“Custodial Naturalness”

Electroweak scale hierarchy from conformal and custodial symmetry

Andreas Trautner

based on:

PLB 861(2025) and arXiv:2502.09699

w/ **Thede de Boer** and Manfred Lindner

HECA Seminar
NCBJ, Warsaw

04.03.25

Outline

- Hierarchy problem
- General idea of "Custodial Naturalness"
- Minimal model
- Numerical analysis, experimental constraints and predictions
- Extensions and embeddings
- Conclusions

Disclaimer: For this talk in 4D, scale invariance $\sim$ conformal invariance.

Electroweak scale hierarchy problem

Not a problem *in* the Standard Model (SM). [Bardeen '95]

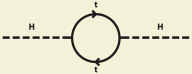
However, in presence of heavy scales Λ_{high}, it remains puzzling that

(see, however, [Mooij, Shaposhnikov '21], [K.-S. Choi '24])

$$m_h^2 \propto \Lambda_{\text{high}}^2 \,,$$

which, e.g. in case $\Lambda_{\text{high}} \sim M_{\text{Pl}}$, is not supported by observation.

Symmetry based solutions:



- Supersymmetry.
- Composite Higgs ($h =$ pNGB of some new strongly coupled sector).

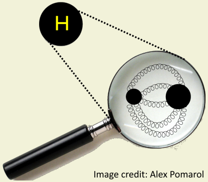


Image credit: Alex Pomarol

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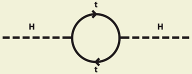

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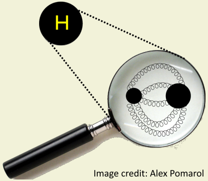

Image credit: Alex Pomarol

However, neither is Nature close-to supersymmetric,
nor do Higgs measurements hint at compositeness.
Also: No top-partners observed.

But: SM *is* close to **scale invariant**, *explicitly* broken only by $\mu_H \, (\sim m_h \sim v_{\text{EW}})_{\text{SM}}$.

Conformal “solution”

- The SM exhibits classical scale symmetry, only explicitly broken by $\mu_H^2\,|H|^2$.
- Quantum corrections *can* spontaneously generate $\mu_H^2 \sim \Lambda_{\mathrm{CW}}^2 \sim \mathrm{e}^{-\frac{\lambda}{g^4}}\Lambda_{\mathrm{high}}^2$,
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New here:

Higgs as pNGB of spontaneously broken **custodial symmetry** avoids this problem.

✓ Technically natural suppression of EW scale.

✓ Only elementary fields, no compositeness, all perturbative.

✓ No top partners, marginal top Yukawa like in SM.

"Custodial Naturalness" – General Idea

Assumptions:

1. Classical scale invariance.
2. New complex scalar Φ + new $\mathrm{U}(1)_\mathrm{X}$ gauge symmetry. $\mathrm{SU}(3)_\mathrm{c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}\times\mathrm{U}(1)_\mathrm{X}$
3. High-scale $\mathrm{SO}(6)$ **custodial** symmetry of scalar potential:

$$\Rightarrow \qquad V(H,\Phi) \;=\; \lambda\left(|H|^2+|\Phi|^2\right)^2 \qquad \text{at } \mu=\Lambda_\mathrm{high}\equiv M_\mathrm{Pl}\,.$$

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Both, scale invariance + $\mathrm{SO}(6)$ are broken by quantum effects.

- ***If*** $\mathrm{SO}(6)$ *were* classically exact $\rightarrow$ [Coleman, Weinberg ’73] $\rightarrow$ VEVs $\langle\Phi\rangle\&\langle H\rangle$.

$\Rightarrow$ $\mathrm{SO}(6)\xrightarrow{\langle\mathbf{6}\rangle}\mathrm{SO}(5)$: massive dilaton + 4 *would-be* NGBs + massless NGB “h”.

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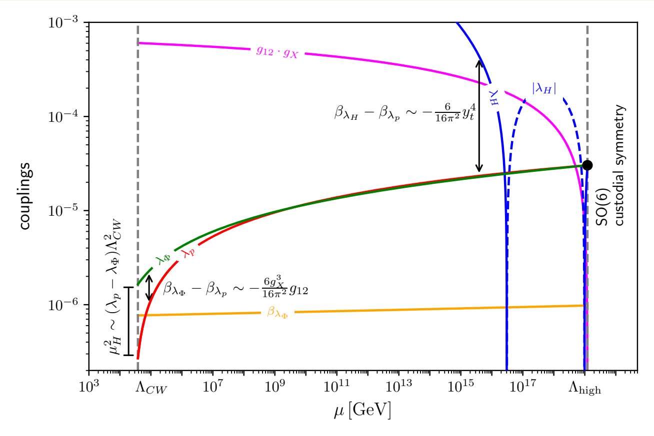
- Realistically: $\mathrm{SO}(6)$ explicitly broken by: y_t, g_Y & g_X, g_{12}, . . . , e.g. y_new

$\Rightarrow$ $\mathrm{SO}(6) \xrightarrow{\langle\mathbf{6}\rangle} \mathrm{SO}(5)$: massive dilaton + 4 *would-be* NGBs + massive pNGB “h”.

General Idea – RGE evolution is key!

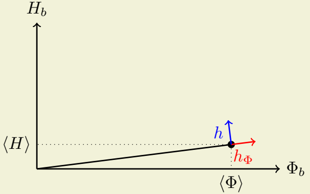
below M_{Pl} : $\quad V_{\text{tree}}(H,\Phi) \;=\; \lambda_H|H|^4 + 2\,\lambda_p|\Phi|^2|H|^2 + \lambda_\Phi|\Phi|^4\;.$



Actual running for a benchmark point. Dashed=negative.
β_i: Beta function coefficients.

Dominant breaking of
Custodial Symmetry (C.S.):
Top Yukawa coupling $\boldsymbol{y_t}$

$\lambda_H \gg \lambda_{p,\Phi} \;\Rightarrow\; \langle H\rangle \ll \langle\Phi\rangle$



Crucially: $\mu_H^2 \sim [\lambda_p-\lambda_\Phi]\; v_\Phi^2$

General Idea – Masses and EW scale

Splitting $\lambda_p - \lambda_\Phi$ **requires** additional (BSM) source of C.S. breaking!*

$$\beta_{\lambda_p} - \beta_{\lambda_\Phi} \simeq \frac{1}{16\pi^2}\lambda_p \left[-\frac{9}{2}g_L^2 - \frac{3}{2}g_Y^2 + 12\lambda_H + 6y_t^2\right]_{\mathrm{SM}} + \frac{g_{12}g_X^2}{16\pi^2}\left[6g_X + \frac{3}{2}g_{12}\right]_{\mathrm{BSM}}$$

$\curvearrowright$ Minimal possibility: $\mathrm{U(1)_X} - \mathrm{U(1)_Y}$ gauge kinetic mixing g_{12}.

Masses of physical real scalars $h_\Phi \subset \Phi$ and $h \subset H$: $\langle\Phi\rangle = \frac{v_\Phi}{\sqrt{2}}, \langle H\rangle = \frac{v_h}{\sqrt{2}}$

Dilaton: $$m_{h_\Phi}^2 \approx \beta_{\lambda_\Phi} v_\Phi^2 \approx \frac{3\,g_X^4}{8\pi^2} v_\Phi^2$$

pNGB Higgs: $$m_h^2 \approx 2\left[\lambda_\Phi\left(1 + \frac{g_{12}}{2\,g_X}\right)^2 - \lambda_p\right] v_\Phi^2 \,.$$

- EW scale VEV gets to keep the SM relation $v_H^2 \approx \dfrac{m_h^2}{2\lambda_H}$

$\Rightarrow$ The **EW scale is *custodially* suppressed** compared to the intermediate scale v_Φ of spontaneous scale and custodial symmetry violation.

Minimal Model

Field	#Gens.	$\mathrm{SU(3)_c \times SU(2)_L \times U(1)_Y}$	$\mathrm{U(1)_X}$	$\mathrm{U(1)_{B-L}}$
Q	3	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	$-\frac{2}{3}$	$+\frac{1}{3}$
u_R	3	$(\mathbf{3}, \mathbf{1}, +\frac{2}{3})$	$+\frac{1}{3}$	$+\frac{1}{3}$
d_R	3	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	$-\frac{5}{3}$	$+\frac{1}{3}$
L	3	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$+2$	-1
e_R	3	$(\mathbf{1}, \mathbf{1}, -1)$	$+1$	-1
ν_R	3	$(\mathbf{1}, \mathbf{1}, 0)$	$+3$	-1
H	1	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	$+1$	0
Φ	1	$(\mathbf{1}, \mathbf{1}, 0)$	$+1$	$q_\Phi^{\mathrm{B-L}} = -\frac{1}{3}$

$$Q^{(\mathrm{X})} \equiv 2\, Q^{(\mathrm{Y})} + \frac{1}{q_\Phi^{\mathrm{B-L}}}\, Q^{(\mathrm{B-L})}$$

- The only free parameter of the charge assignment is $q_\Phi^{\mathrm{B-L}}$.
- Constrained to $\frac{1}{3} \lesssim |q_\Phi^{\mathrm{B-L}}| \lesssim \frac{5}{11}$; special value: $q_\Phi^{\mathrm{B-L}} = -\frac{16}{41}$. **Let us fix** $q_\Phi^{\mathrm{B-L}} = -\frac{1}{3}$.

Note: Our model is very similar to "classical conformal extension of minimal $B-L$ model", but $q_\Phi^{\mathrm{B-L}} \neq -2$.
[Iso, Okada, Orikasa '09]

Numerical analysis

- SM parameters $G_{\rm F}$, $m_h \longleftrightarrow$ parameters λ, g_X ($@\Lambda_{\rm high} \sim M_{\rm Pl}$).
- Remaining free parameter: g_{12}. Can fix $g_{12}|_{M_{\rm Pl}} = 0 \quad \Leftrightarrow \quad$ C.S. fixes all d.o.f.'s.

Minimal model has the same number of parameters as the SM!

$\rightarrow$ Properties of Z' and h_Φ are predictions of the model.

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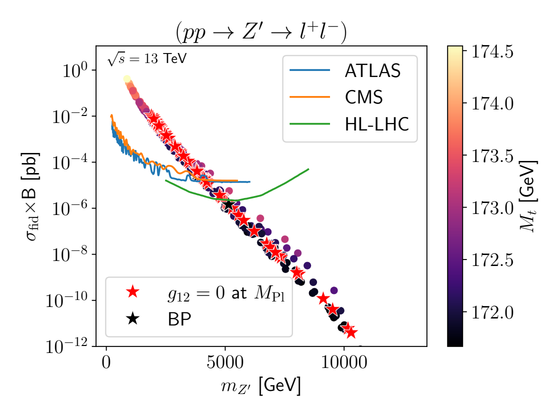
Parameter scan

- Impose $\mathrm{SO}(6)$ symmetric BC's @M_Pl: $\lambda_{H,\Phi,p}|_{M_\mathrm{Pl}} = \lambda|_{M_\mathrm{Pl}}$ and $g_{12}|_{M_\mathrm{Pl}} = 0$.
- 2-loop running with `PyR@TE`. [Sartore, Schienbein '21]
- Iteratively determine intermediate scale Φ_0, match to SM at $\mu_0 \sim \mathcal{O}(g_X \Phi_0)$.
- Numerically minimize 1-loop V_eff (at μ_0), compute v_Φ and v_H, m_{h_Φ}, m_h, $\lambda_{H,\Phi,p}$, match to 1-loop $V_\mathrm{eff}^\mathrm{SM}$ (+dilaton hidden scalar, corrections negligible).
- From μ_0 down to m_t 2-loop running.
- Require $v_H^\mathrm{exp} = 246.2 \pm 0.1\,\mathrm{GeV}$, as well as g_L, g_Y, g_3 and y_t within SM errors.
- Low scale new couplings g_X, g_{12} and masses $m_{Z'}$, m_{h_Φ} are predictions.

Phenomenological constraints



- $Z' \to l^+l^-$ resonance searches require $m_{Z'} \gtrsim 4\,\mathrm{TeV}$. (di-jets are weaker)
- EW precision: Additional custodial breaking shifts m_Z,

$$\Delta m_Z \propto -m_Z \langle H\rangle^2/(2\langle\Phi\rangle^2)\ .$$

  Constraint: $\langle\Phi\rangle \gtrsim 18\,\mathrm{TeV}$, weaker than direct Z' searches.

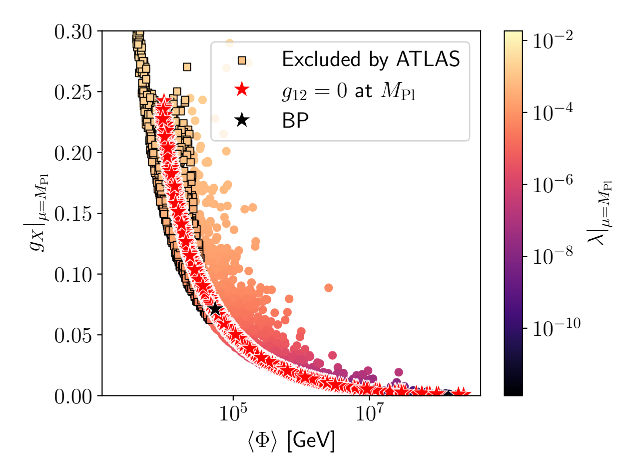
- Dilaton-higgs mixing:

$$\mathcal{O}_{h_\Phi} \approx \sin\theta \times \mathcal{O}^{\rm SM}_{h\to h_\Phi}\ .$$

  For $m_{h_\Phi} \sim 75\,\mathrm{GeV}$, $\sin\theta \lesssim 10^{-1}$ is a-OK.
  (typical values for us are BP: $\sin\theta \sim 10^{-2.5}$)

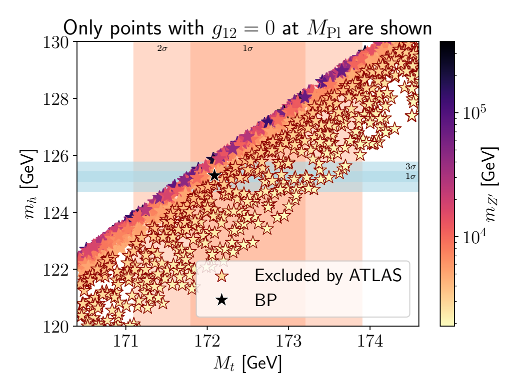
- Neglect dilaton-gauge-gauge coupling from trace anomaly, suppressed by $\frac{v_h}{v_\Phi}$.

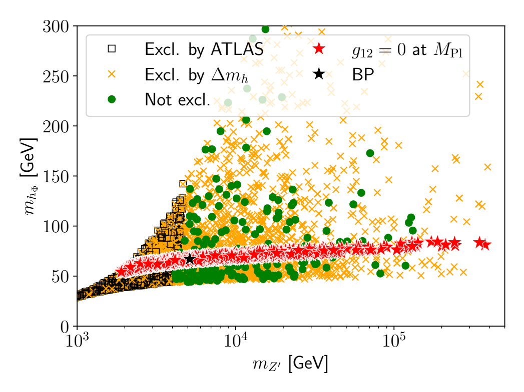
Parameter space $(q_\Phi = -\frac{1}{3})$



Parameters at $\mu = M_{\rm Pl}$. All points shown reproduce the correct EW scale.
New scale $\langle\Phi\rangle = v_\Phi/\sqrt{2}$ is prediction. (m_h, M_t not imposed as constraint).

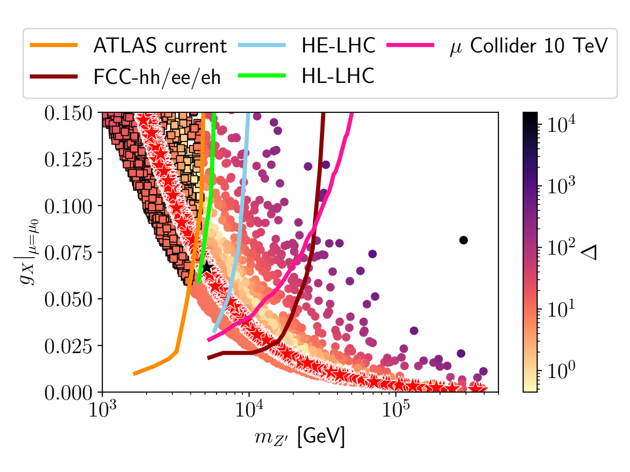
Reproductions and predictions $(q_\Phi = -\frac{1}{3})$





All points shown reproduce the correct EW scale. M_t: top pole mass.

Fine tuning and Future collider projections $(q_\Phi = -\frac{1}{3})$



Fine tuning:

$$\Delta \ := \ \max_{g_i} \left| \frac{\partial \ \ln \frac{\langle H \rangle}{\langle \Phi \rangle}}{\partial \ln g_i} \right| \ .$$

Barbieri-Giudice measure. [Barbieri, Giudice '88]

The choice of $\langle H \rangle / \langle \Phi \rangle$ automatically subtracts the shared sensitivity of VEVs to variation of g_i. [Anderson, Castano '95]

Red stars: $g_{12}|_{M_{\rm Pl}} = 0$.

Black star: benchmark point.

Projections are for hypercharge universal Z' from [R.K. Ellis et al. '20]

Prime target: Z' at FC, Dilaton production(+displaced dec.) at Higgs factories.

Extensions of most minimal model

Minimal model portals: $|\Phi|^2|H|^2$ and $X^{\mu\nu}Y_{\mu\nu}$.

In extensions also neutrino portal and new Yukawa portals $y_{\rm new}$.

Additional fermions can: (already known from $B-L$ model.)

- Provide ingredients for neutrino mass generation, [Iso, Okada, Orikasa '09] [Foot, Kobakhidze, McDonald, Volkas '07]
- Be part of the dark matter, [S. Okada '18]
- "Cure" SM vacuum instability. [(Das), Oda, Okada, Takahashi '15('16)]

"Custodial Naturalness" is reasonably stable under variation of boundary conditions, charge assignments, addition of extra particles.

[de Boer, Lindner, AT 2502.09699]

Next-to-Minimal Models: (M1) $q_\Phi \equiv q_\Phi^{\mathrm{B-L}} \neq -\frac{1}{3}$

Field	#Gens.	$\mathrm{SU}(3)_\mathrm{c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$	$\mathrm{U}(1)_\mathrm{X}$	$\mathrm{U}(1)_{\mathrm{B-L}}$
(M2) Minimal set of additional fermions with Φ Yukawa couplings				
ψ_L	1	$(\mathbf{1},\mathbf{1},0)$	$-(\frac{1}{q_\Phi}+1)$	$-(1+q_\Phi)$
ψ_R	1	$(\mathbf{1},\mathbf{1},0)$	$-(\frac{1}{q_\Phi}+1)$	$-(1+q_\Phi)$
(M3) Minimal additional set of fermions that allow for DM				
ψ_L	1	$(\mathbf{1},\mathbf{1},0)$	$\frac{p}{q_\Phi}$	p
ψ_R	1	$(\mathbf{1},\mathbf{1},0)$	$\frac{p}{q_\Phi}+1$	$p+q_\Phi$
ψ'_L	1	$(\mathbf{1},\mathbf{1},0)$	$\frac{p}{q_\Phi}+1$	$p+q_\Phi$
ψ'_R	1	$(\mathbf{1},\mathbf{1},0)$	$\frac{p}{q_\Phi}$	p

Designed such as to allow new Φ-Yukawa couplings

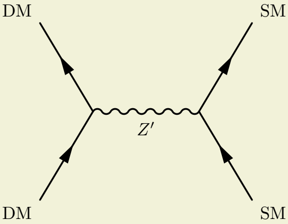
$$\mathcal{L}_\mathrm{Yuk} \supset y_\psi\,\overline{\psi}_L\Phi^\dagger\nu_R^\alpha \quad \text{(M2)} \qquad \text{or} \qquad y_\psi\,\overline{\psi}_L\Phi^\dagger\psi_R + y_{\psi'}\,\overline{\psi'}_L\Phi\psi'_R \quad \text{(M3)}$$

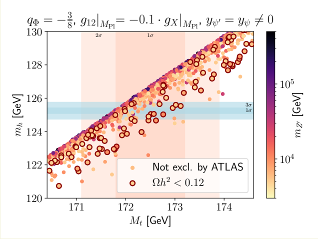
Mechanism for ν-mass generation (M2), **or** multi-component fermion Dark Matter (M3).
Additional contribution to custodial symmetry breaking:

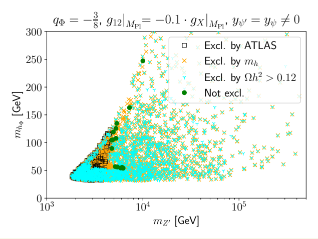
$$\beta_{\lambda_p} - \beta_{\lambda_\Phi}\Big|_{y_\psi} \simeq \frac{\sum_k 2y_{\psi_k}^4}{16\pi^2}\,.$$

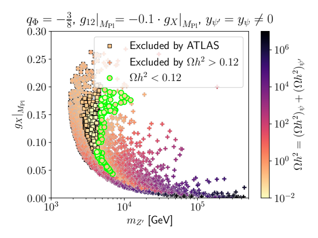
Dark Matter model (M3), $(q_\Phi = -\frac{3}{8})$

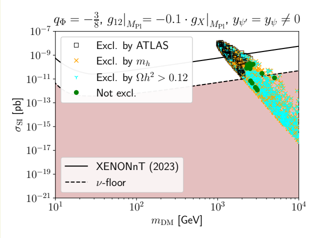
Two-component DM: new VL fermions $\psi_{L,R}$, $\psi'_{L,R}$.

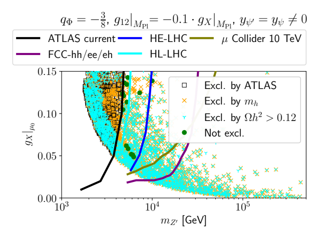










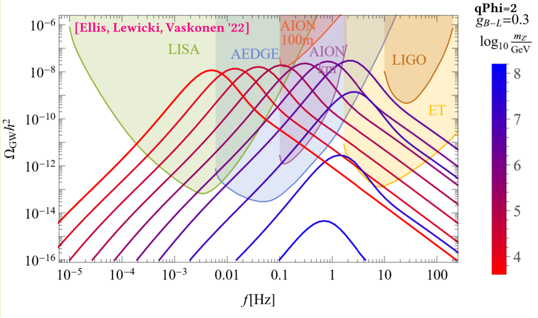


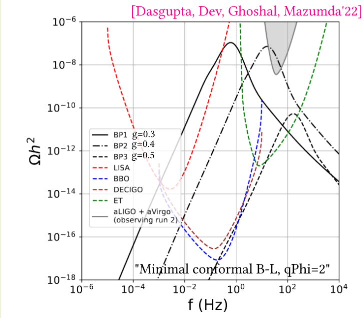
Most flexible scenario, g_{12} *and* y_ψ's, still predictive and very constrained.

Requires $m_{Z'} \approx 2m_\psi \approx 2m_{\psi'}$.

Gravitational wave signals?

- We have ignored finite-T effects so far. This is yet to be done.
- CW transition is known to be first order $\rightarrow$ Gravitational wave signals.
  see e.g. [Litim, Wetterich, Tetradis '97], [Dasgupta, Dev, Ghoshal, Mazumdar '22],[Huang, Xie '22]
- In fact, the "minimal conformal $B-L$ model" is prototype for **strong supercooling** $\rightarrow$ strong GW signal from bubble collisions. see e.g. [Ellis,Lewicki,Vaskonen'20]





Quantitative predictions for our specific case have yet to be worked out!

Embeddings and variations of Custodial Naturalness

- Further reduction of parameters:
  Embeddings into a unified group $G_{\mathrm{cust.}} \subset G_{\mathrm{GUT}}$ would allow to constrain $q^{\Phi}_{\mathrm{B-L}}$ and compute the size of gauge-kinetic mixing g_{12}.
- Flavor dependent Fermion charge assignments?
- Custodial symmetry could originate from UV fixed point $\leftrightarrow$ quantum criticality.
  e.g. [Litim, Sannino '14]
- *There is a possibility to realize large enough splitting of $\lambda_p - \lambda_\Phi$ without new sources of CS breaking; this requires $\mu \approx 10^{11}\mathrm{GeV}$.
- Link dynamical scale generation to inflation, with Φ as inflaton.
  e.g. [Kubo et al. '18], [Kubo, AT et al. '20]
- …

Conclusions

- Classical scale invariance + extended custodial symmetry, here $\mathrm{SO}(6)$

$\Rightarrow$ New mechanism to explain large scale separation and little hierarchy problem.

- Minimal model: Φ + $\mathrm{U}(1)_\mathrm{X}$ gauge: same number of parameters as the SM.
- Predicts light scalar dilaton $m_\Phi \sim 75\,\mathrm{GeV}$ + Z' at $4 - 100\,\mathrm{TeV}$.
- Top mass at lower end of currently allowed 1σ region.
- Predictions reasonably stable under extensions, e.g. m_ν or particle DM.
- Perfect model to motivate new colliders + Higgs factory + GR waves.
- Many extensions and details to explore, e.g. extension to flavor.

Thank You!

Image credit: kidsinthecity.pl

Backup slides

Details of the potential and matching

Effective potential for background fields H_b and Φ_b @1-loop $\overline{\text{MS}}$:

$(-1)^{2s_i} \equiv {}_{(-)}^{+}1$ for bosons(fermions), $n_i \equiv \#\text{d.o.f}$
$C_i = \frac{5}{6}\left(\frac{3}{2}\right)$ for vector bosons(scalars/fermions).

$$V_{\text{eff}} = V_{\text{tree}} + \sum_i \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu^2}\right) - C_i\right] .$$

Two different *analytical* expansions: First

$$V_{\text{EFT}}(H_b) := V_{\text{eff}}\left(H_b, \tilde{\Phi}(H_b)\right) , \qquad \text{with} \qquad \left.\frac{\partial V_{\text{eff}}}{\partial \Phi_b}\right|_{\Phi_b = \tilde{\Phi}(H_b)} = 0 .$$

Using $\Phi_0 := \Phi(H_b/\Phi_b = 0)$, we expand V_{EFT} in $H_b \ll \Phi_0$, $\curvearrowright$ RG-scale independent expression

$$V_{\text{EFT}} \approx 2\left[\lambda_p - \left(1 + \frac{g_{12}}{2\, g_X}\right)^2 \lambda_\Phi\right] \Phi_0^2 H_b^2 + \frac{\lambda_p \lambda_H}{16\pi^2}[...] .$$

This expression illustrates the origin of the Higgs mass and EW scale suppression.

Alternatively, take $\mu = \mu_0 := \sqrt{2} g_X \Phi_0 \mathrm{e}^{-1/6} \sim \langle\Phi\rangle$ and “t Hooft-like” expansion $\frac{\lambda_p}{\lambda_H} \sim \frac{H_b^2}{\Phi_0^2} \sim \epsilon^2 \to 0$,

$$V_{\text{EFT}} = -\frac{6\, g_X^4}{64\pi^2}\Phi_0^4 + 2\,\lambda_p \Phi_0^2 H_b^2 + \lambda_H H_b^4 + \sum_{i=\text{SM}} \frac{n_i(-1)^{2s_i}}{64\pi^2} m_{i,\text{eff}}^4 \left[\ln\left(\frac{m_{i,\text{eff}}^2}{\mu_0^2}\right) - C_i\right] .$$

This expression facilitates matching to the SM at scale μ_0.

Details of the potential and matching II

For all practical purpose the usual CW relation holds:

$$\Phi_0^2 \approx \exp\left\{-\frac{16\pi^2\lambda_\Phi}{3g_X^4} - \ln(2g_X^2) + \frac{1}{3} + \ldots\right\}\mu^2 \ .$$

Analytically we can use $H_b \ll \tilde{\Phi}(0) := \Phi_0$ and the leading order expression for Φ_0 reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) \ = \ -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right] + 4\,\lambda_p^2\left(\ln 2\lambda_p - 1\right)\right\}}{3\,q_\Phi^4 g_X^4 + 4\,\lambda_p^2} \ .$$

Alternatively, we can use the ϵ expansion, and Φ_0 at $\mathcal{O}(\epsilon^0)$ reads

$$\frac{1}{16\pi^2}\ln\left(\frac{\Phi_0^2}{\mu^2}\right) \ = \ -\frac{\lambda_\Phi + \frac{1}{16\pi^2}\left\{q_\Phi^4 g_X^4\left[3\ln\left(2q_\Phi^2 g_X^2\right) - 1\right]\right\}}{3\,q_\Phi^4 g_X^4} \ .$$

This is an example for the difference between the two expansion schemes. Note that our quantitative analysis is not based on any of these expansions but uses a fully numerical minimization of the effective potential to compute $\langle\Phi\rangle$ and $\langle H\rangle$.

Integrating out scalar in non-conformal model

Consider a simple two complex scalar system with a potential given by

$$V = -m_H^2|H|^2 - m_\Phi^2|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_p|H|^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4.$$

For $m_\Phi^2 > 0$ and $-m_H^2 + m_\Phi^2 \frac{\lambda_p}{\lambda_\Phi} > 0$, this potential has a minimum at $\langle \Phi \rangle := \frac{v_\Phi}{\sqrt{2}} = \sqrt{\frac{m_\Phi^2}{\lambda_\Phi}}, \langle H \rangle = 0$.

Integrating out the heavy field Φ at tree level, we find the low energy potential

$$\begin{aligned} V_{\text{EFT}} &= \left(-m_H^2 + \lambda_p \frac{v_\Phi^2}{2}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4 \\ &= \left(-m_H^2 + \lambda_p \frac{m_\Phi^2}{\lambda_\Phi}\right)|H|^2 + \frac{1}{2}\left(\lambda_H + \frac{\lambda_p^2}{\lambda_\Phi}\right)|H|^4. \end{aligned}$$

The light field is massless at tree level if $\lambda_\Phi\, m_H^2 = \lambda_p\, m_\Phi^2$.

A special point fulfilling this condition is $m_H^2 = m_\Phi^2 := m^2$ and $\lambda_p = \lambda_\Phi := \lambda$. At this point the original potential is given by

$$V = -m^2\left(|H|^2 + |\Phi|^2\right) + \frac{\lambda}{2}\left(|H|^2 + |\Phi|^2\right)^2 + \frac{\lambda_H - \lambda}{2}|H|^4$$

This potential is symmetric up to the quartic term of H which can violate the symmetry badly without affecting the light mass term at tree level.

Benchmark point 1 (BP)

μ [GeV]	g_X	g_{12}	λ_H	λ_p	λ_Φ	y_t	m_{h_Φ} [GeV]	$m_{Z'}$ [GeV]	m_h [GeV]	v_H [GeV]
$1.2 \cdot 10^{19}$	0.0713	0.	$\lambda_H = \lambda_p = \lambda_\Phi = 3.3030 \cdot 10^{-5}$			0.377	-	-	-	-
4353	0.0668	0.0093	**0.084**	$-1.6 \cdot 10^{-6}$	$-2.5 \cdot 10^{-11}$	**0.795**	67.0	5143	**132.0**	**263.0**
172	-	-	0.13	-	-	0.930	-	-	125.3	246.1

Table: Input parameters of an example benchmark point (BP) at the high scale (top) and corresponding predictions at the matching scale μ_0 (middle) and M_t (bottom). At μ_0 the bold parameters also correspond to the parameters of the one-loop SM effective potential. The numerical result for the VEV of Φ is $\langle \Phi \rangle = v_\Phi/\sqrt{2} = 54407\,\mathrm{GeV}$.

One-loop RGE's

Neglect all Yukawas besides y_t and take general $\mathrm{U}(1)_\mathrm{X}$ charges $q_{H,\Phi}$.

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\Bigg[+\frac{3}{2}\left(\left(\frac{g_Y^2}{2}+\frac{g_L^2}{2}\right)+2\left(q_H g_X+\frac{g_{12}}{2}\right)^2\right)^2+\frac{6}{8}g_L^4-6y_t^4$$
$$+24\lambda_H^2+4\lambda_p^2+\lambda_H\left(12y_t^2-3g_Y^2-12\left(q_H g_X+\frac{g_{12}}{2}\right)^2-9g_L^2\right)\Bigg],$$

$$\beta_{\lambda_\Phi} = \frac{1}{16\pi^2}\left(+6q_\Phi^4 g_X^4+20\lambda_\Phi^2+8\lambda_p^2-12\lambda_\Phi q_\Phi^2 g_X^2\right),$$

$$\beta_{\lambda_p} = \frac{1}{16\pi^2}\Bigg[+6q_\Phi^2 g_X^2\left(q_H g_X+\frac{g_{12}}{2}\right)^2+8\lambda_p^2$$
$$+\lambda_p\left(8\lambda_\Phi+12\lambda_H-\frac{3}{2}g_Y^2-6q_\Phi^2 g_X^2-6\left(q_H g_X+\frac{g_{12}}{2}\right)^2-\frac{9}{2}g_L^2+6y_t^2\right)\Bigg],$$

$$\beta_{g_{12}} = \frac{1}{16\pi^2}\left[-\frac{14}{3}g_X g_Y^2-\frac{14}{3}g_X g_{12}^2+\frac{41}{3}g_Y^2 g_{12}+\frac{179}{3}g_X^2 g_{12}+\frac{41}{6}g_{12}^3\right].$$

The dominant splitting of $\lambda_\Phi-\lambda_p$ via running (for benchmark charges) is given by

$$\beta_{\lambda_\Phi}-\beta_{\lambda_p} = -\frac{6\,g_{12}\,g_X^2}{16\pi^2}\left(g_X+\frac{g_{12}}{4}\right)-\frac{\lambda_p}{16\pi^2}\left[6y_t^2-\frac{9}{2}g_L^2-\frac{3}{2}g_Y^2+12(\lambda_H-\lambda_p)\right]+\dots,$$

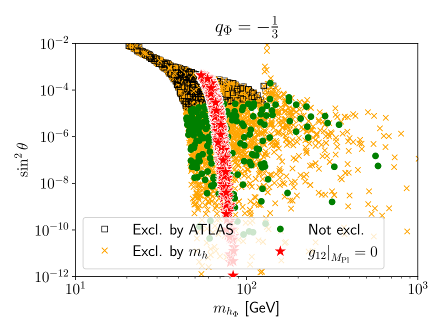
We do the numerical running with the full two-loop beta functions computed with PyR@TE.

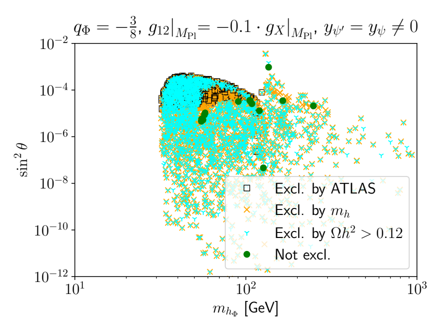
Higgs-dilaton mixing

A crude analytic expression for the Higgs-dilaton mixing angle is

$$\tan\theta \approx \frac{2\left[\lambda_p - \left(1+\frac{g_{12}}{2g_X}\right)^2\left(\lambda_\Phi - \frac{3g_X^4}{16\pi^2}\right)\right] v_H v_\Phi}{m_h^2 - m_{h_\Phi}^2}\,.$$

Note: We use a fully numerical evaluation of all masses and mixings for our analysis which also confirms the analytic approximations.





Gauge-kinetic mixing

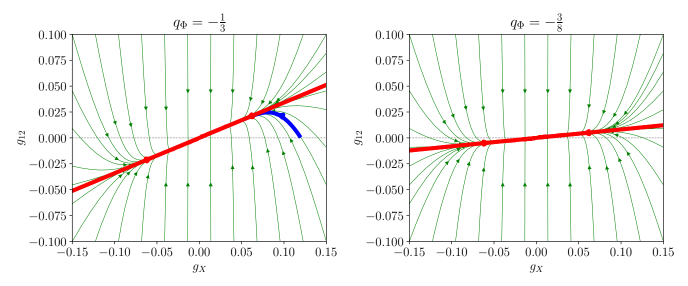
Gauge kinetic mixing parameter in $B-L$ basis, $\tilde{g} := \varepsilon g_Y/\sqrt{1-\varepsilon^2}$ with $\varepsilon F^{\mu\nu}F'_{\mu\nu}$. The $\mathrm{U}(1)$ part of the gauge covariant derivative acting on generic field ϕ is given by

$$\left[\partial_\mu + i\left(Q^{(\mathrm{Y})}, Q^{(\mathrm{B-L})}\right)\begin{pmatrix} g_Y & \tilde{g} \\ 0 & g_{\mathrm{B-L}} \end{pmatrix}\begin{pmatrix} A_\mu^{(\mathrm{Y})} \\ A_\mu^{(\mathrm{X})} \end{pmatrix}\right]\phi.$$

$A_\mu^{(\mathrm{Y})}$ and $A_\mu^{(\mathrm{X})}$ are the $\mathrm{U}(1)$ gauge fields. Rewriting this in terms of $\mathrm{U}(1)_\mathrm{X}$ charge $Q^{(\mathrm{X})}$:

$$\left[\partial_\mu + i\left(Q^{(\mathrm{Y})}, Q^{(\mathrm{X})}\right)\begin{pmatrix} g_Y & \tilde{g} - 2q_\Phi g_{\mathrm{B-L}} \\ 0 & q_\Phi g_{\mathrm{B-L}} \end{pmatrix}\begin{pmatrix} A_\mu^{(\mathrm{Y})} \\ A_\mu^{(\mathrm{X})} \end{pmatrix}\right]\phi.$$

Hence, we define $g_{12} := \tilde{g} - 2q_\Phi g_{\mathrm{B-L}}$ and the gauge coupling $g_X := q_\Phi g_{\mathrm{B-L}}$. Running as function of scale:



Neutrino mass generation (M2)

Minimal extension with new Φ-Yukawa interactions, $\mathcal{L}_{\text{Yuk}} \supset y_\psi\,\overline{\psi}_L \Phi^\dagger \nu_R^\alpha$ (M2).
After SSB, Dirac mass terms:

$$\mathcal{L}_{\text{mass}} \supset \begin{pmatrix} \overline{\nu}_L^\alpha & \overline{\psi}_L \end{pmatrix} \begin{pmatrix} y_\nu^{\alpha\beta} \frac{v_H}{\sqrt{2}} & 0 \\ y_\psi^\beta \frac{v_\Phi}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \nu_R^\beta \\ \psi_R \end{pmatrix} + \text{h.c.} \equiv \begin{pmatrix} \overline{\nu}_L^\alpha & \overline{\psi}_L \end{pmatrix} M_N \begin{pmatrix} \nu_R^\beta \\ \psi_R \end{pmatrix} + \text{h.c.} \tag{1}$$

Majorana masses not generated due to unbroken (accidental) lepton number. Fermion masses^2 are eigenvalues of ($\alpha, \alpha' = 1, 2, 3$, sum over β implicit)

$$M_N M_N^\dagger = \begin{pmatrix} y_\nu^{\alpha\beta} (y_\nu^\dagger)^{\beta\alpha'} \frac{v_H^2}{2} & y_\nu^{\alpha\beta} (y_\psi^*)^\beta \frac{v_H v_\Phi}{2} \\ y_\psi^\beta (y_\nu^\dagger)^{\beta\alpha'} \frac{v_H v_\Phi}{2} & y_\psi^\beta (y_\psi^*)^\beta \frac{v_\Phi^2}{2} \end{pmatrix}.$$

The mass matrix has rank 3, implying the lightest active neutrino is predicted to be massless. There is a heavy sterile (w.r.t. SM gauge int's.) state with mass $\approx \sqrt{y_\psi^\beta y_\psi^\beta}\,\frac{v_\Phi}{\sqrt{2}}$ and field content

$$\Psi \sim \begin{pmatrix} \cos(\alpha_\psi)\psi_L + \sin(\alpha_\psi)\nu_L \\ \nu_R' \end{pmatrix}.$$

Mixing angle $\sin(\alpha_\psi) \approx y_\nu v_H/(y_\psi v_\Phi)$ is automatically suppressed ($v_H \ll v_\Phi$) and ν_R' is a linear combination of ν_R's not involving ψ_R.