi$  = DM: complete gauge singlet  $\tilde{q}$  = mediator: heavy colored & charged scalar

## Including transitions: "add an additional U(1)"

Assume a <u>colored</u> & <u>charged</u> heavy scalar:  $\tilde{q} \in (\mathbf{3}, \mathbf{1})_{1/3}$  ("b-squark")



 $\rightarrow$  QCD dominates the potential and BSF  $(\widetilde{q}^{\dagger}\widetilde{q})^{[8]} \rightarrow B_{i}^{[1]} + g$ 

$$\kappa \equiv \frac{V_{[8]}}{V_{[1]}} = -\frac{1}{8}$$

 $\rightarrow$  QED allows transitions  $B_i \rightarrow B_j$ 

## Abundance without excited states



## Abundance with an excited state (n=1)



## Abundance with some excited states



## Abundance with many excited states



## Abundance with enough excited states



## Abundance with too many excited states

- Cross-section converged in the pertubative regime.
- Bound-to-bound transitions give strong enhancement:  $x^{1.1} \rightarrow x^{1.6}$



## Eternal annihilation – how does it end?

- Respect unitarity bounds
- Avoid non-perturbative regime  $\alpha \sim 1$  ?!



## superWIMP production with bound states

Adds a finite life-time:  $\chi = DM$ ,  $\tilde{q} = ,t$ -channel mediator"





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### <u>Summary</u>

Importance of **bound states** for dark matter:

1. Repulsive potentials show enhanced BSF.

2. Dominant effect at small temperatures.

3. Non-Abelian excited states can not be neglected.

4. Large effects from transitions between bound states.

5. Bound state formation can source eternal depletion.

6. Unitarity is systematically violated in BSF at leading order.

Using intuition from QED is dangerous & bound states are exciting!