[arXiv:2502.02291]

# Primordial black hole formation via inverted bubble collapse


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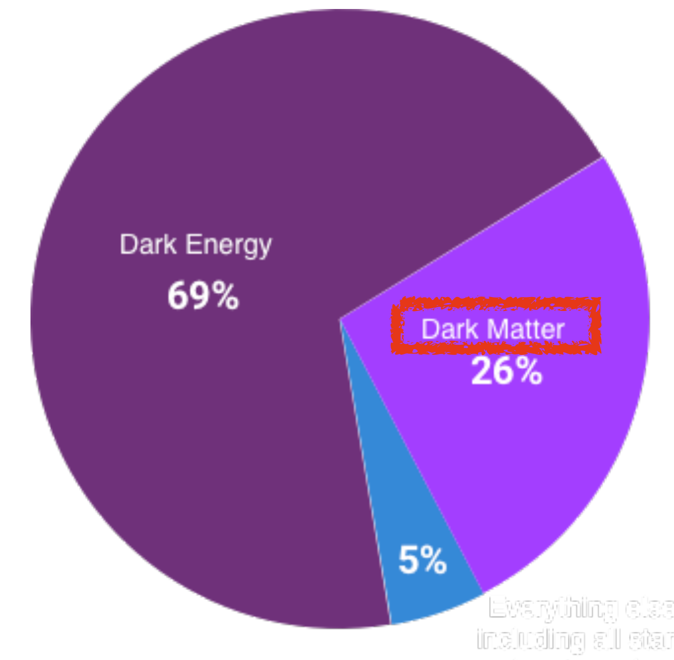
# Contents

# Dark matter

- A lot of observational evidence of dark matter (DM):

  Galaxy rotation curves, Gravitational lensing,
  Cosmic Microwave Background (CMB) anisotropies, etc.



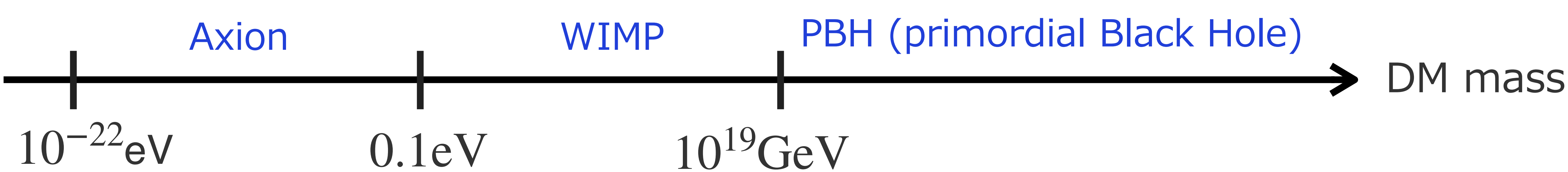

[https://chandra.harvard.edu]

- However, we do not identify what DM is.

- The nature of DM that we know:

  Electrically neutral, weak interactions, cold and attractive.

- Mass range of DM:

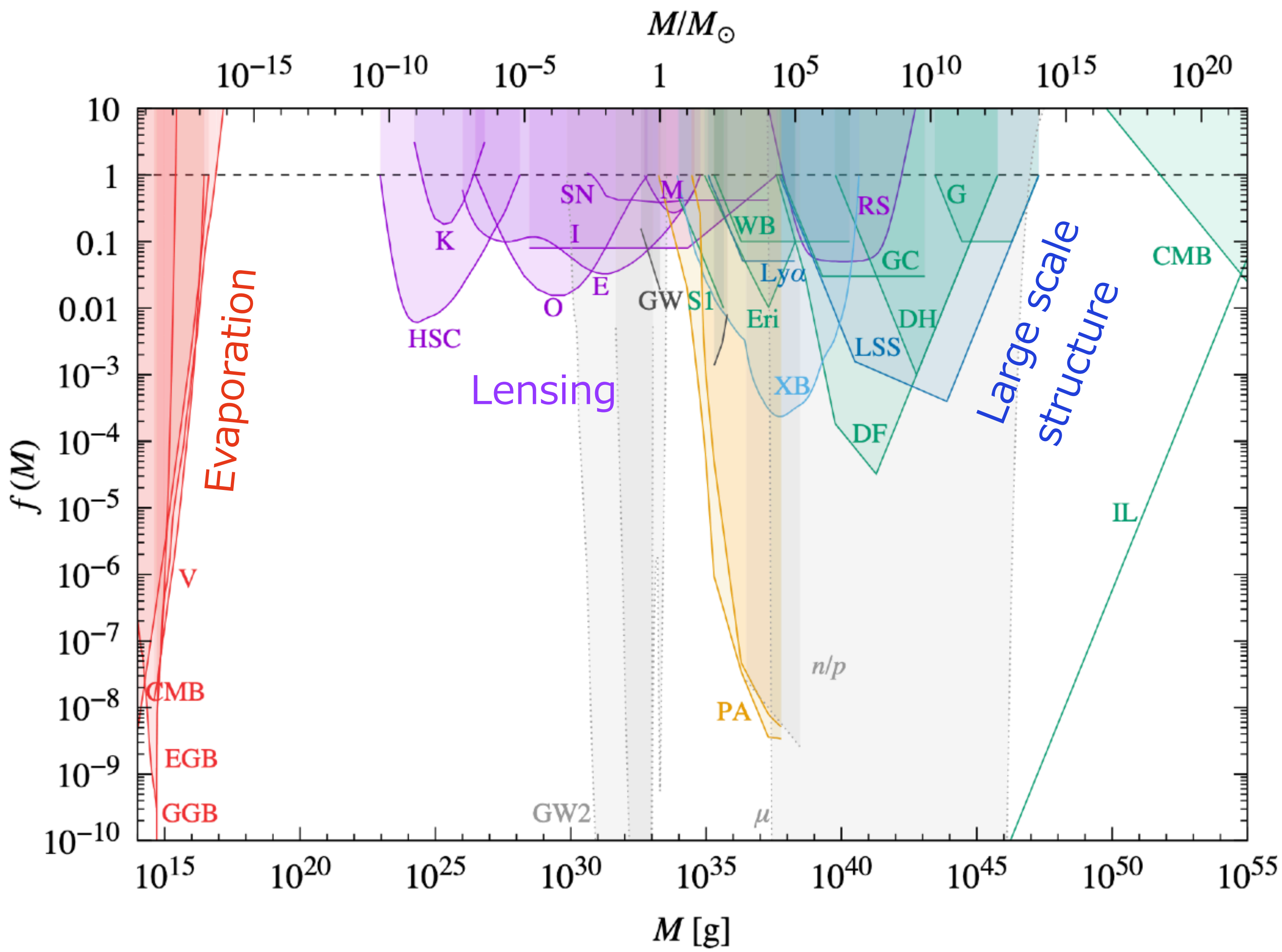



- Atractive candidate of DM: PBH (primordial black Hole)

  - It does not require new particle physics beyond the standard model.
  - It naturally arises from early universe density fluctuations.

# PBH mass regions

[Carr, Kohri, Sendouda, Yokoyama, Rept.Prog.Phys. 84 (2021) 11, 116902]




$$f = \frac{\rho_{\rm PBH}}{\rho_{\rm CDM}}$$

- many of mass region is strongly constrained.

- PBH can be the main component of DM at $M \sim 10^{-10} M_\odot$.

# Hints of PBH in the Earth mass region

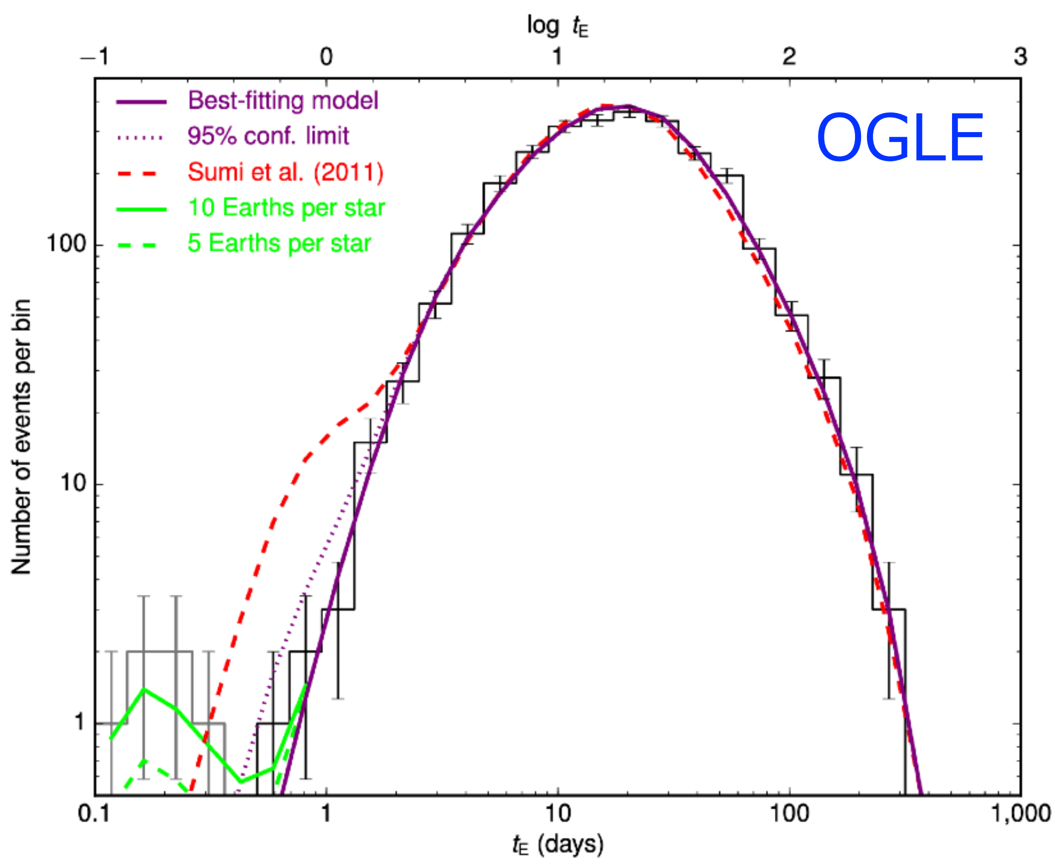
- OGLE has detected six events through its 5-year observations of stars in the Galactic bulge, hinting at the presence of Earth-sized PBH.(While they conflict with the bound from OGLE under the asumpition of null-results.)

- One possible microlensing event is found in the observation data of M31 with Subaru HSC ($M \sim 10^{-8} M_\odot$).

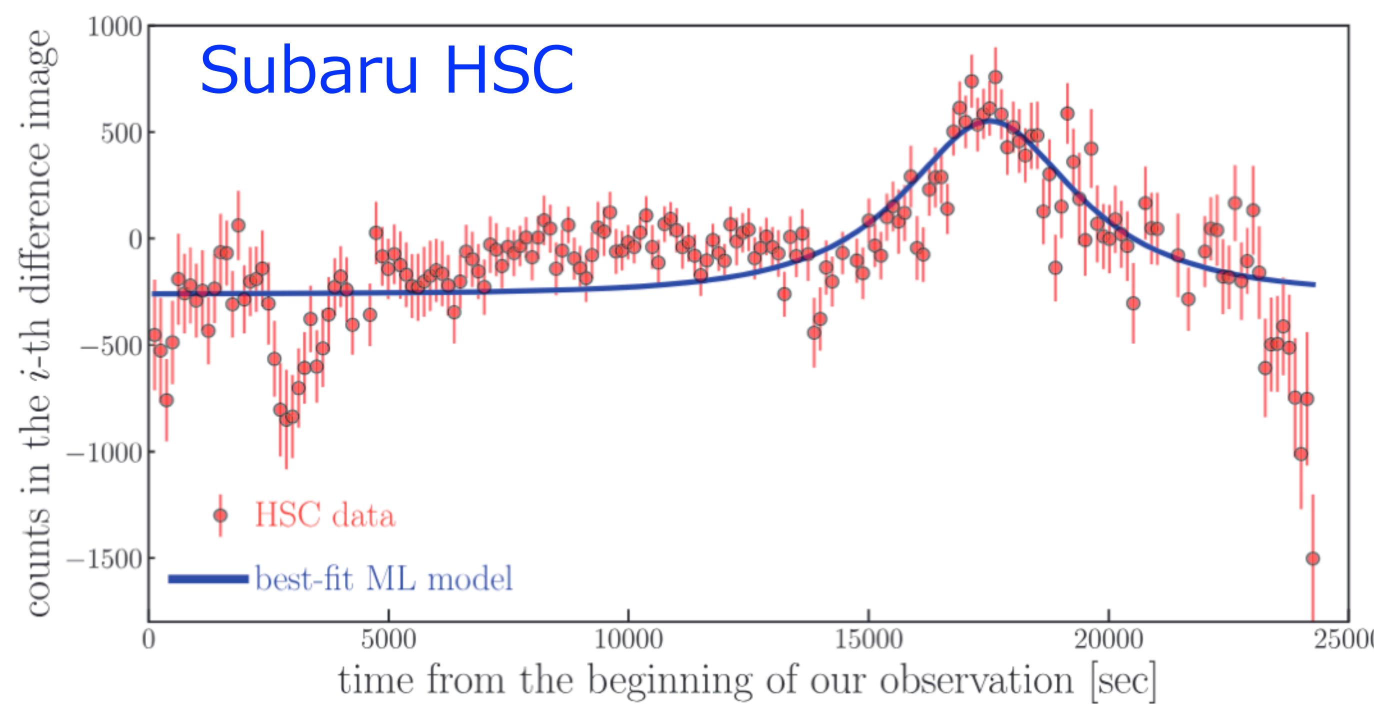
- The favored parameter region of DM is $(M, f) \sim (10^{-5} M_\odot, 10^{-1})$.

[Sugiyama, Takada, Kusenko, PLB 840 (2023)]

[Mroz, et. al., Nature 548 (2017)]




[Niikura, et. al., Nature Astron. 3 (2019)]



# Some production mecanisms for PBHs

PBHs can be formed by the gravitational collapse of over-density regions in the early Universe.

- **Inflation:** [S. W. Hawking, Commun. Math. Phys. 25, 152 (1972); B. J. Carr, S. W. Hawking, Mon. Not. Roy. Astron. Soc. 168, 399 (1974); B. J. Carr, Astrophys. J. 201, 1 (1975); Y. B. Zel'dovich, I. D. Novikov, Sov. Astron. 10, 602 (1967), etc.]
  - Generating large curvature perturbations during inflation, such a perturbation reenters the horizon and collapes into PBHs.
  - Challenges: Such scenarios often require fine-tuning of the inflaton potential.
- **Topological defects (e.g., domain wall):** [F. Ferrer, E. Masso, et. al., PRL 122, 101301 (2019); S. Ge, Phys. Dark Univ. 27, 100440 (2020); J. Liu, Z.-K. Guo,R.-G. Cai, PRD 101, 023513 (2020); G. B. Gelmini, J. Hyman, et. al., JCAP 06, 055 (2023); N. Kita jima, J. Lee, et. al., PLB 851, 138586 (2024); etc.]
  - If domain walls are created in the early universe, they can collapse into PBHs.
- **First order phase transitions(FOPT):** [K. Sato, M. Sasaki, et. al., Prog. Theor. Phys. 65, 1443 (1981), Prog. Theor. Phys. 66, 2052 (1981), PLB 108, 98; H. Kodama, M. Sasaki, K. Sato, Prog. Theor. Phys. 68, 1979 (1982); K. Jedamzik, J. C. Niemeyer, PRD 59,124014 (1999); etc. ]
  - By FOPT, energy contrast can be created. Such an overdense region collapses into PBHs.

Challenges for topological defects and FOPTs: overdense regions deviate from spherical symmetry. It is unclear how much deviation is allowed for PBH formation.

# New mecanisms for PBH formation

We propose a new mechanism for PBH formation:

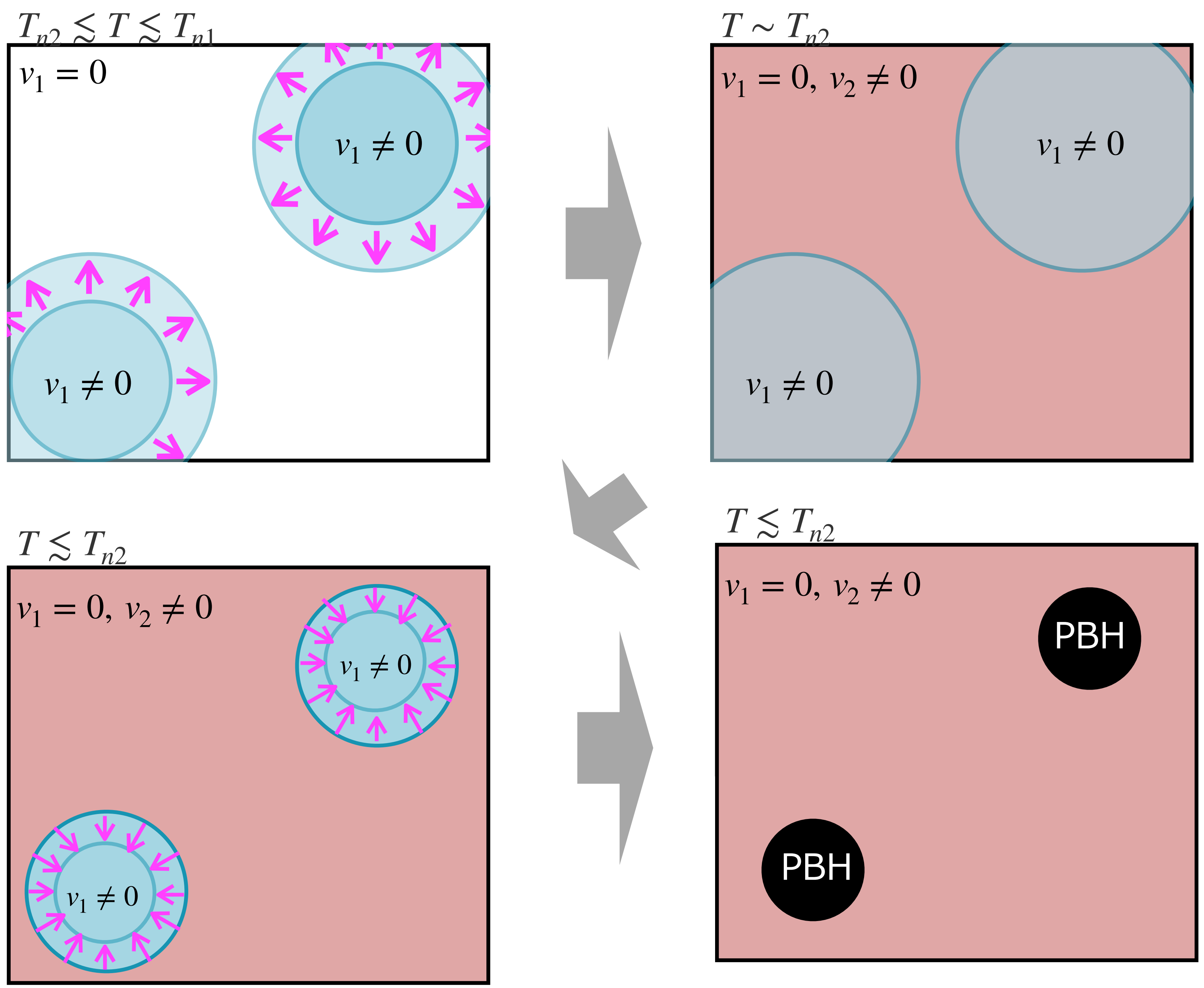
Inverted bubble collapse (IBC) mechanism

- Use of two kinds of phase transitions.
- Overdence regions are sphecially symmetric.
- Bubble walls should be runaway.
- PBH mass scale strongly depends on formation time of PBH.

  (in other mechanisms it is determined by horizon mass)

# Contents

# Dynamics of bubbles




- Bubbles do not colide with each other.
- Created bubbles are spherical symetric.
- Second PT can be both of 1st order and 2nd order.

# Set up for inverted bubbele collapse mecanism

Set up

Two scalar fields that develop VEV.

- Any scale of scalar bosons is possible.

- Toward application to the singlet model, let us consider VEVs close to EW scale. one obtains PBH mass $\mathcal{O}(10^{-6}\text{-}10^{-5})M_\odot$ :

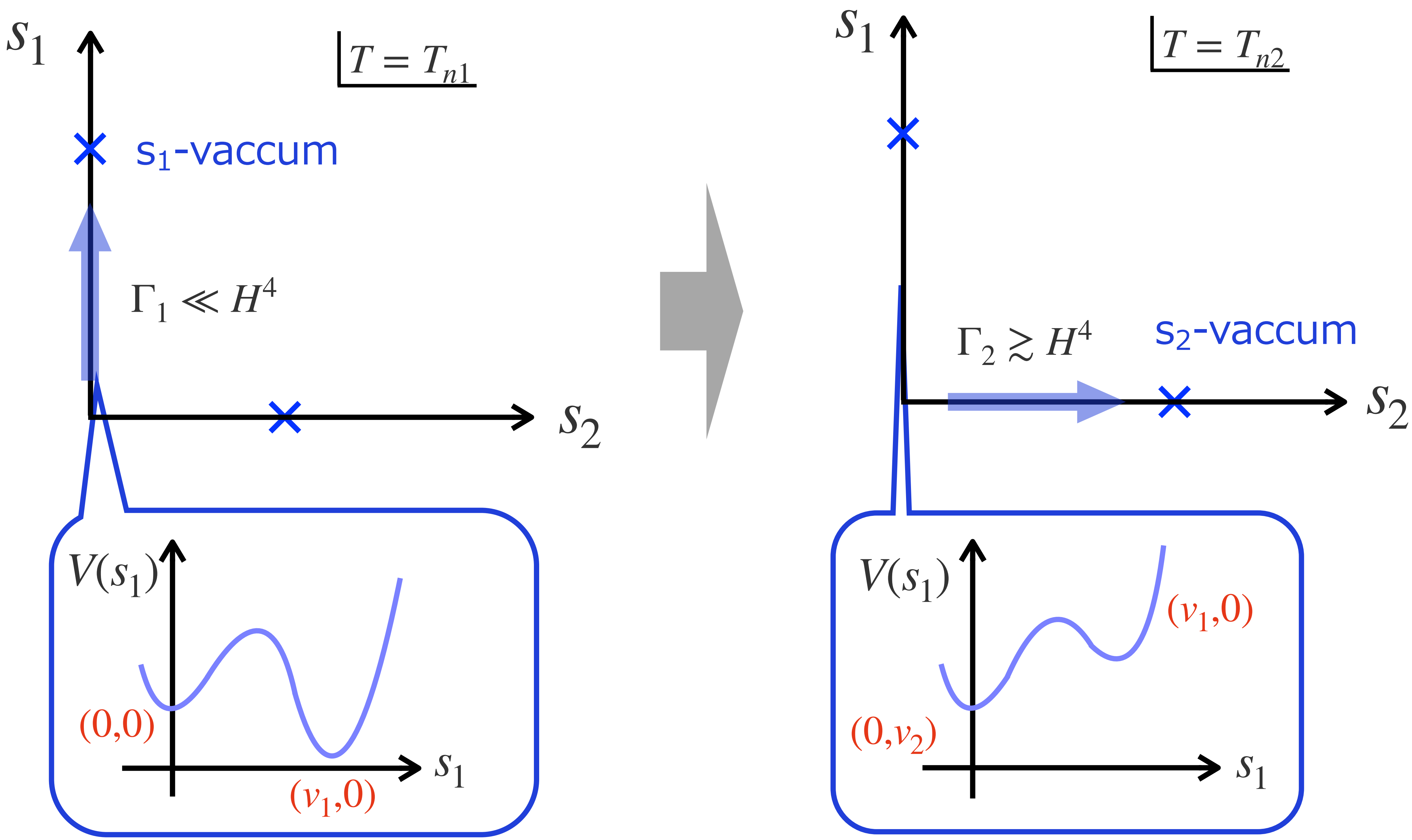
$$M_{\rm PBH} \sim M_{\rm H} \sim \frac{M_{\rm pl}^2}{T^2} \sim 10^{-5}M_\odot$$

[$\because$ Horizon mass $M_H \simeq \frac{4\pi}{3}R_H^3\rho$]

[$\because$ Asuuming radiation dominated Universe $\rho = \rho_r = \frac{\pi^2}{30}g_* T^4$]

$\longleftrightarrow$ Microlensing events in Subaru and OGLE

# Scematic picture for phase transions



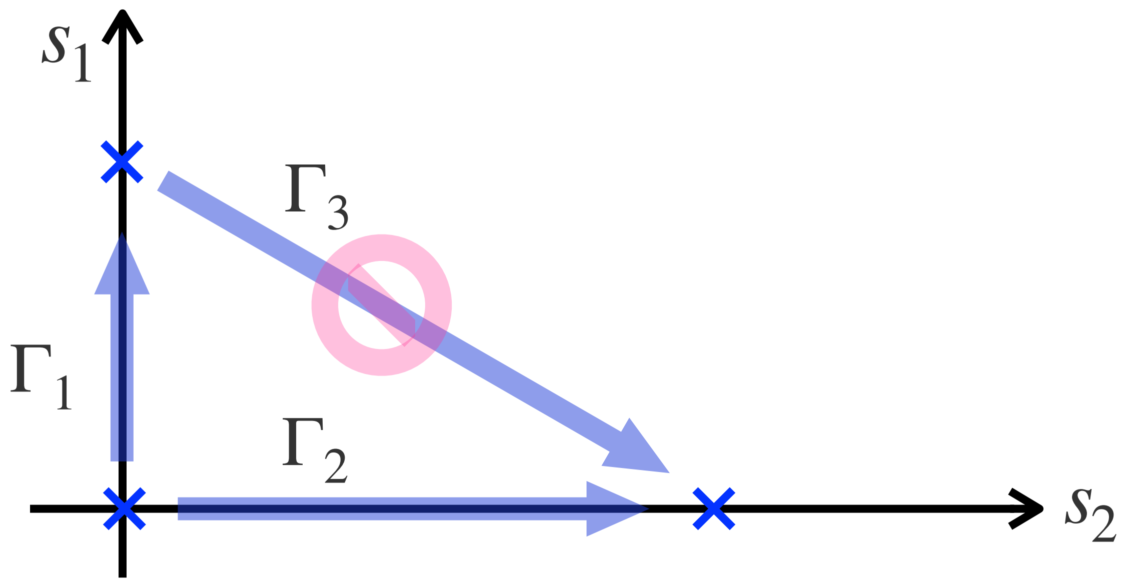

①: 1st PT happens in the $s_1$ direction

- It is **incomplete**.
- $(v_1,0)$ is true vaccum.

②: 2nd PT happens in the $s_2$ direction

- It is completed.
- $(v_1,0)$ is **false vaccum**.

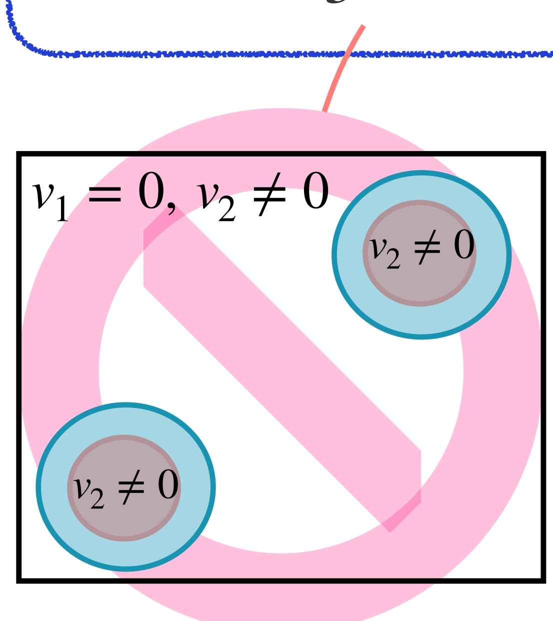
# Conditions for inverted bubbele collapse mecanism




$$(i):\ \Gamma_1(T_{n1}) \ll H^4(T_{n1})$$

$$(ii):\ \Gamma_2(T_{n2}) \gtrsim H^4(T_{n2})$$

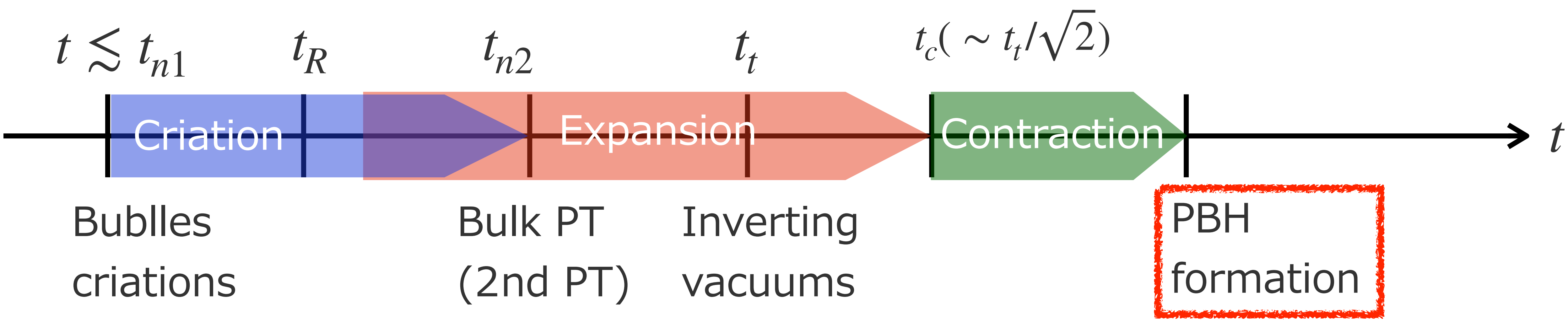
$$(iii):\ \Gamma_3(T) \lesssim H^4(T)\,, \qquad T_E \lesssim T \lesssim T_t$$




$T_t$: at this temperature $\Delta V$ becomes $V_{\rm eff}([v_1,0]) - V_{\rm eff}([0,v_2]) \gtrsim 0$ i.e., $[v_1,0]$ becomes False vacuum.

$T_E$: PBH formation temperature

# Time evolution in inverted bubbele collapse mecanism




$t_{n1}$ : 1st PT happens. ($T_{n1}$: correspoinding critical temerature)

$t_R$ : Nucreation time, at which bubble size grows up to $R$.

$t_{n2}$ : Bulk phase transition (2nd PT) happens. ($T_{n2}$: nucleation temperature)

$t_t$ : $s_1$- vacuum tunrns into falae vacuum.

$t_c$ : Bubbles start to shrink.

# Conditions for PBH formation

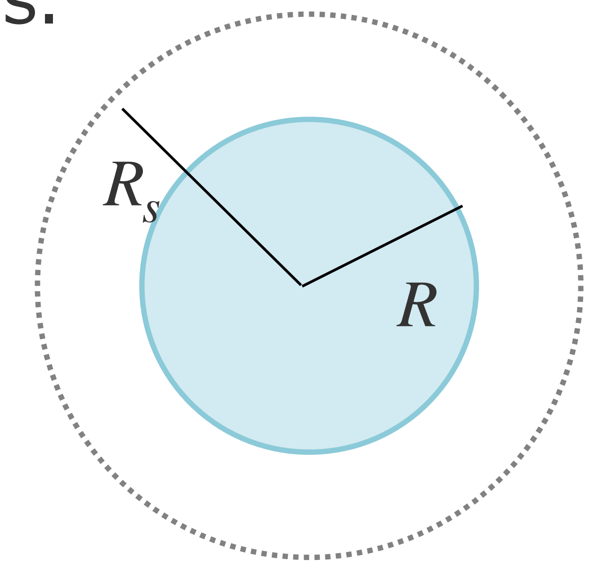
- The bubble wall undergoes a runaway.
  - Vacuum energy is mainly translated into wall kinetic energy.
  - Bubble walls are accelerated.
  - Efficiency factor $\epsilon(\lesssim 1)$ $\quad \epsilon = \dfrac{E^{\mathrm{W}}_{\mathrm{kin}}}{M_b}$ $\quad$ ( $M_b$ : Whole energy of bubble )

→ Runaway case: $\epsilon = 1$

- Bubble size becomes smaller than Schawarzschild radius.

$$R_s = 2G\epsilon M_b \gtrsim \delta$$

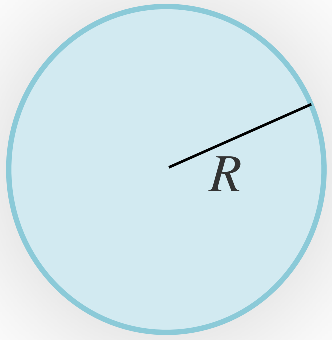
( $\delta$ :Width of bubble wall )



# Imporatant quantity for PBH formation

- Energy of bubble:

$$M_b \simeq \frac{4\pi}{3} R^3 \Delta V$$




  - $R$ highly depens on $T_c$.

- Imporatant quantity for PBH formation:

$\Delta V$: Difference of potential between false vacuum and true vacuum

$T_c$: temperature bubble starts to shrink

# Bubble size distributions

- Number density:

$$\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} = -3Hn_\mathrm{b} + \Gamma(T(t))$$

$$\rightarrow \quad n_\mathrm{b}(t)a^3(t) = e^{\int^t \mathrm{d}t' a^3(t')\Gamma(T(t'))}$$

- Time evolution of bubble size:

$t_R$ : Nucreation time, the created buble grows up to $R$.

$t_c$ : time, at which bubbles start to shrink.

$$R = a(t_c)\left(\frac{R_\mathrm{in}}{a(t_R)} + \int_{t_R}^{t_c} \mathrm{d}\tilde{t}\,\frac{v}{a(\tilde{t})}\right)$$

$$\rightarrow \quad t_R \simeq t_c\left(1 - \frac{R}{2vt_c}\right)^2$$

$$[\because \rho = \rho_r = \frac{\pi^2}{30}g_* T^4, \quad R_\mathrm{in} \ll 1]$$

- Bubble size distribuion:

$$\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}R}(t_c) = \left|\frac{\mathrm{d}t_R}{\mathrm{d}R}\right| \frac{1}{a(t_c)^3}\frac{\mathrm{d}n_\mathrm{b}a^3}{\mathrm{d}t}(t_R) = \frac{\Gamma(T_R)}{v}\left(1 - \frac{R}{2vt_c}\right)^4$$

$$[\because \frac{dt_R}{dR} = \frac{1}{v}\sqrt{\frac{t_R}{t_c}}]$$

# PBH mass function

- Around the bulk PT, we fit the tunneling rate $\Gamma$ as follows

$$\Gamma(T) \simeq cH_{\rm PT}^4 \, e^{-\alpha \frac{T-T_{\rm PT}}{T_{\rm PT}}} \quad \text{for} \quad T \geq T_{\rm PT} \qquad [\because S_3/T \approx \alpha(T-T_{\rm PT})/T_{\rm PT}]$$

$c, \alpha$ : Constant parameters   $T_{\rm PT}$ : the temperature at which bulk PT happens

- Relation between rudius $R$ and the PBH mass M:

$$M \simeq M_b \simeq \frac{4\pi}{3} R^3 \Delta V$$

- PBH mass function

$$\frac{\mathrm{d}\rho_{\rm PBH}}{\mathrm{d}\ln M}(t) = M \frac{\mathrm{d}n_{\rm b}}{\mathrm{d}R}(t_c) \frac{\mathrm{d}R}{\mathrm{d}\ln M_{\rm b}} \left(\frac{a(t_c)}{a(t)}\right)^3 = \frac{MR}{3} \frac{\Gamma(T_R)}{v} \frac{t_R^2}{t_c^2} \left(\frac{a(t_c)}{a(t)}\right)^3 ,$$

$[\alpha = 0]$

$$\frac{\mathrm{d}f_{\rm PBH}}{\mathrm{d}\ln M} \simeq \frac{1}{0.44\,\mathrm{eV}} \frac{45}{2\pi^2 g_{*s}(T_c) T_c^3} \frac{cMH_{\rm PT}^4}{3v} \left(\frac{3M}{4\pi\epsilon\Delta V}\right)^{1/3} \left[\frac{T_c}{T_R}\right]^4 \qquad \propto M^{4/3}$$

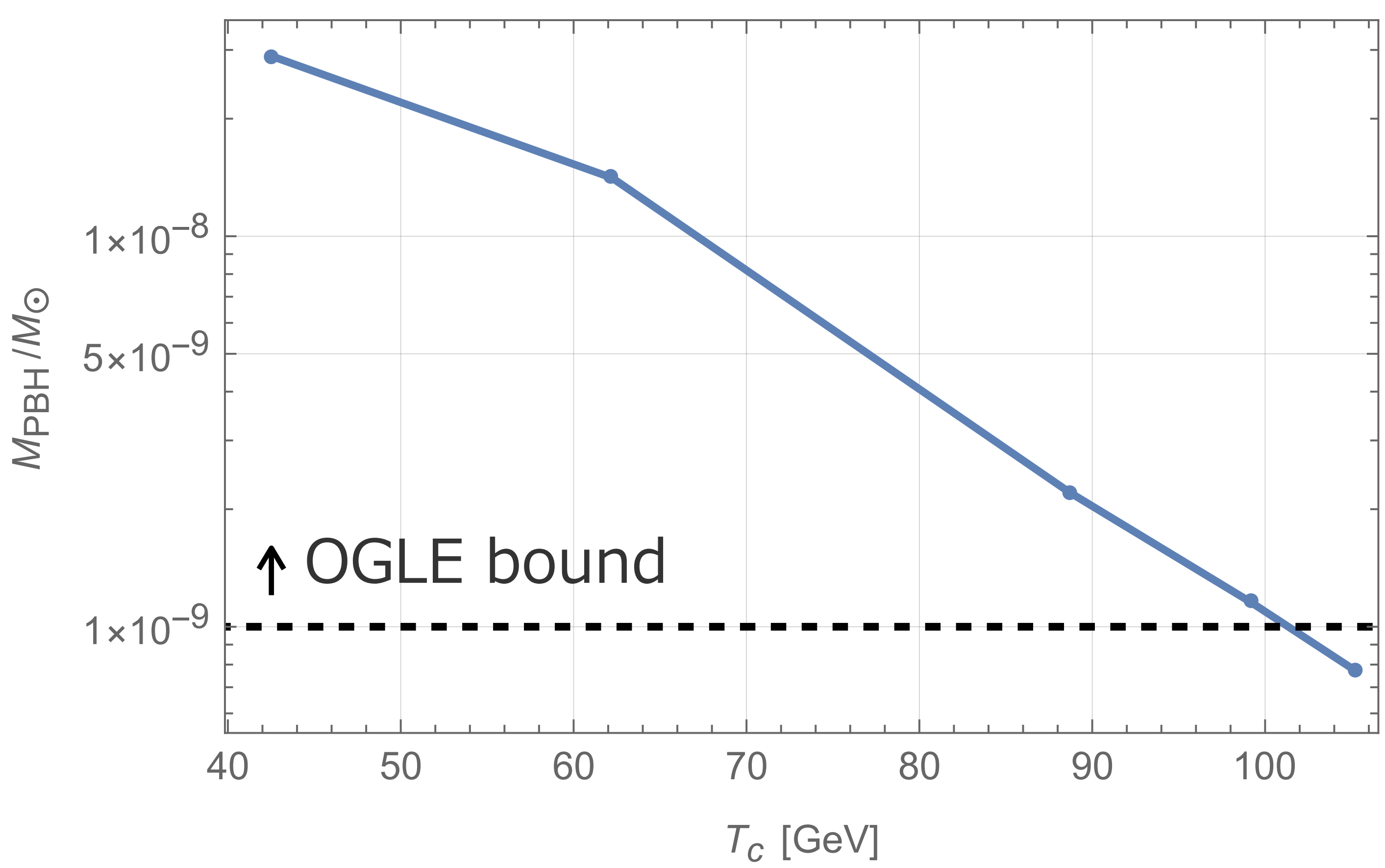
# PBH mass

PBH mass M as a function of $T_c$ and $T_R$:

$$M \simeq M_b \simeq \frac{4\pi}{3} R^3 \Delta V \simeq \frac{4\pi}{3} \left\{ \frac{1}{H_c} \left( 1 - \frac{T_c}{T_R} \right) \right\}^3 \Delta V, \qquad H_c \simeq \left( \frac{\pi^2}{90 m_{\rm pl}^2} g_* \right)^{1/2} T_c^2$$

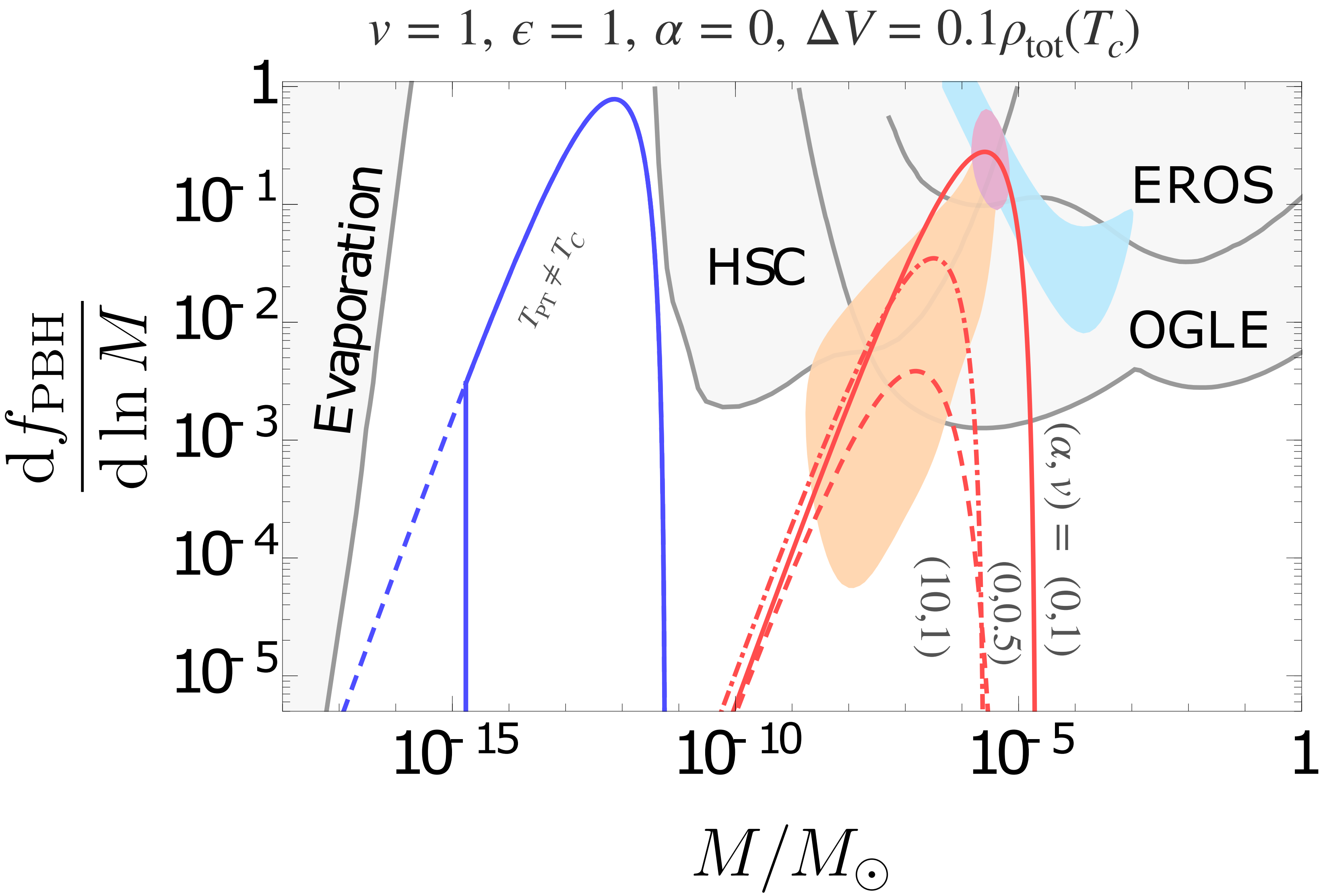
PBH mass depends on $T_R$ and $T_C$ .

With small $T_c$, $M$ growth by $T_c^{-6}$



# Numerical result (model independent)

[Murai, KS, Takahashi]




| | $(a)$ | $(b)$ |
|---|---|---|
| $T_{\rm PT}[{\rm GeV}]$ | $15$ | $3 \times 10^4$ |
| $T_{\rm c}[{\rm GeV}]$ | $15$ | $2.8 \times 10^4$ |
| $c$ | $10^{-8}$ | $10^{-11}$ |

Lensing events in Subaru HSC

Lensing events in OGLE

Combined results

[Sugiyama, Takada, Kusenko, PLB 840 (2023)]

- Small $v$ or nonzero $\alpha$ reduces $f_{PBH}$.
- With $T_C = \mathcal{O}(10)\mathrm{GeV}$ (scenario a), the lensing event can be explained.
- If $T_{PT} \neq T_C$, M reaches $M_{\rm min}$. This corresponds to $t_R = t_{\rm PT}$. $\Gamma \simeq 0\ (t > t_{\rm PT})$
- With $T_C = \mathcal{O}(10^3)\mathrm{GeV}$, DM can be explained.

# Contents

# Concrete model: two real singlet scalar models

- Particle contents: $Z_2$ symmetry is imposed.

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+\phi+iG^0) \end{bmatrix}$$ : triggers EWPT, $Z_2$ even

$$S_1 = v_1 + s_1$$ : triggers PT in singlet directions, $Z_2$-even

$$S_2 = v_2 + s_2$$ : triggers PT in singlet directions , $Z_2$-odd

- Physical states: $h,\ H_1,\ H_2$

$$\begin{pmatrix} \phi \\ s_1 \\ s_2 \end{pmatrix} = R_\theta \begin{pmatrix} h \\ H_1 \\ H_2 \end{pmatrix},$$

- Higgs potential: $V$ is invariant under the shift $s_1 \to s_1 + v_1'$. ⟹ $v_1 = 0$

$$\left.\begin{aligned} V(\Phi, S_1, S_2) = & -m_\Phi^2|\Phi|^2 + \lambda|\Phi|^4 + \mu_{\Phi 1}|\Phi|^2 S_1 + \lambda_{\Phi 1}|\Phi|^2 S_1^2 \\ & + t_1 S_1 - m_1^2 S_1^2 + \mu_1 S_1^3 + \lambda_1 S_1^4 \end{aligned}\right] \Phi + S_1$$

$$\left. - m_2^2 S_2^2 + \lambda_2 S_2^4 + \lambda_{\Phi 2}|\Phi|^2 S_2^2 + \lambda_{12} S_1^2 S_2^2 + \mu_{12} S_1 S_2^2, \right]$$ terms with $S_2$

- Responsible for EWPT.
- Responsible for PT in ($S_1$,$S_2$) plane.
- Determines thermal effects to the potential.

# Implementation of IBC mechanism

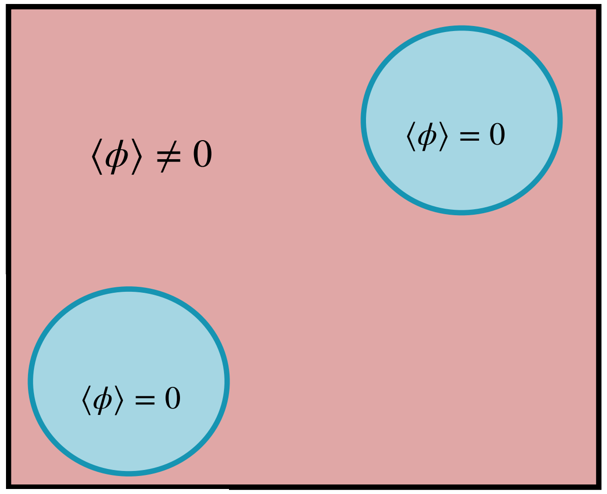
- One has three dimensional directions for PTs: $(\ \phi,\ s_1,\ s_2\ )$

- One should require the bubble walls undergo runaway.
  - Otherwise, bubble energy is dissipated by plasma.
  - Created bubbles disappear without collapsing PBHs.

- If bulk PT is realized by EWPT, runway does not happen.

[Bödeker, Moore, JCAP 05 (2017) 025]

  - Vector bosons pass bubble walls, obtaining masses.
  - Soft vector bosons are radiated. It induces additional friction.




Incomplete PT : $S_1$- direction

Bulk PT : $S_2$- direction

# Effective potential

$$V_{\text{eff}}(s_1, s_2, T) = \underbrace{V^0_{\text{eff}}(s_1, s_2)}_{\text{Tree level}} + \underbrace{V^{\text{cw}}_{\text{eff}}(s_1, s_2)}_{\text{CW potential}} + \underbrace{V^T_{\text{eff}}(s_1, s_2, T)}_{\text{Thermal corrections to the potential}} + \underbrace{V^{\text{ring}}_{\text{eff}}(s_1, s_2, T)}_{\text{Resummed thermal corrections to masses}}$$

- PBH formation in IBC mecanism happens in the $(s_1, s_2)$-plane. We analyise $V(s_1, s_2, T)_{\text{eff}} \equiv V_{\text{eff}}(\phi = 0, s_1, s_2, T)$.
- Tree level potential:

$$V_0 = -m_1^2 s_1^2 + \mu_1 s_1^3 + \lambda_1 s_1^4 - m_2^2 s_2^2 + \lambda_2 s_2^4 + \lambda_{12} s_1^2 s_2^2, \qquad [\because \ t_1 s_1 = \mu_{12} s_1 s_2^2 = 0]$$

  - $s_1$ direction: The potential barrier is easily obtained by $\mu_1 s_1^3$.
  - $s_2$ direction: similar to SM Higgs potential.
  - $\lambda_{12}$ controls potential barrier between $s_1$ and $s_2$ directions.

- CW potential: one-loop corrections to the potential. $\overline{\text{MS}}$ scheme is applyied.
- Thermal effects to masses ($V^{\text{ring}}$) : [R. Parwani, Phys. Rev. D 45, 4695 (1992)]

$$\bar{m}(s_1, s_2) \to M = \bar{m}(s_1, s_2) + \Pi(T) \qquad \Pi(T) : \text{thermal mass}$$

# Benchmark points for IBC mechanism [1/2]

$$\theta_i = 0,\ m_{H_1} \simeq 223\,\mathrm{GeV},\ m_{H_2} = 246\,\mathrm{GeV},$$
$$\lambda_{12} = 0.5, \mu_1 \simeq -190\,\mathrm{GeV},\ \lambda_1 = 0.335,$$
$$v_2 = 70\mathrm{GeV}, \lambda_{\Phi 1} = -0.14$$

Tree level potential:

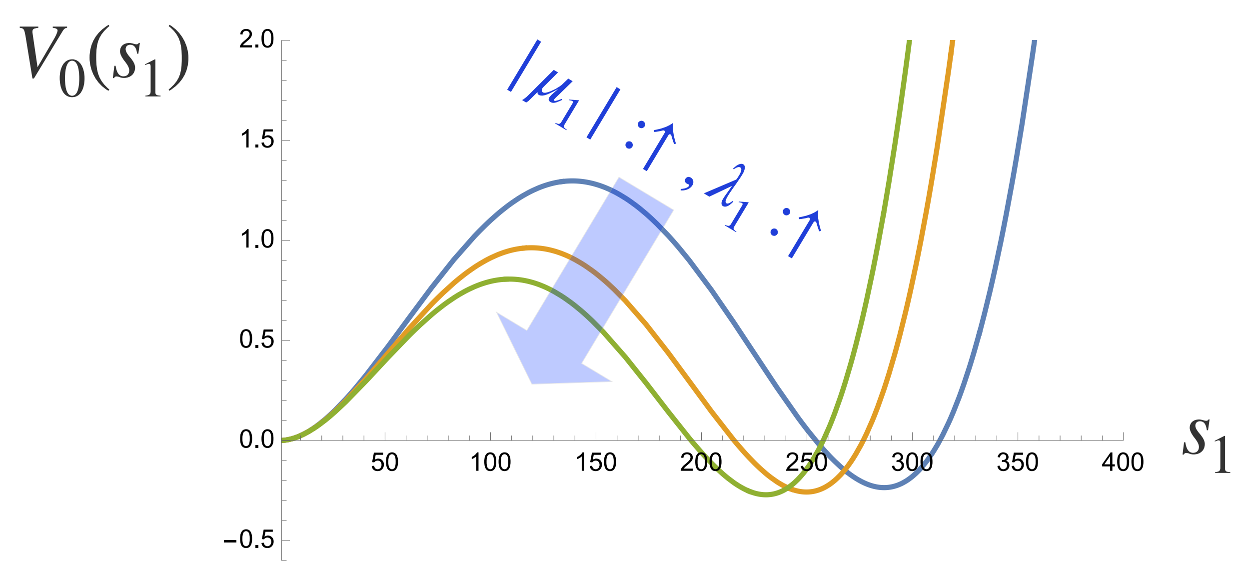
$$V_0 = -m_1^2 s_1^2 + \mu_1 s_1^3 + \lambda_1 s_1^4 - m_2^2 s_2^2 + \lambda_2 s_2^4 + \lambda_{12} s_1^2 s_2^2,$$

- Alignment limit to avoid collider bounds and reduce the potential $\Rightarrow$ $t_1 s_1 = \mu_{12} s_1 s_2^2 = 0$, $\mu_{12} s_1 |\Phi|^2 = 0$
- To obtain the vaccums in the EW scale. $\Rightarrow$ $T_{n1}, T_{n2} \sim \mathcal{O}(100)\mathrm{GeV}$
- To reduce the temperature dependence of $V_{\mathrm{eff}}$. $\Rightarrow$ Abundant PBH due to $\Gamma_{n1}(T_{n1}) \approx \mathrm{const}$.

$$V_{\mathrm{eff}} \approx V_0 + (c_1\lambda_{12} + d_1\lambda_1)T^2 s_1^2 + c_2\lambda_{12}T^2 s_2^2$$

- $\mu_1 \approx m_2^2$, $\lambda_1 \sim \mathcal{O}(0.1)$ makes bariear high and the vacuum shallow.

$\Rightarrow$ $\Gamma_{n1}(T_{n1}) \ll H^4$



# Benchmark points for IBC mechanism [2/2]

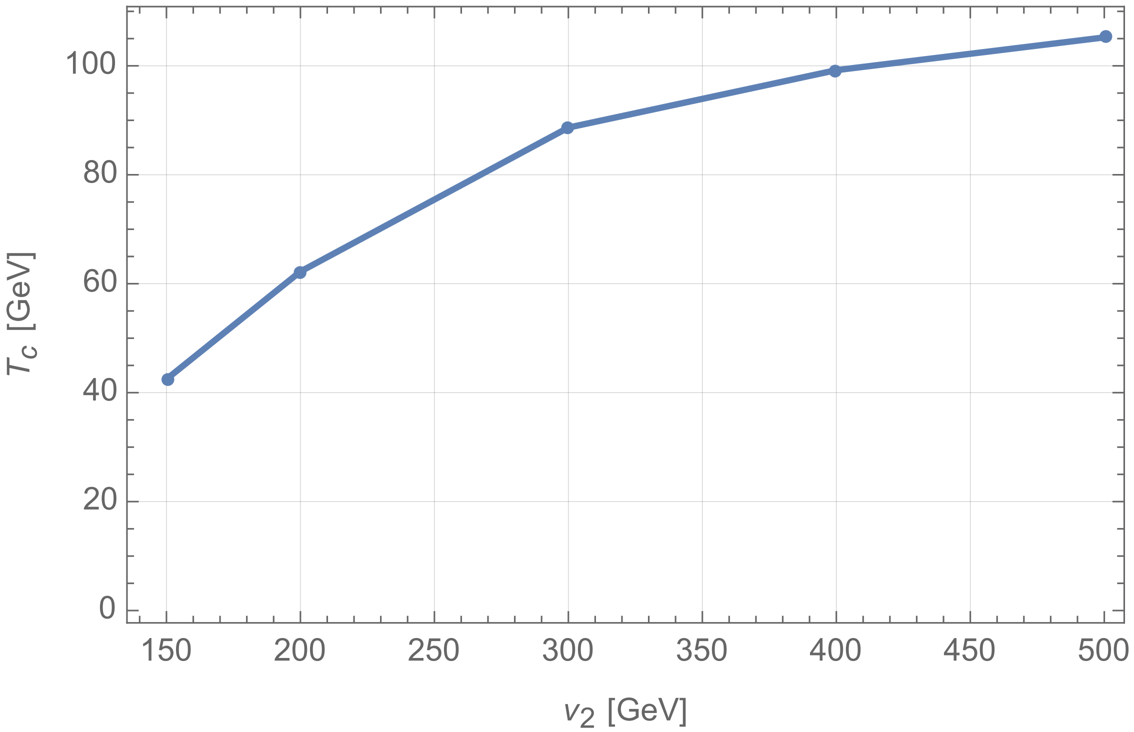
$$\theta_i = 0,\ m_{H_1} \simeq 223\,\mathrm{GeV},\ m_{H_2} = 246\,\mathrm{GeV},$$
$$\lambda_{12} = 0.5, \mu_1 \simeq -190\,\mathrm{GeV},\ \lambda_1 = 0.335,$$
$$v_2 = 70\mathrm{GeV}, \lambda_{\Phi 1} = -0.14$$

Tree level potential:

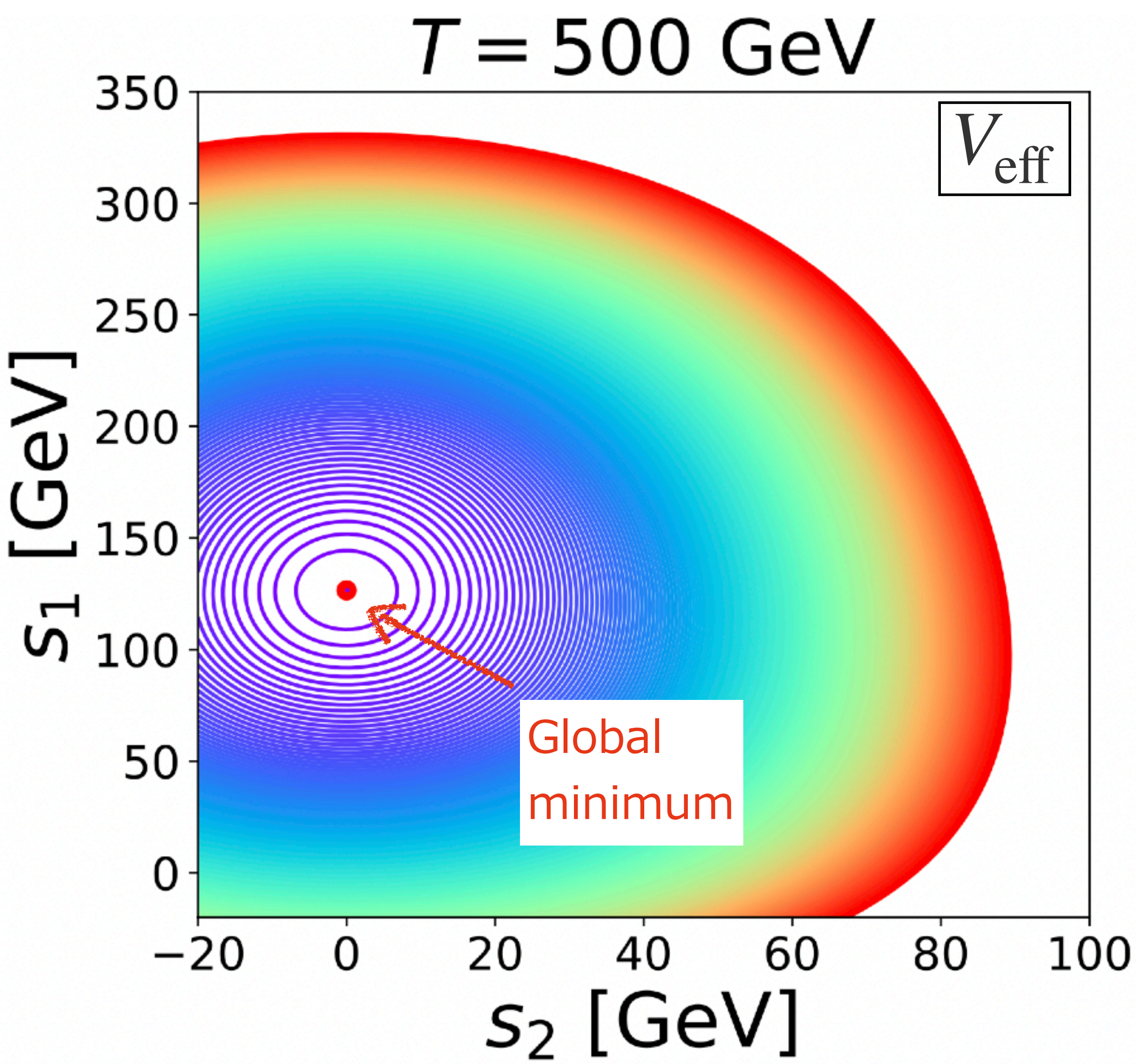
$$V_0 = -m_1^2 s_1^2 + \mu_1 s_1^3 + \lambda_1 s_1^4 - m_2^2 s_2^2 + \lambda_2 s_2^4 + \lambda_{12} s_1^2 s_2^2,$$

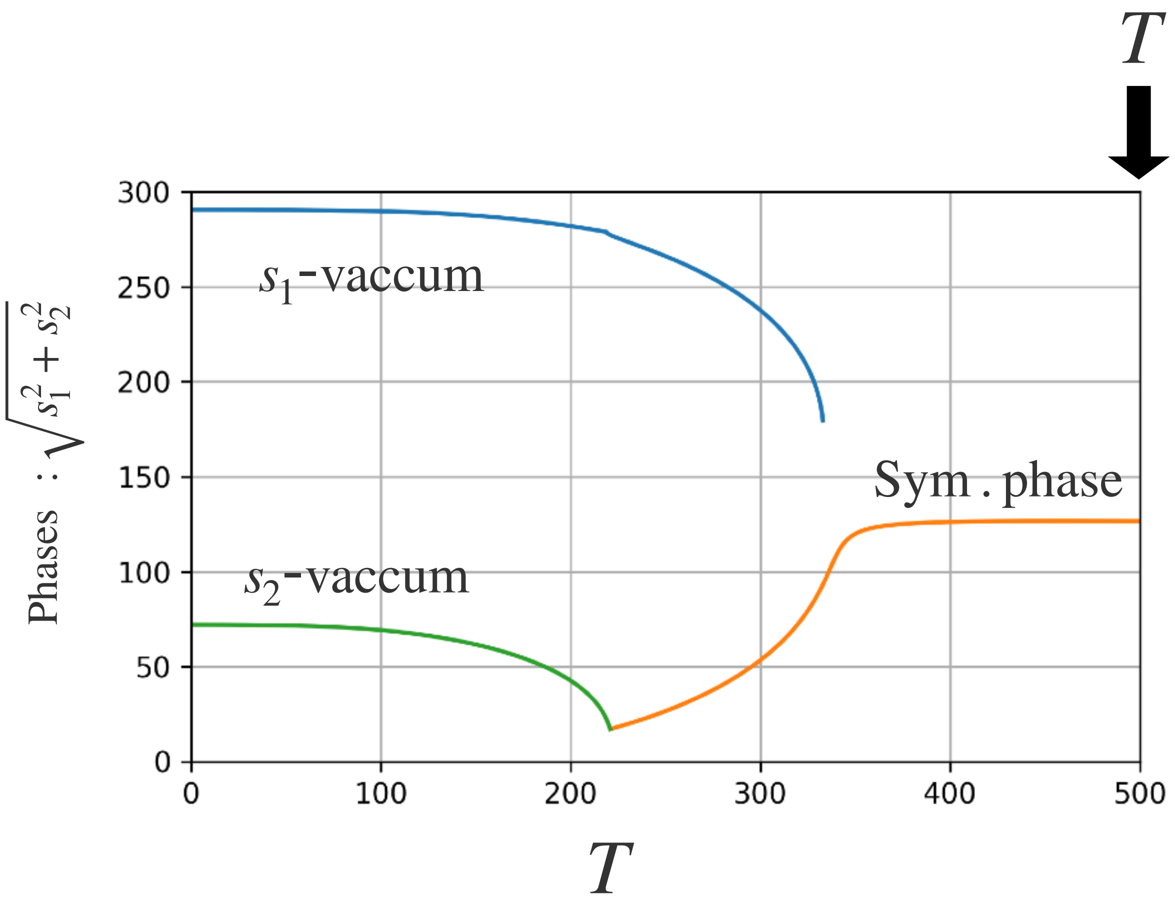
- With smaller v2, low Tc is obtained.

Increasing PBH masses and the abundance.



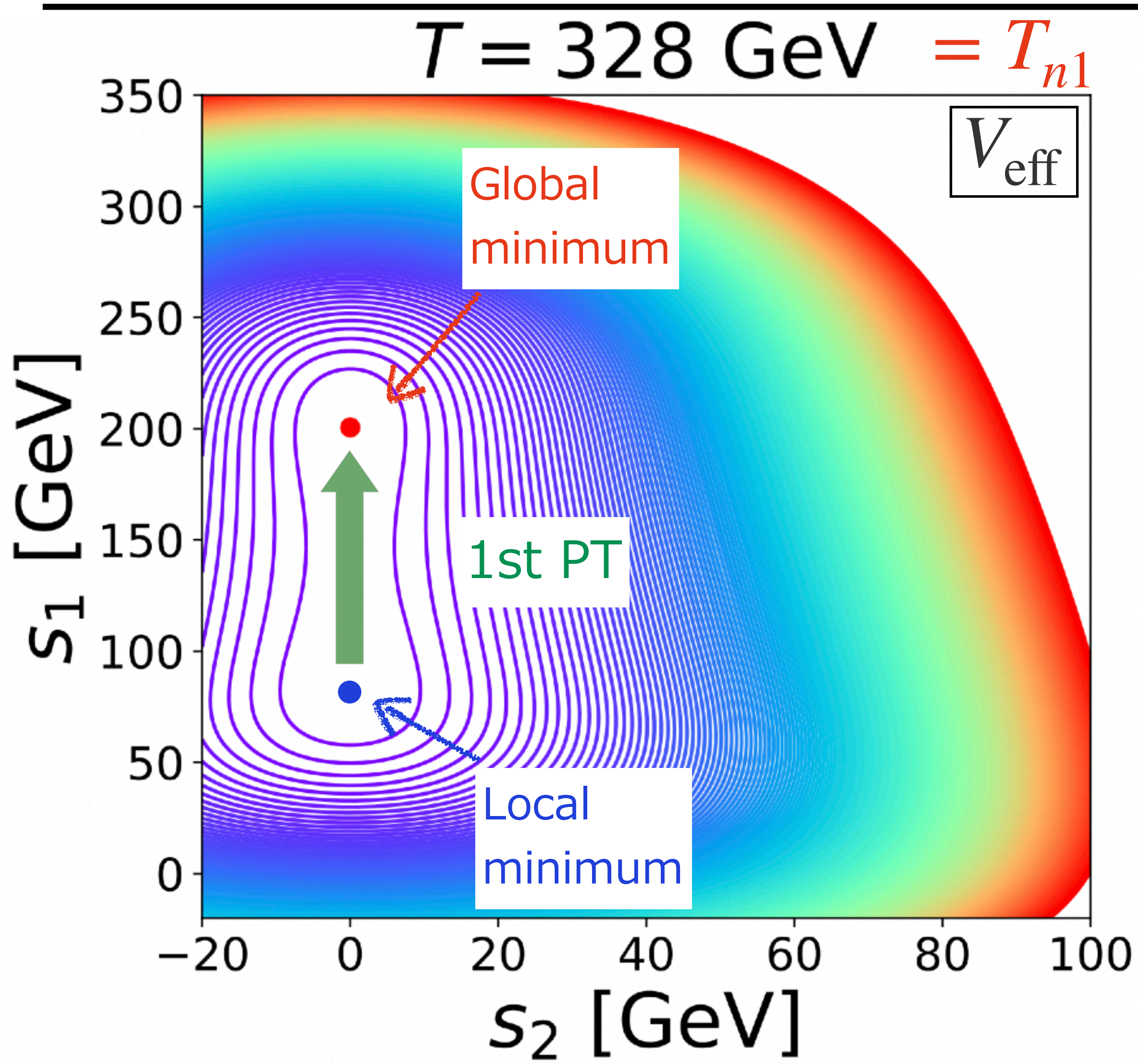
# Temperature dependence of the Effective potential in IBC [1/4]

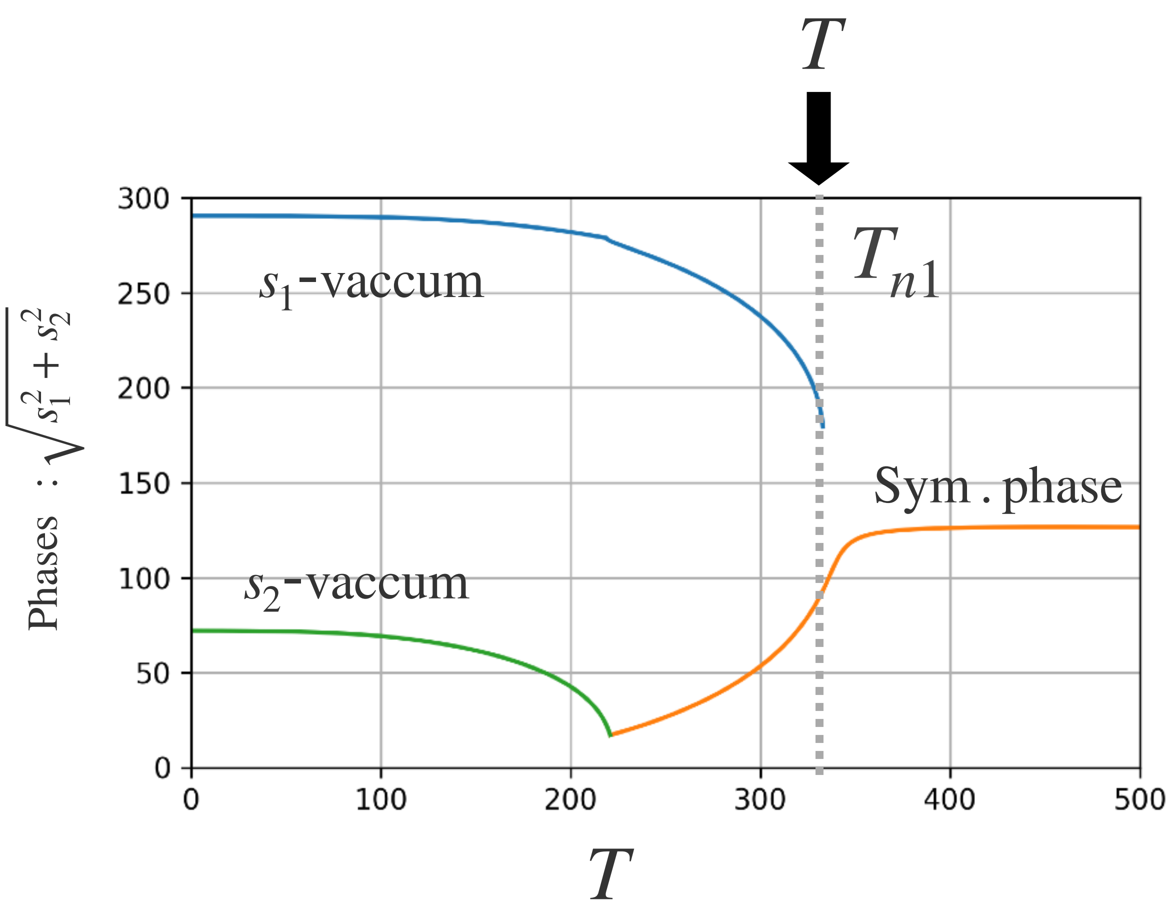



- At high temperature, the trivial vacuum only exists.

# Temperature dependence of the Effective potential in IBC [2/4]



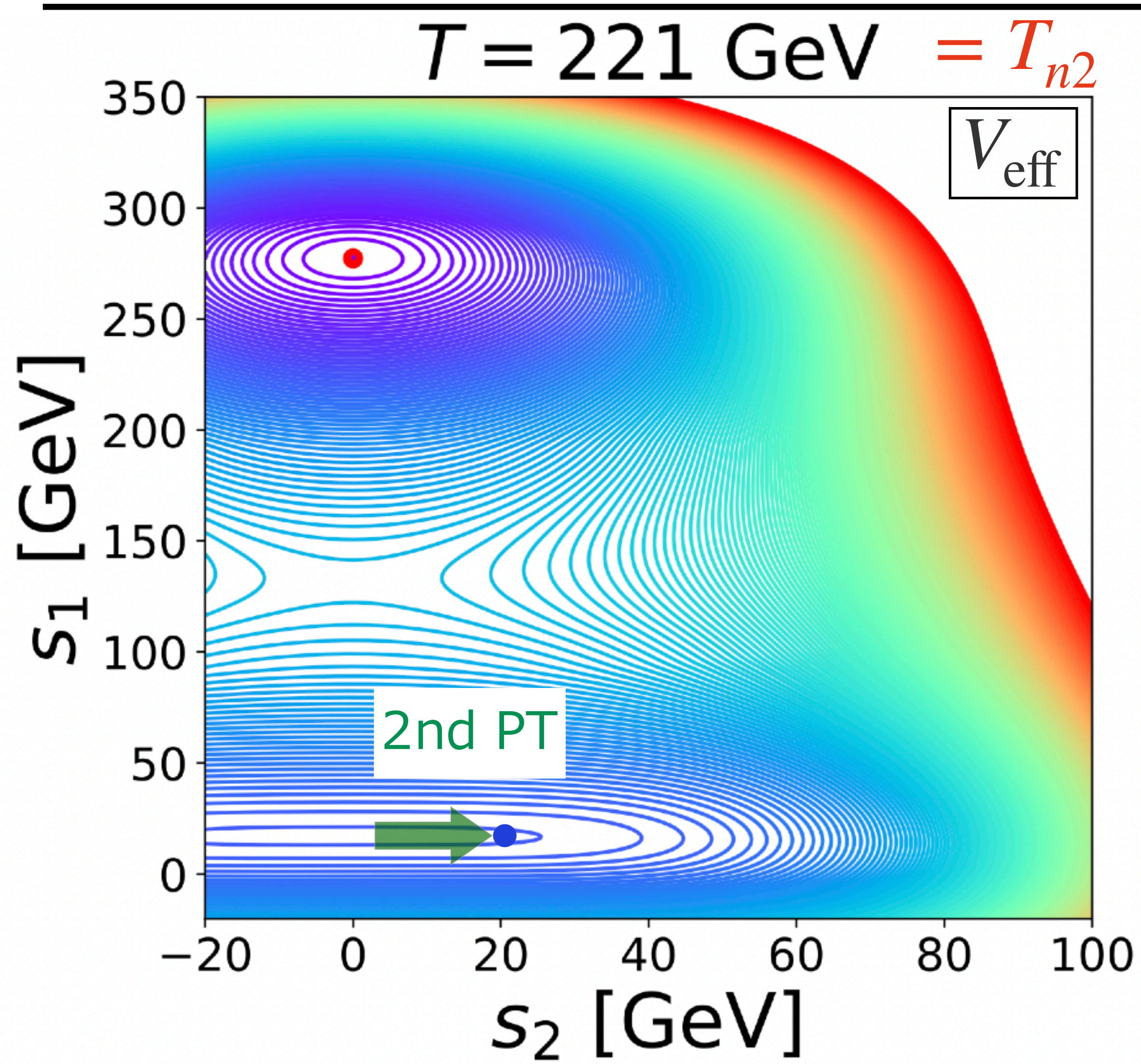



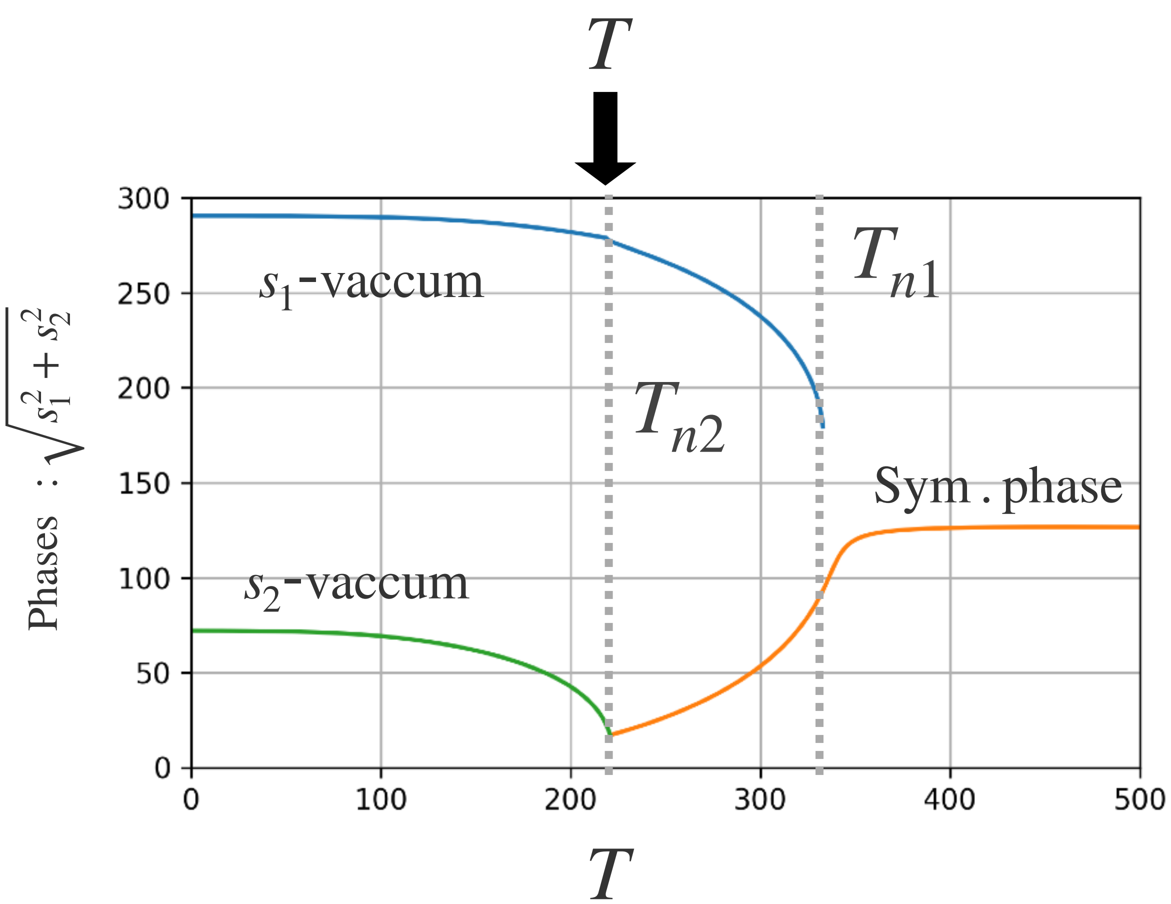

- The original exisiting vaccum becomes false vaccum.

- Transition of Sym. $\to$ $s_1$-vaccum happens:

$$\Gamma_{n1}/H^4\Big|_{T=T_{n1}} \sim \mathcal{O}(10^{-10})$$

# Temperature dependence of the Effective potential in IBC [3/4]

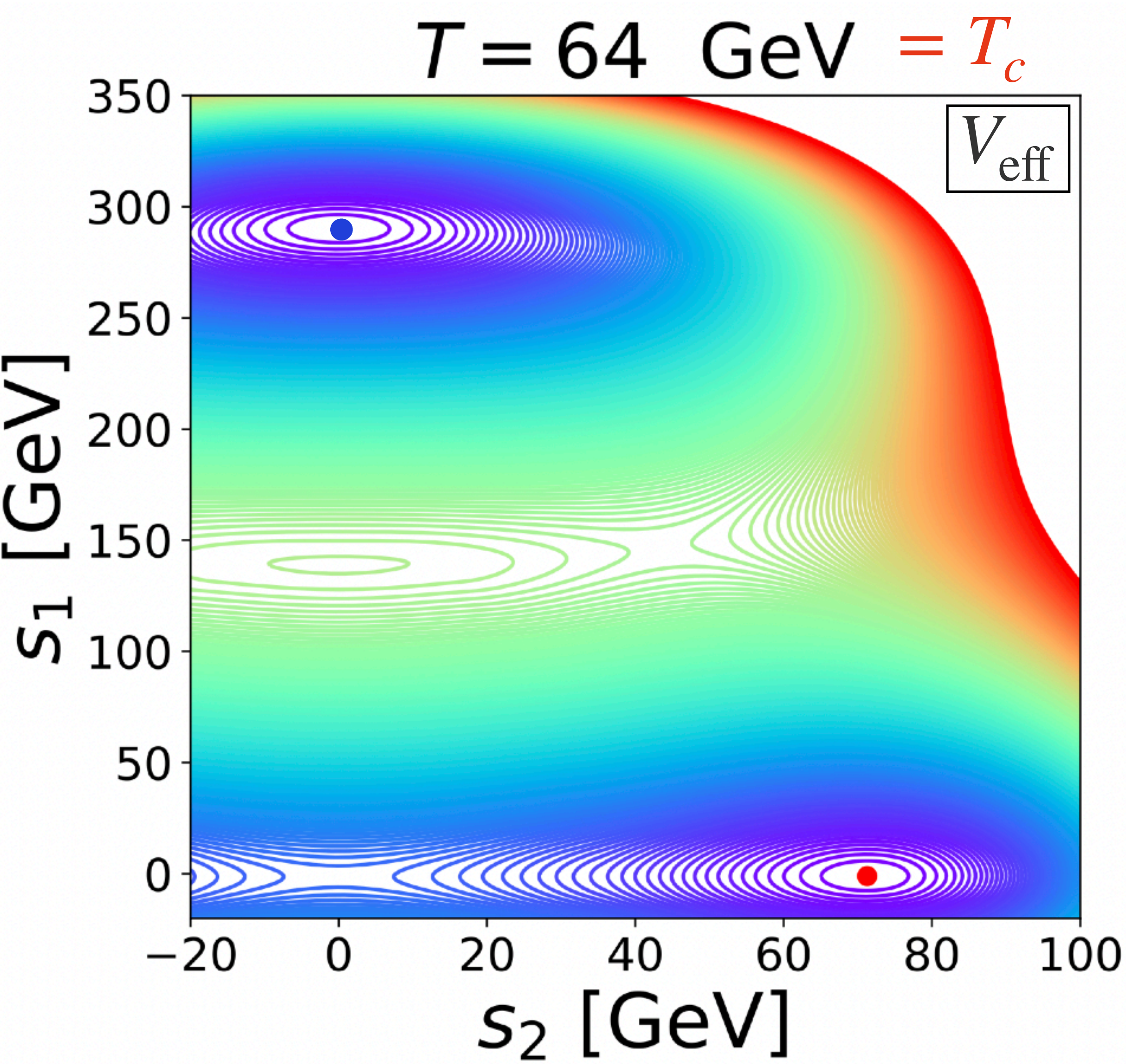


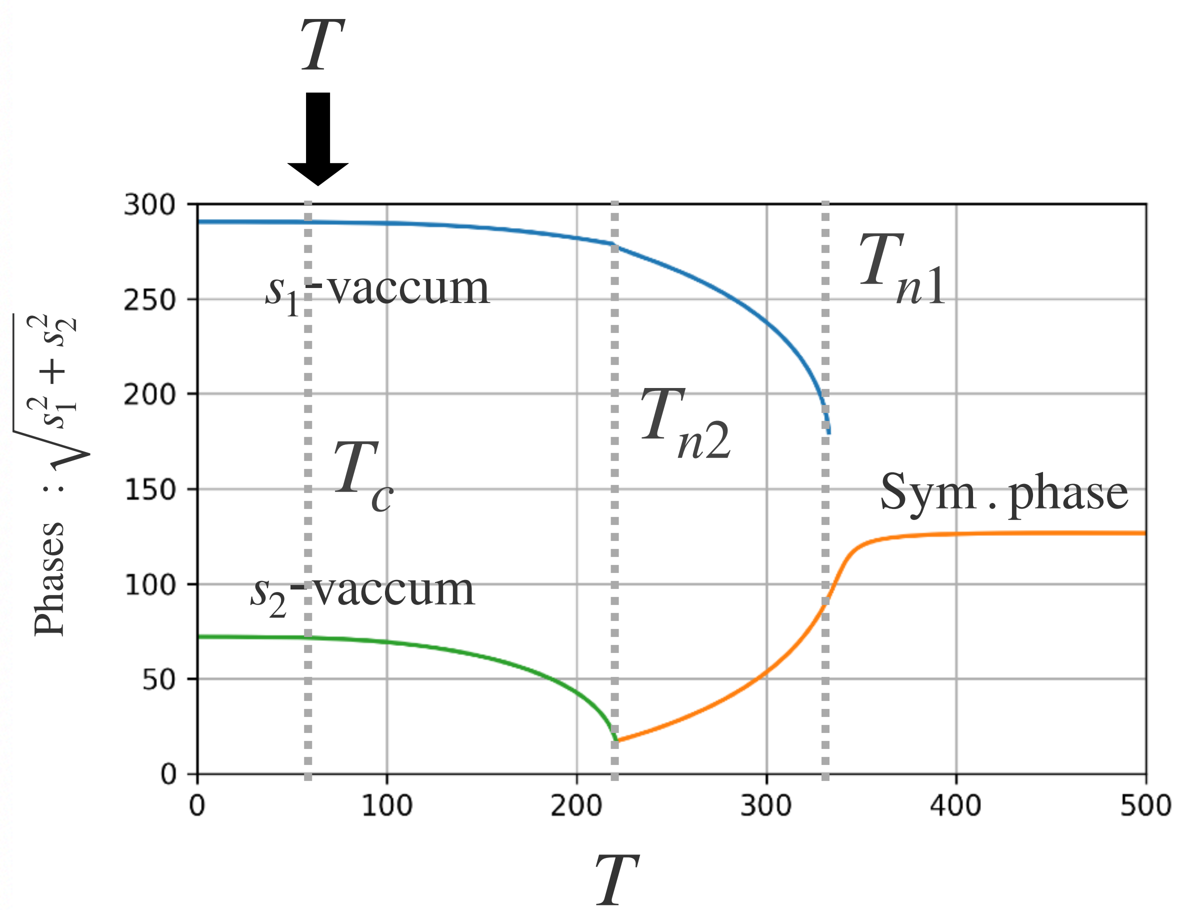




- Due to the high barrier, transition of Sym. $\rightarrow$ $s_1$-vaccum is prohibited.

- Second-order PT happens at $T_{n2}$ in the $s_2$ direction.

# Temperature dependence of the Effective potential in IBC [4/4]




- Soon after the 2nd PT, s1-vaccum: false vaccum, s2-vaccum: true vaccum.

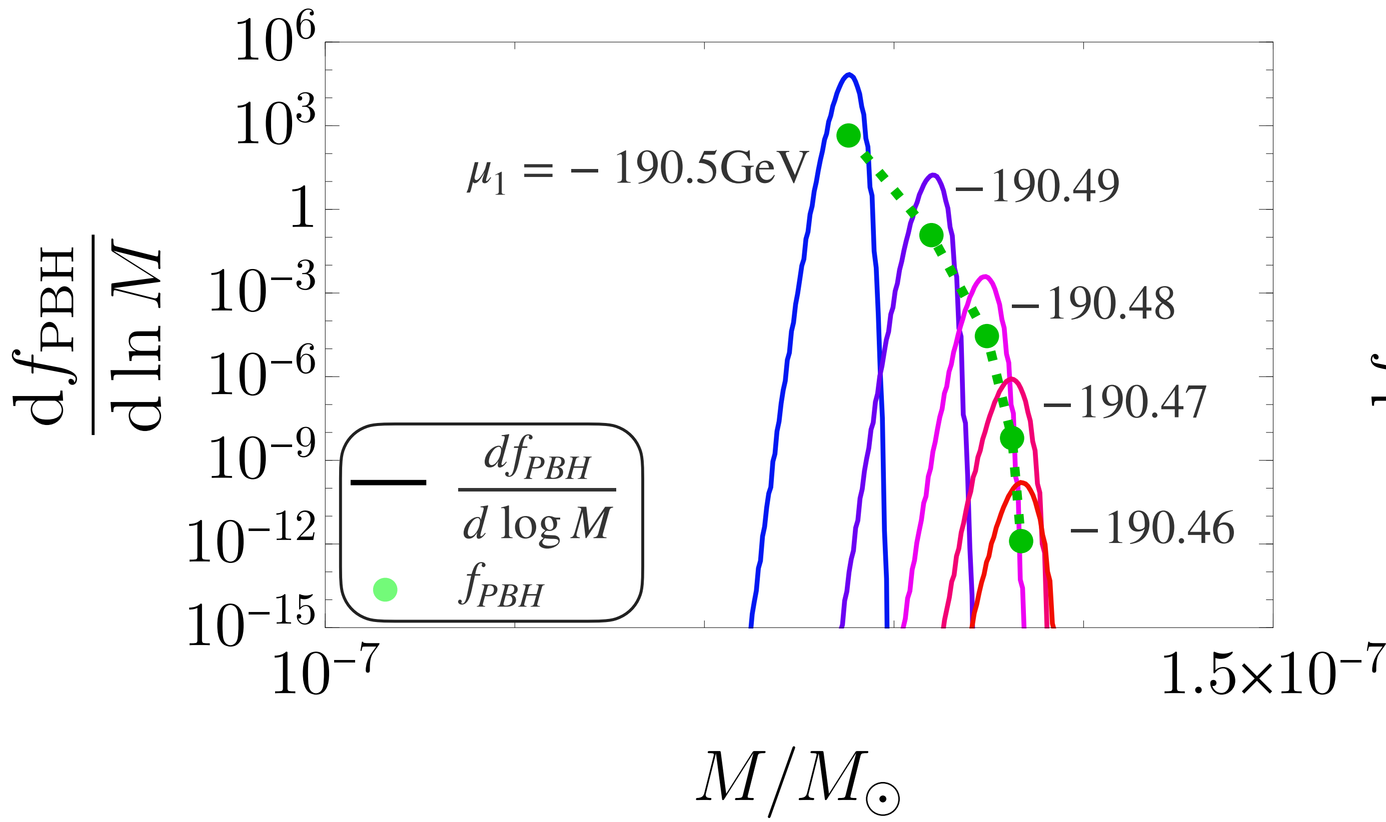
- At T=Tc, false and true vaccums are inverted.

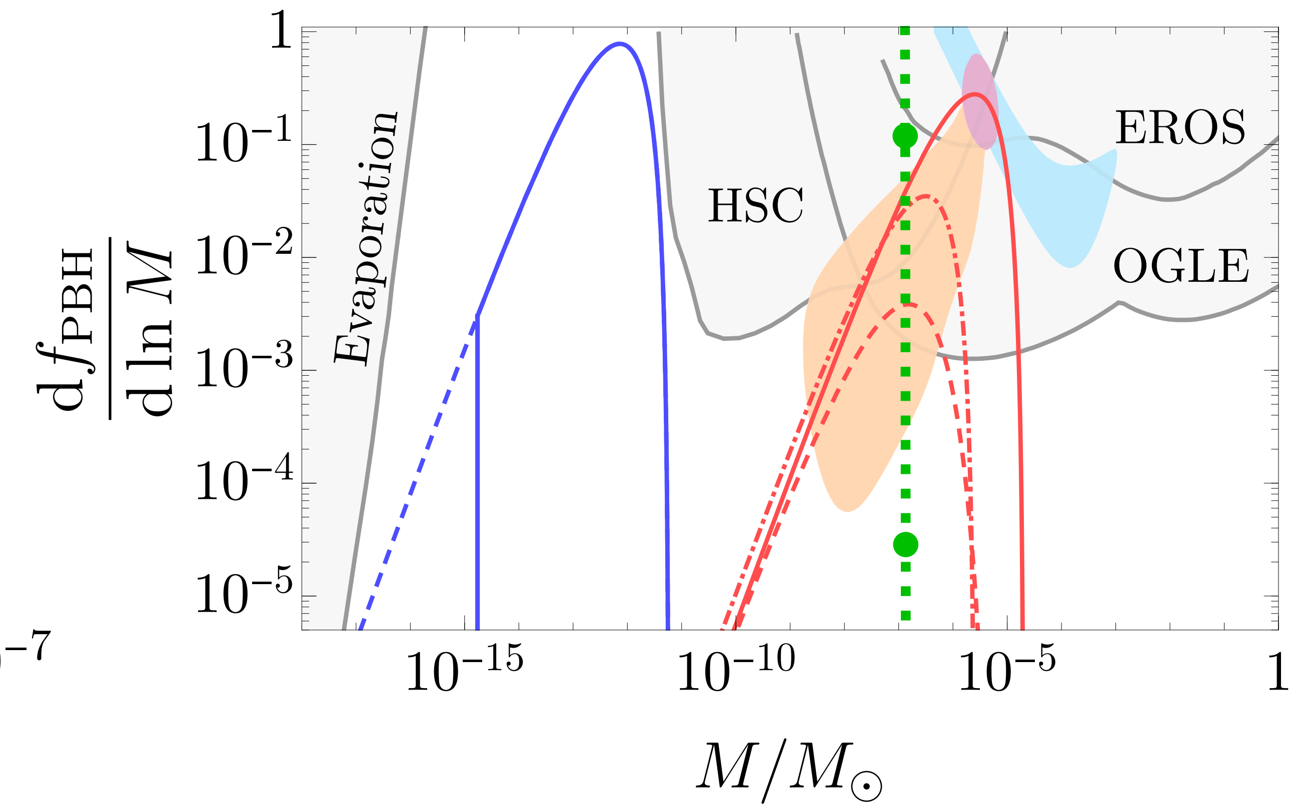
- The IBC mechanism works in the chosen benchmark.

- From the PT analysis, $\Gamma_{n1}$, $\Delta V$ and $T_c$ are obtained. ➡ PBH in IBC mecanism can be calcurated.

# Numerical results for the singlet model

[Murai, KS, Takahashi]




- A highly monochomatic PBH mass spectrum is obtained.

- The lensing events observed in Subaru HSC can be explained.
  With a smaller Tc, OGLE results can be fit.

- Obtained PBH strongly depends on the shape of the potential.

➡ Higgs sector can be reconstructed by PBH mesurments.

# Summary

- We have proposed a new mechanism for PBH formations: inverted bubble collapse mechanism.

- First incomprete PT creates an overdense region. Second (bulk) PT collapses it to produce PBHs.

- In this scenario, overdence regions are sphecially symmetric.

- This mechanism itself is a general PBH formation mechanism.

- The mechanism in a model with two real singlet scalars can accommodate microlensing events by OGLE and Subaru HSC.