Using gauge/gravity duality for strongly interacting models

Werner Porod

(Uni. Würzburg)

Work with Johanna Erdmenger, Nick Evans, Yang Liu, Kostas Rigatos

PRL **126** (2021), 071602; JHEP **02** (2021), 058; Universe **9** (2023), 289; JHEP **07** (2024), 169

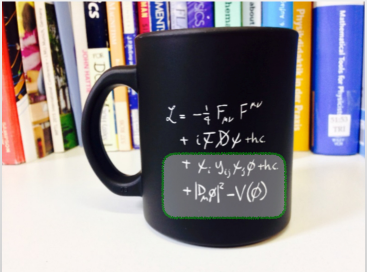
Warsaw, 1 April 2025

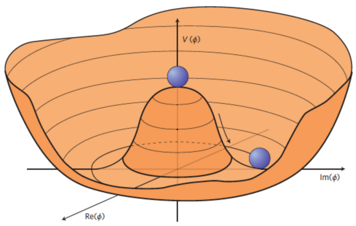
Overview

Jobs of the SM-Higgs Multiplet

$$\phi(x) = \frac{1}{\sqrt{2}} e^{i\tau^a \chi^a(x)/v} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- its non-zero vacuum expectation value v spontaneously breaks the electroweak gauge group $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$
- gives masses to $W^\pm$, Z
- gives masses to the fermions
- bonus: provides one physical scalar h ('the Higgs boson')

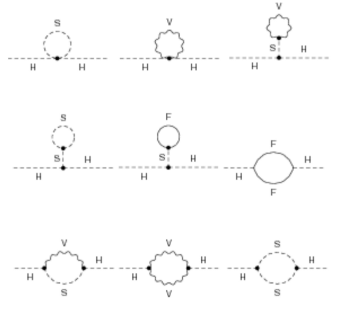




Hierarchy problem

In the absence of new symmetries/dynamics: Higgs condensate and Higgs mass are

unstable to quantum corrections & dragged-up to very large energy scales

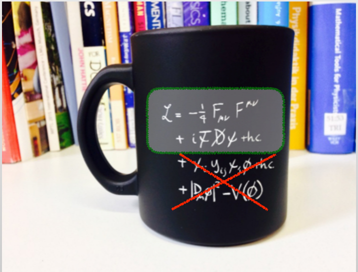


$$\frac{\delta v^2}{v^2} = \sum_i \pm \frac{g_i^2}{16\pi^2} \frac{M_i^2}{v^2} \gg 1$$

M_i: proxy for unknown heavy mass scales (flavour, GUTs, gravity, ...)

What if there were no Higgs?

QCD breaks electroweak symmetry! just wrongly



Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

1^{st} family quarks: q_L, u_R and d_R

Global symmetry: $SU(2)_l \times SU(2)_r$
(of QCD sector)

At QCD scale: condensation

$$\langle \bar{q}_L q_R \rangle = -f_\pi B_0 \simeq (200 \text{ MeV})^3$$

$SU(2)_l \times SU(2)_r \to SU(2) \Rightarrow$ 3 Nambu-Goldstone bosons: $\pi^{0,+,-}$

Problems

- $m_W = m_Z \simeq$ O(100 MeV)
- no Higgs d.o.f. at the scale of $m_{W,Z}$
- $U(1)_{em} = U(1)_Y$
- a priori no masses for quarks and leptons (but could be induced via 4-Fermi operators, (as in Nambu-Jona-Lasinio model (NJL-model))

…but the hierarchy problem would be solved!

Experimentalist

Phenomenologist

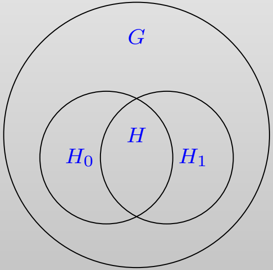
Model-Builder

Formal Theorist

'Minimal Composite Higgs framework'

K. Agashe, R. Contino and A. Pomarol, NPB **719** (2005), 165
R. Contino, TASI lectures 2009

Assumes there is an additional strong force, often called hyper-color, and new 'quarks'



G: $SO(5) \times U(1)_X$, global symmetry of the strong sector above confinement scale

H_1: $SO(4) \times U(1)_X \sim SU(2)_L \times SU(2)_R \times U(1)_X$, global symmetry group in confined phase

H_0: $SU(2)_L \times U(1)_Y$, SM electroweak gauge group

H: $U(1)_{em}$, unbroken gauge group

- $SO(5) \to SO(4)$ breaking $\Rightarrow$ 4 Nambu-Goldstone bosons in $(2,2)$ of $SU(2)_L \times SU(2)_R$
- $Y = T^{3R} + X$, $U(1)_X$ needed to get correctly the hypercharges of the fermions

'Minimal Composite Higgs framework'

Fermion masses and couplings: partial compositeness

Higgs transforms non-linearly under G.
$\Rightarrow$ no Yukawa interaction if fermion are elementary (transform linearly).
Possible solution: mix elementary fermions with composite resonances.

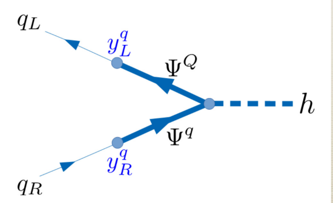
Elementary fermions (in $SO(5)$) rep.)

$$q_L = \frac{1}{\sqrt{2}}(\mathrm{i}\,d_L, d_L, \mathrm{i}\,u_L, -u_L, 0)^T$$

$$q_R = (0,0,0,0,u_R)^T$$

Composite fermions (in $SO(5)$) rep.)

$$\Psi = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{i}B - \mathrm{i}X_{5/3} \\ B + X_{5/3} \\ \mathrm{i}T + \mathrm{i}X_{2/3} \\ -T + X_{2/3} \\ \sqrt{2}\tilde{T} \end{pmatrix}$$



Generic Composite Higgs set-up

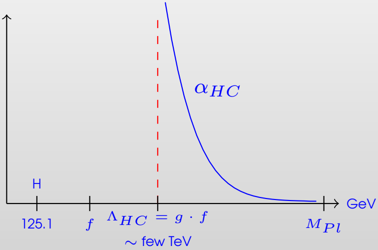
Possible solution to hierarchy problem

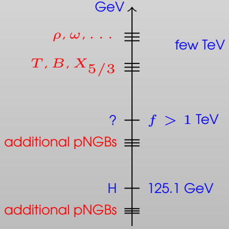
- Generate a scale $\Lambda_{HC} \ll M_{pl}$ through a new confining gauge group
- Interpret Higgs as a pseudo-Nambu-Goldstone boson (pNGB) of a spontaneously broken global symmetry of the new strong sector, G/H

(Georgi, Kaplan, PLB **136** (1984), 136)

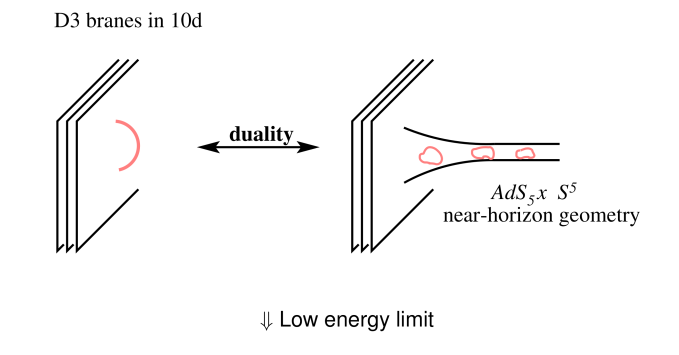
'Price' to pay

- additional resonances at the scale Λ_{HC} (vectors, vector-like fermions, scalars)
- additional light pNGBs/ extended scalar sector
- deviations of the Higgs couplings from their SM values of O(v/f)





String theory origin of the gauge/gravity duality

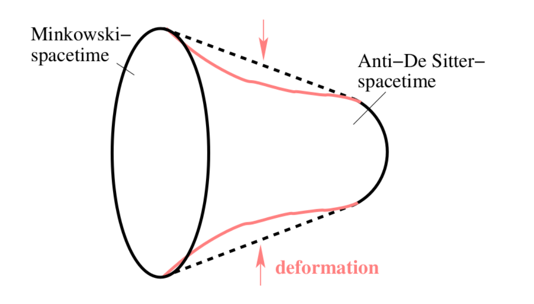


$\mathcal{N} = 4$ Supersymmetric $SU(N)$ gauge theory in four dimensions ($N \to \infty$ limit) $\Leftrightarrow$ Supergravity on the space $\mathrm{AdS}_5 \times S^5$

J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998), 231

Breaking of conformal symmetry and supersymmetry

Deformation of AdS_5



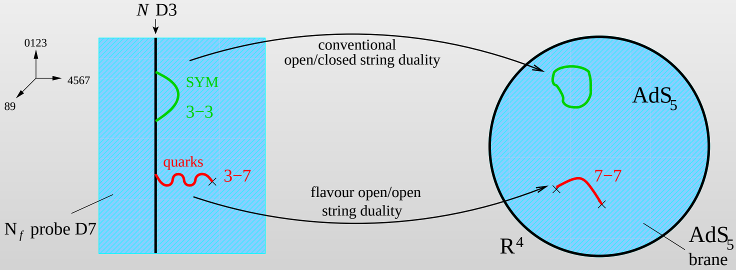
5^{th} dimension $\Leftrightarrow$ energy scale

Deformation of S^5: breaking of supersymmetry

$\Rightarrow$ Generalized AdS/CFT Correspondence: Gauge/Gravity Duality

see e.g. J. Erdmenger et al, Eur. Phys. J. A **35** (2008), 81 for a review

Quarks in the gauge/gravity duality



(from J. Erdmenger et al, Eur. Phys. J. A **35** (2008), 81)

$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

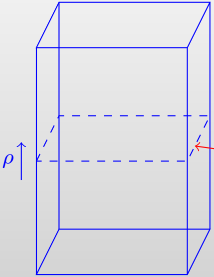
duality acts twice

$\mathcal{N}=4$ $SU(N)$ Super Yang-Mills theory coupled to $\mathcal{N}=2$ fundamental hypermultiplet $\Leftrightarrow$ IIB supergravity on AdS$_5 \times S^5$ + Probe brane DBI on AdS$_5 \times S^3$

A. Karch and E. Katz, JHEP **06** (2002), 043

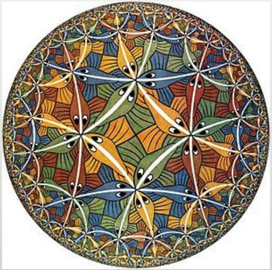
(DBI: Dirac-Born-Infeld)

How does gauge/gravity duality work?



$$d^2 s = \frac{d^2\rho}{\rho^2} + \rho^2 d^2 x_{3+1}$$

3+1d slice parallel to D3 brane on which field theory lives



dilation ⇔ running of gauge couplings

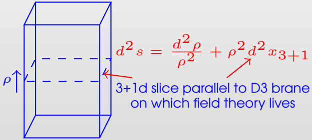
$$\int d^4 x \partial_\mu \phi \partial^\mu \phi \,, \quad x \to e^{-\alpha} x \quad \phi \to e^{\alpha} \phi$$

dilaton become spacetime symmetry of AdS

$$\rho \to e^{\alpha} \rho$$

ρ is a continuos mass dimension → RG scale

How does gauge/gravity duality work?



$$\sqrt{-\det g} = \left|\begin{pmatrix} -\rho^2 & 0 & 0 & 0 & 0 \\ 0 & \rho^2 & 0 & 0 & 0 \\ 0 & 0 & \rho^2 & 0 & 0 \\ 0 & 0 & 0 & \rho^2 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\rho^2} \end{pmatrix}\right|^{1/2} = \rho^3$$

Field theory side

Operators and sources appear as fields in the bulk, e.g.

$$\int d^4x \, m_f \, \bar\psi\psi$$

m is the quark mass and c the condensate

$$c = \langle \bar\psi\psi \rangle$$

AdS side

A field for the mass/condensate

$$\int d^4x \int d\rho \frac{1}{2} \rho^3 (\partial_\rho L)^2$$

$$\Rightarrow \quad \partial_\rho \left(\rho^3 \partial_\rho L\right) = 0$$

$$\Rightarrow L = m_f + \frac{c}{\rho^2}$$

Running Dimensions in Holography

Holographically we can change the dimension of our operator by adding a mass term

$$\partial_\rho\left(\rho^3\partial_\rho L\right) - \rho\,\Delta m^2 L = 0 \quad , \quad \gamma(\gamma-2) = \Delta m^2$$
$$\Rightarrow L = \frac{m_f}{\rho^\gamma} + \frac{c}{\rho^{2-\gamma}}$$

$\Delta m^2 = -1$ corresponds to $\gamma = 1$ and is special – the Breitenlohner Freedman bound instability ...

So we can include a running coupling by a running mass squared for the scalar.

Top down derivation:

- several string constructions e.g. probe D7 branes in D3 backgrounds
- neglect back reaction of probe branes on metric '$\Leftrightarrow$' quenched approximation on lattice

R. Alvares, N. Evans, K.-Young arXiv:1204.2474 (hep-ph); M. Jarvinen, E. Kiritsis arXiv:1112.1261 (hep-ph)

Dynamic AdS/YM

Action on gravity side:

$$S = \int d^4x \int d\rho\, \rho^3 \,\text{Tr}\left[\frac{1}{\rho^2 + |X|^2}|DX| + \frac{\Delta m^2}{\rho^2}|X|^2\right]$$

$$X = L(\rho)\mathrm{e}^{2\mathrm{i}\xi^a(x)T^a}\,, \qquad d^2s = \frac{d^2\rho}{\rho^2 + |X|^2} + (\rho^2 + |X|^2)d^2x$$

$\xi^a = \pi^a/f_\pi$

We use the top-down IR boundary condition on mass-shell: $L'(\rho = L) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate - no hard wall.
gauge/gravity duality: $X \Leftrightarrow \bar{q}q$

The gauge dynamics is input through a guess for $\Delta m = \gamma(\gamma - 2)$ with

$$\gamma = \frac{3(N_c^2 - 1)}{4\pi N_c}\alpha$$

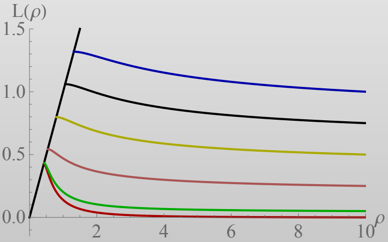
in case of $SU(N_c)$. The only free parameters are N_c, N_f, m, Λ_{UV}

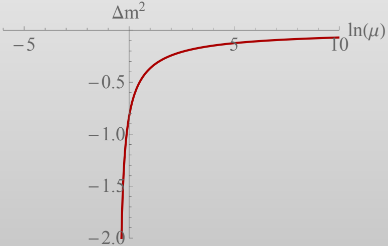
T. Alho, N. Evans, K. Tuominen arXiv:1307.4896 (hep-ph)

Formation of the Chiral Condensate

We solve for the vacuum configuration of $L_0 = |X|$

$$\partial_\rho \left(\rho^3 \partial_\rho L_0(\rho)\right) - \rho\, \Delta m^2(\rho) L_0(\rho) = 0\,, \quad L_0'(\rho = L_0) = 0 \quad \text{and} \quad L_0(\rho) = \rho$$





$N_c = 3, N_f = 2, \mu = \sqrt{\rho^2 + L_0^2}$

quark masses m_f from bottom to top: 0, 0.05, 0.25, 0.5, 0.75, 1

Meson Fluctuations

$$S = \int d^4x \int d\rho\, \rho^3 \mathrm{Tr}\left[\frac{1}{\rho^2 + |X|^2}|DX|^2 + \frac{\Delta m^2}{\rho^2}|X|^2\right] + \frac{1}{2g_5^2}(F_V^2 + F_A^2)$$

$$X = L_0 + \delta(\rho)e^{ikx}\,, \quad k^2 = -M^2$$

$$\partial_\rho(\rho^3\partial_\rho\delta) - \Delta m^2\,\rho\,\delta - \rho\,L_0\,\delta\frac{\partial\Delta m^2}{\partial L}\bigg|_{L_0} + M^2\frac{\rho^3}{(L_0^2+\rho^2)^2}\delta = 0$$

with $\delta(\rho_{IR}) = 1$ and δ being normalizable

The source free solutions δ pick out particular mass states, e.g. in QCD f_0 and its radial excited states

The gauge fields let us also study the operators and states

$$V_\mu \Leftrightarrow \bar{u}\gamma_\mu u \to \rho \text{ meson}\,, \quad A_\mu \Leftrightarrow \bar{u}\gamma_\mu\gamma_5 u \to a \text{ meson}$$

Non-abelian set-up

Basic idea: mass differences correspond to D7 brane separation
⇒ starting point non-abelian DBI action

$$S_{N_f} = -\tau_p \int d^{p+1}\xi e^{-\phi} \, \mathrm{STr} \left(\sqrt{-\det(P[G_{rs} + G_{ra}(Q^{-1} - \delta)^{ab} G_{sb}] + T^{-1} F_{rs})} \sqrt{\det Q^a{}_b} \right)$$

$$Q^a{}_b = \delta^a{}_b + \mathrm{i}\, T\, [X^a, X^c] G_{cb}$$

$$\mathrm{STr}(A_1 \ldots A_n) \equiv \frac{1}{n!} \sum \mathrm{Tr}\,(A_1 \ldots A_n + \text{ all permutations}) \,,$$

$G_{ab}(r^2)$ have matrix structure, e.g., for the case of diagonal real masses in SU(2)

$$G_{\rho\rho} = \frac{1}{r^2} \quad \text{with} \quad r^2 = \begin{pmatrix} r_u^2 & 0 \\ 0 & r_d^2 \end{pmatrix} .$$

⇒ coupled Sturm-Liouville problem

† J. Erdmenger, N. Evans, Y. Liu and W.P., Universe **9** (2023) no.6, 289; JHEP **07** (2024), 169

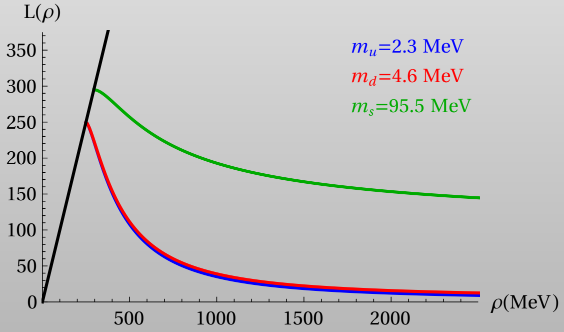
QCD, $N_f = 3$, Embeddings

$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2 \qquad b_0 = \frac{1}{6\pi}\left(11 N_c - (N_f + \bar{N}_f)\right) , \qquad \gamma = \frac{3({N_c}^2 - 1)}{4 N_c \pi}\alpha \, .$$

Two-loop contribution included as well

$$G_{\rho\rho} = \frac{1}{r^2} \quad \text{with} \quad r^2 = \begin{pmatrix} r_u^2 & 0 & 0 \\ 0 & r_d^2 & 0 \\ 0 & 0 & r_s^2 \end{pmatrix} \quad \text{and} \quad r_i^2 = \rho^2 + L_i^2$$

⇒ eqs. for L_i ($i = u, d, s$) decouple:



QCD, $N_f = 3$, masses

mass	QCD (MeV)	$N_f = 3$ –Split Masses (MeV)	Deviation
$M_{\rho(770)}$	775	775*	fitted
$M_{K^*(892)}$	892	888	-0.5%
$M_{\phi(1020)}$	1019	940	-7.8%
$M_{p,n}$	939	1079	+14.9%
$M_{\Lambda,\Sigma}$	1174	1257	+7.1%
M_{Ξ}	1318	1435	+8.9%
M_{π}	139	254	+83%
M_K	496	544	+9.7%
M_{η}	548	570	4.0%

quark masses: $m_u = m_d = 3.5$ MeV and $m_s = 93.5$ MeV

Example $SU(4)/Sp(4)$, I

Example: $Sp(2N_c)$ gauge group with 2 Dirac fermions in the fundamental representation
The pseudo-reality means the flavour symmetry is U(4) on the 4 Weyl fermions
(we neglect the anomaly)

The vacuum condensate is anti-symmetric in spin (2×2=1A+3S), anti-symmetric in colour, so anti-symmetric in flavour . . . possible vacua

$$L = \begin{pmatrix} 0 & L_0(\rho) & 0 & 0 \\ -L_0(\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & L_0(\rho) \\ 0 & 0 & -L_0(\rho) & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 0 & 0 & -Q(\rho) \\ 0 & 0 & Q(\rho) & 0 \\ 0 & -Q(\rho) & 0 & 0 \\ Q(\rho) & 0 & 0 & 0 \end{pmatrix}$$

$$X' = U^T X U$$

$$\Rightarrow L' = \begin{pmatrix} 0 & L_0 + Q & 0 & 0 \\ -L_0 - Q & 0 & 0 & 0 \\ 0 & 0 & 0 & L_0 - Q \\ 0 & 0 & -L_0 + Q & 0 \end{pmatrix} = \begin{pmatrix} 0 & L_p & 0 & 0 \\ -L_p & 0 & 0 & 0 \\ 0 & 0 & 0 & L_m \\ 0 & 0 & -L_m & 0 \end{pmatrix}$$

Both break $U(4) \to Sp(4)$ . . . The first is $SU(2)_L$ invariant, the second breaks $SU(2)_L$

Example SU(4)/Sp(4), II

Sp(4) vacuum & $SU(2)_L$ preserving fermion masses ($Sp(4)$ preserving if $m_1 = m_2$)

$$L = \begin{pmatrix} 0 & L_0(\rho) & 0 & 0 \\ -L_0(\rho) & 0 & 0 & 0 \\ 0 & 0 & 0 & L_0(\rho) \\ 0 & 0 & -L_0(\rho) & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & m_1 & 0 & 0 \\ -m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & -m_2 & 0 \end{pmatrix}$$

There are 2×6 real meson fields in here corresponding to the A_2 of $SU(4)$ which we parameterize as

$$X_f = \begin{pmatrix} 0 & \sigma - Q_5 + iS - i\pi_5 & Q_2 - \pi_2 + i\pi_1 - iQ_1 & -Q_4 + \pi_4 + iQ_3 - i\pi_3 \\ -\sigma + Q_5 + i\pi_5 - iS & 0 & Q_4 + \pi_4 + iQ_3 + i\pi_3 & Q_2 + \pi_2 + iQ_1 + i\pi_1 \\ \pi_2 - Q_2 + iQ_1 - i\pi_1 & -Q_4 - \pi_4 - iQ_3 - i\pi_3 & 0 & \sigma + Q_5 + iS + i\pi_5 \\ Q_4 - \pi_4 + i\pi_3 - iQ_3 & -Q_2 - \pi_2 - iQ_1 - i\pi_1 & -\sigma - Q_5 - iS - i\pi_5 & 0 \end{pmatrix}.$$

6 pNGBs:

- π_{1-4} transform as $(2,2)$ under $SU(2)_L \times SU(2)_R$, gets eventually massive due to $m_{1,2}$, ew. loops and top mass
- π_5 potential DM candidate, see Cacciapaglia/Sannino, and S (anomaly)

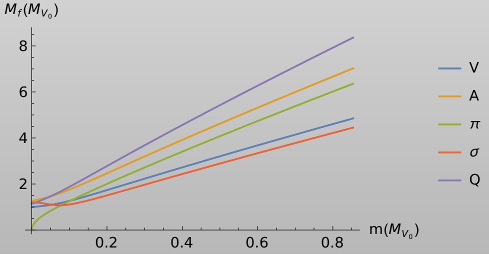
Example SU(4)/Sp(4), III

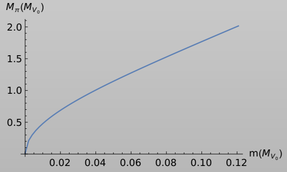
$m_1 = m_2 = 0$

Observables	$SU(2)^{(a)}$	Lattice $SU(2)$	$Sp(4)$	Lattice $Sp(4)^{(b)}$	$Sp(6)$	$Sp(8)$	$Sp(10)$
m_V (10)	1^*	1.00(3)	1^*	1.00(33)	1^*	1^*	1^*
m_A (6)	1.66	1.11(46)	1.26	1.61(17)	1.18	1.14	1.12
m_σ (1)	1.26	1.5(1.1)	1.20		1.22	1.23	1.23
m_Q (5)	1.13		1.13		1.13	1.13	1.13
$m_{\pi,S}$ (6)	0.02		0.01		0.01	0.01	0.01
F_V	0.38		0.53	0.52(10)	0.59	0.64	0.67
F_A	0.48		0.54	0.673(92)	0.59	0.63	0.66
f_π	0.06		0.10	0.122 (99)	0.12	0.12	0.13

(a) R. Arthur et al, arXiv:1602.06559, arXiv:1607.06654
(b) Ed Bennet et al, arXiv:1909.12662

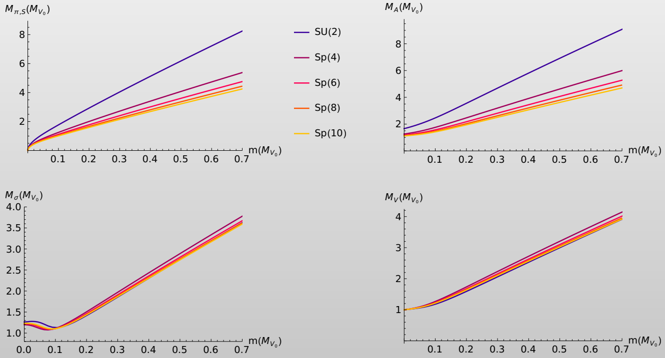
Sp(4) gauge group, $m_1 = m_2 = m$





Example SU(4)/Sp(4), IV

Spectrum dependence on gauge group and fermion mass



In addition one can study

- effects of mass splitting $m_1 \neq m_2$
- Composite Higgs to Techni-color transition using higher dimensional operators
- add other representations to get top-partners

List of "minimal" CHM UV embeddings

$G_{\rm HC}$	ψ	χ	Restrictions	$-q_\chi/q_\psi$	Y_χ	Non Conformal	Model Name
	Real	Real	SU(5)/SO(5) × SU(6)/SO(6)				
$SO(N_{\rm HC})$	$5 \times \mathbf{S}_2$	$6 \times \mathbf{F}$	$N_{\rm HC} \geq 55$	$\frac{5(N_{\rm HC}+2)}{6}$	1/3	/	
$SO(N_{\rm HC})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$N_{\rm HC} \geq 15$	$\frac{5(N_{\rm HC}-2)}{6}$	1/3	/	
$SO(N_{\rm HC})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\rm HC} = 7, 9$	$\frac{5}{6}, \frac{5}{12}$	1/3	$N_{\rm HC} = 7, 9$	M1, M2
$SO(N_{\rm HC})$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\rm HC} = 7, 9$	$\frac{5}{6}, \frac{5}{3}$	2/3	$N_{\rm HC} = 7, 9$	M3, M4
	Real	Pseudo-Real	SU(5)/SO(5) × SU(6)/Sp(6)				
$Sp(2N_{\rm HC})$	$5 \times \mathbf{Ad}$	$6 \times \mathbf{F}$	$2N_{\rm HC} \geq 12$	$\frac{5(N_{\rm HC}+1)}{3}$	1/3	/	
$Sp(2N_{\rm HC})$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$2N_{\rm HC} \geq 4$	$\frac{5(N_{\rm HC}-1)}{3}$	1/3	$2N_{\rm HC} = 4$	M5
$SO(N_{\rm HC})$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$N_{\rm HC} = 11, 13$	$\frac{5}{24}, \frac{5}{48}$	1/3	/	
	Real	Complex	SU(5)/SO(5) × SU(3)^2/SU(3)				
$SU(N_{\rm HC})$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$N_{\rm HC} = 4$	$\frac{5}{3}$	1/3	$N_{\rm HC} = 4$	M6
$SO(N_{\rm HC})$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$N_{\rm HC} = 10, 14$	$\frac{5}{12}, \frac{5}{48}$	1/3	$N_{\rm HC} = 10$	M7
	Pseudo-Real	Real	SU(4)/Sp(4) × SU(6)/SO(6)				
$Sp(2N_{\rm HC})$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$2N_{\rm HC} \leq 36$	$\frac{1}{3(N_{\rm HC}-1)}$	2/3	$2N_{\rm HC} = 4$	M8
$SO(N_{\rm HC})$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$N_{\rm HC} = 11, 13$	$\frac{8}{3}, \frac{16}{3}$	2/3	$N_{\rm HC} = 11$	M9
	Complex	Real	SU(4)^2/SU(4) × SU(6)/SO(6)				
$SO(N_{\rm HC})$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$N_{\rm HC} = 10$	$\frac{8}{3}$	2/3	$N_{\rm HC} = 10$	M10
$SU(N_{\rm HC})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$N_{\rm HC} = 4$	$\frac{2}{3}$	2/3	$N_{\rm HC} = 4$	M11
	Complex	Complex	SU(4)^2/SU(4) × SU(3)^2/SU(3)				
$SU(N_{\rm HC})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}}_2)$	$N_{\rm HC} \geq 5$	$\frac{4}{3(N_{\rm HC}-2)}$	2/3	$N_{\rm HC} = 5$	M12
$SU(N_{\rm HC})$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{S}_2, \overline{\mathbf{S}}_2)$	$N_{\rm HC} \geq 5$	$\frac{4}{3(N_{\rm HC}+2)}$	2/3	/	

G. Ferretti, JHEP **06** (2016), 107; A. Belyaev et al. JHEP **01** (2017), 094

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

- The sextet quarks A_2 are expected to condense first and break $SU(6) \to SO(6)$. Their main job: form FA_2F baryon top partners.
- Then the fundamentals F break $SU(4) \to Sp(4)$ – this is where the Higgs is generated as just discussed.

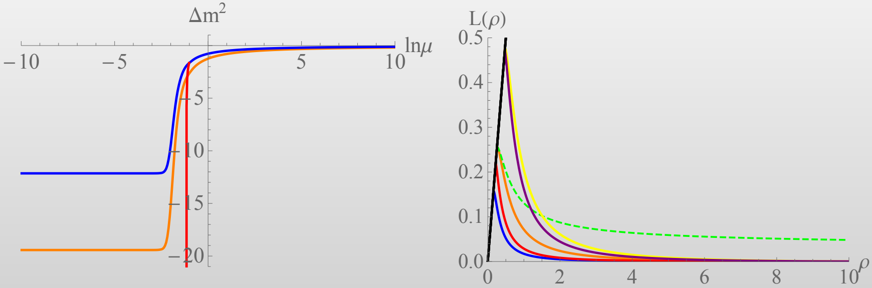
$$b_0 = \frac{1}{6\pi}\left(11(N+1) - N_{f_1} - 2(N-1)N_{f_2}\right)$$
$$\gamma_{A_2} = \frac{3}{2\pi} N\alpha\,,$$
$$\gamma_F = \frac{3}{2\pi}\frac{2N+1}{4}\alpha\,,$$

with $N = 4$, $n_{f_2} = 4$ and $N_{f_2} = 6$ (two-loop contributions included as well)

These fix Δm^2 and hence the model

Quenching: set $N_{f_1} = N_{f_2} = 0$ in b_i

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$



- blue line: F
- orange line: A_2
- red line: F but A_2 integrated out when it condensates
- dashed green: F + additional HDO-terms such that it matches in the IR the A_2 representation.
- yellow line: quenched models for the A_2
- purple line: quenched models for the F

How you decouple the quarks is important and unknown

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

	AdS/$Sp(4)$ no decouple	AdS/$Sp(4)$ A2 decouple	AdS/$Sp(4)$ quench	lattice[a] quench	lattice[b] unquench
$f_{\pi A_2}$	0.120	0.120	0.103	0.1453(12)	
$f_{\pi F}$	0.0569	0.0701	0.0756	0.1079(52)	0.1018(83)
$M_{V A_2}$	1*	1*	1*	1.000(32)	
$f_{V A_2}$	0.517	0.517	0.518	0.508(18)	
$M_{V F}$	0.61	0.814	0.962	0.83(19)	0.83(27)
$f_{V F}$	0.271	0.364	0.428	0.411(58)	0.430(86)
$M_{A A_2}$	1.35	1.35	1.28	1.75 (13)	
$f_{A A_2}$	0.520	0.520	0.524	0.794(70)	
$M_{A F}$	0.938	1.19	1.36	1.32(18)	1.34(14)
$f_{A F}$	0.303	0.399	0.462	0.54(11)	0.559(76)
$M_{S A_2}$	0.375	0.375	1.14	1.65(15)	
$M_{S F}$	0.325	0.902	1.25	1.52 (11)	1.40(19)
$M_{B A_2}$	1.85	1.85	1.86		
$M_{B F}$	1.13	1.53	1.79		

[a] Ed Bennett et al., PRD **101** (2020), 074516; [b] Ed Bennett et al., JHEP **12** (2019), 053

- We set the scale in the A_2 sector
  the pattern of mass scales is right
  A, S meson sectors are a little light
- The F sector is lighter than the A_2s and then also in the right pattern

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

	AdS/$Sp(4)$ no decouple	AdS/$Sp(4)$ A2 decouple	AdS/$Sp(4)$ quench	lattice[a] quench	lattice[b] unquench
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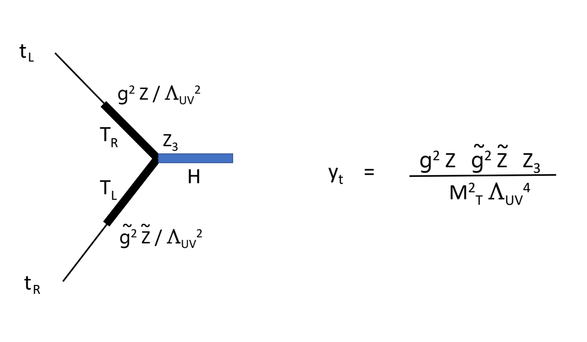
[a] Ed Bennett et al., PRD **101** (2020), 074516; [b] Ed Bennett et al., JHEP **12** (2019), 053

- holographic modeling useful to study effect of changes in the running
- the gap between the F and A_2 sector grows (in particular if A_2 not decoupled)
- the slower the running the lighter the scalar mass becomes – is the biggest change
- prediction for baryons: between FFF and $A_2A_2A_2$ values

Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

Top Yukawa coupling:



Plausible forms for the Z factors up to O(1) couplings
(this is beyond quadratic order in the holographic model)

$$Z_3 \simeq \int d\rho\, \rho^3 \frac{\partial_\rho \pi(\rho)\, \psi_B(\rho)^2}{(\rho^2 + L^2)^2}\,, \qquad Z = \tilde{Z} \simeq \int d\rho\, \rho^3 \partial_\rho \psi_B(\rho)$$

$\Rightarrow Y_t \simeq 0.01 - 0.1$ naturally if $\Lambda_{UV} \simeq$ few TeV

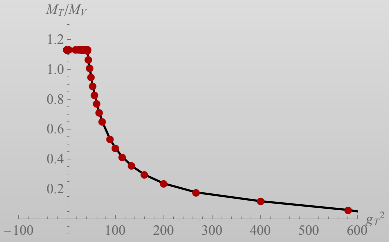
Confirmed on the lattice

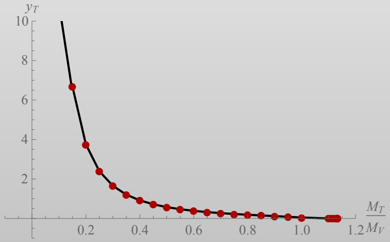
Gauge group $Sp(4)$, 4F, $6A_2$, global: $SU(4) \times SU(6)/Sp(4) \times SO(6)$

Important: We can lower the top partner mass using a HDO

$$\mathcal{L}_{HDO} = \frac{g_T^2}{\Lambda_{UV}^5} |F A_2 F|^2 \,.$$

which raises Y_t:





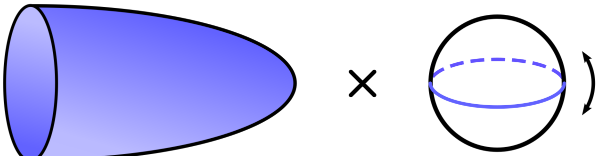
This is a new mechanism to generate the large top mass in these models – drives the top partner baryon mass to half the vector meson mass

- We have holographic models that describe chiral symmetry breaking due to the running of γ
- QCD: good agreement between data and calculations
- non-Abelian DBI to get mass splitting between different representations for mesons and baryons
- Composite Higgs: compare to lattice results and look for changes as we unquench, and extra flavours beyond the lattice
- We have proposed a new HDO method to raise the top Yukawa coupling in these models
- What next
  - get an understanding of the uncertainties
  - proper treatment of 2-scale condensation
  - inclusion of two-index representations in brane picture

Quarks in the gauge/gravity duality

Add D7-Branes (eight-dimensional surfaces) to ten-dimensional space

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		



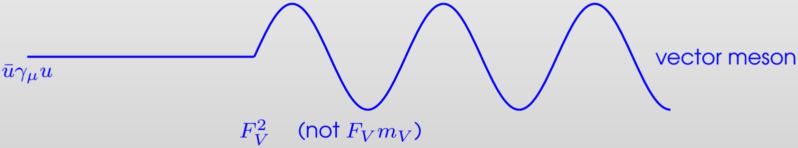
(thanks to J. Erdmenger)

Quarks: Low energy limit of open strings between D3- and D7-Branes

Decay Constants

a la AdS/QCd, see J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, PRL **95** (2005), 261602

Decay constants are determined by allowing a source to couple to a physical state



Now we need to fix the normalizations of the holographic linear perturbations ...

For the physical states we canonically normalize the kinetic terms...

For the source solutions we fix κ and the norms so that we match perturbative results for e.g. Π_{VV} in the UV

$$N_V^2 = N_A^2 = \frac{g_5^2 d(R)\, N_f(R)}{48\pi^2}$$

Baryons

In D3/D7 system some quark-gaugino-quark tri-fermion states are described by world volume fermions on the D7 – it does not seem unreasonable to include three quark states in this way therefore

$$S_{1/2} = \int d^4x \int \rho\, \rho^3 \bar{\Psi}(\not{D}_{AdS} - m)\Psi$$

The four component fermion satisfies the second order equation

$$\left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{M_B^2}{r^4} + \mathcal{P}_2 \frac{1}{r^4} - \frac{m^2}{r^2} - \mathcal{P}_3 \frac{m}{r^3}\, \gamma^\rho\right) \psi = 0\,,$$

where M_B is the baryon mass and

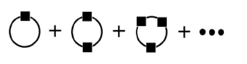
$$\mathcal{P}_1 = \frac{6}{r^2}\left(\rho + L_0\, \partial_\rho L_0\right)\,,$$
$$\mathcal{P}_2 = 2\left((\rho^2 + L_0^2)L\partial_\rho^2 L_0 + (\rho^2 + 3L_0^2)(\partial_\rho L_0)^2 + 4\rho L_0 \partial_\rho L_0 + 3\rho^2 + L_0^2\right)\,,$$
$$\mathcal{P}_3 = \left(\rho + L_0\, \partial_\rho L_0\right)\,.$$

G. F. de Teramond and S. J. Brodsky, PRL **94** (2005), 201601; R. Abt, J. Erdmenger, N. Evans and K. S. Rigatos, JHEP **11** (2019), 160

Higher Dimension/Nambu Jona-Lasinio Operators

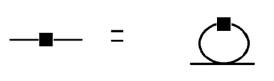
$$\mathcal{L} = \bar{\psi}_L \partial\!\!\!/ \psi_L + \bar{\psi}_R \partial\!\!\!/ \psi_R + \frac{g^2}{\Lambda_{UV}} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L$$

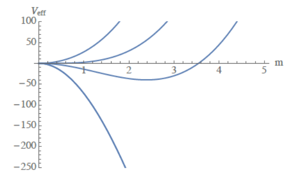
Calculate effective potential

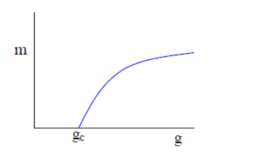


$$\Delta V_{eff} = -\int_0^{\{} \Lambda_{UV} \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\log(k^2 + m^2)$$

$$\frac{g^2}{\Lambda_{UV}} \bar{\psi}_L \psi_R \bar{\psi}_R \psi_L \to \frac{g^2}{\Lambda_{UV}} \langle \bar{\psi}_L \psi_R \rangle \bar{\psi}_R \psi_L \to \frac{m^2 \Lambda_{UV}}{g^2}$$







Witten's Multi-Trace Operator Prescription

E. Witten hep-th/0112258; N. Evans + K. Kim arXiv:1601.02824 (hep-th)

$$\frac{g^2}{\Lambda_{UV}} \bar\psi_L \psi_R \bar\psi_R \psi_L \to \frac{g^2}{\Lambda_{UV}} \langle \bar\psi_L \psi_R \rangle \bar\psi_R \psi_L \to \frac{m^2 \Lambda_{UV}}{g^2}$$

so add

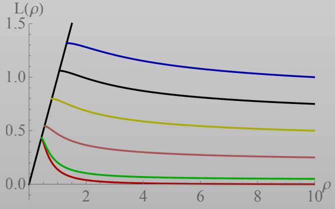
$$S = \int \mathcal{L} + \frac{L^2 \rho^2}{g^2}\Big|_{\Lambda_{UV}}$$

On variation

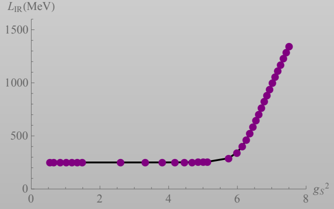
$$0 = \text{E.-L. eqn} + \frac{\partial \mathcal{L}}{\partial L'} \delta L \Big|_{\Lambda_{IR,UV}} + \frac{2 L \rho^2}{g^2} \delta L \Big|_{\Lambda_{UV}}$$

The Euler Lagrange equation solutions are left unchanged but we pick those that satisfy the UV and IR boundary conditions. Now we let the mass vary in the UV and need

$$m = \frac{g^2}{\Lambda_{UV}} c$$



Read off m, c and compute g



$SU(4), 3F, 3\,\bar{F}, 5A_2$; $SU(5)\times SU(3)_L \times SU(3)_R/SO(5)\times SU(3)$

(G. Ferretti, JHEP **06** (2014), 142)

Here the A_2 symmetry breaking generates the SM Higgs; FA_2F top partners

	Lattice[a] $4A_2, 2F, 2\bar{F}$ unquench	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ no decouple	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ no decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
M_{VA_2}	1.00(4)	1*	1*	1*	1*	1*
f_{VA_2}	0.68(5)	0.489	0.489	0.516	0.516	0.517
M_{VF}	0.93(7)	0.933	0.939	0.890	0.904	0.976
f_{VF}	0.49(7)	0.458	0.461	0.437	0.491	0.479
M_{AA_2}		1.37	1.37	1.32	1.32	1.28
f_{AA_2}		0.505	0.505	0.521	0.521	0.522
M_{AF}		1.37	1.37	1.21	1.23	1.28
f_{AF}		0.501	0.504	0.453	0.509	0.492
M_{SA_2}		0.873	0.873	0.684	0.684	1.18
M_{SF}		1.03	1.02	0.811	0.798	1.25
M_{JA_2}	3.9(3)	2.21	2.21	2.21	2.21	2.22
M_{JF}	2.0(2)	2.07	2.08	1.97	2.00	2.17
M_{BA_2}	1.4(1)	1.85	1.85	1.85	1.85	1.86
M_{BF}	1.4(1)	1.74	1.75	1.65	1.68	1.81

[a] V. Ayyar et al., PRD **97** (2018), 074505: (unquenched) $SU(4)\ 2F, 2\bar{F}, 4A_2$

- pattern agree quite well in particular M_{VF} and M_{JF}
- M_{JA_2} off and also M_{BF} on the lattice below our estimate

$SU(4), 3F, 3\bar{F}, 5A_2;\ SU(5) \times SU(3)_L \times SU(3)_R/SO(5) \times SU(3)$

	Lattice[a] $4A_2, 2F, 2\bar{F}$ unquench	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ no decouple	AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ no decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)	0.0997	0.0997	0.111	0.111	0.102
$f_{\pi F}$	0.11(2)	0.0949	0.0953	0.0844	0.109	0.892
$M_{V A_2}$	1.00(4)	1*	1*	1*	1*	1*
$f_{V A_2}$	0.68(5)	0.489	0.489	0.516	0.516	0.517
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$M_{J F}$	2.0(2)	2.07	2.08	1.97	2.00	2.17
$M_{B A_2}$	1.4(1)	1.85	1.85	1.85	1.85	1.86
$M_{B F}$	1.4(1)	1.74	1.75	1.65	1.68	1.81

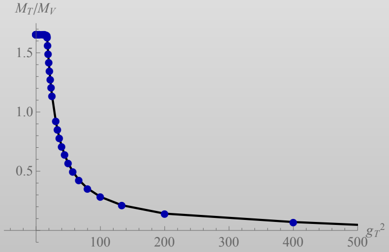
- Adding extra flavours is not a huge change
- Scalar masses get lighter by adding extra flavours

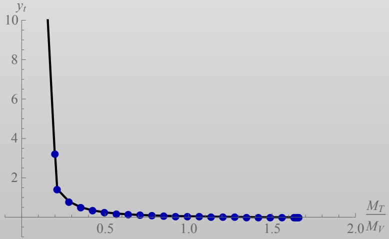
$SU(4), 3F, 3\ \bar{F}, 5A_2; SU(5) \times SU(3)_L \times SU(3)_R/SO(5) \times SU(3)$

Top Yukawa coupling: similar as before, need additional HDO

$$\mathcal{L}_{HDO} = \frac{g_T^2}{\Lambda_{UV}^5} |F A_2 F|^2 \,.$$

which raises Y_t:





This is a new mechanism to generate the large top mass in these models – we drive the top partner baryon mass to about $1/3$ the vector meson mass