How to calculate the terminal velocity of a bubble wall?

Based on:

Phys.Rev.D 108 (2023) 10, 103523,

JHEP 05 (2024) 011,

arXiv:2411.15094,

arXiv:2411.16580

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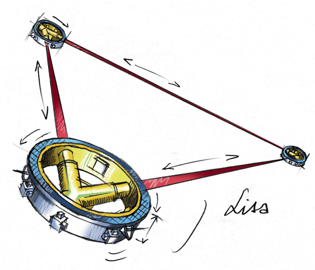
HECA (High Energy, Cosmology and Astro-Particle Physics)

Motivation

- Cosmological first order phase transitions (FOPT) are a common feature of particle physics models.
- FOPT are characterized by departure from thermal equilibrium (third Sakharov condition), thus may provide a proper environment for electroweak baryogenesis,
- Strong FOPT results in production of primordial gravitational waves. Observations of GW signal may give strong constraints on such models and will be possible soon with LISA.
- Evaluation of the bubble-wall velocity in the stationary state, which has a crucial impact both on amplitude of GW signal and baryon asymmetry production, remains to be one of the most problematic issues.



Cosmological first order phase transitions

Let us consider theory of scalar order parameter given by Lagrangian density:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi, T),$$

leading to the equation of motion in the form:

$$\frac{\partial^2 \phi}{\partial t^2} - \Delta\phi = \frac{dV}{d\phi}(\phi, T),$$

where T is temperature.

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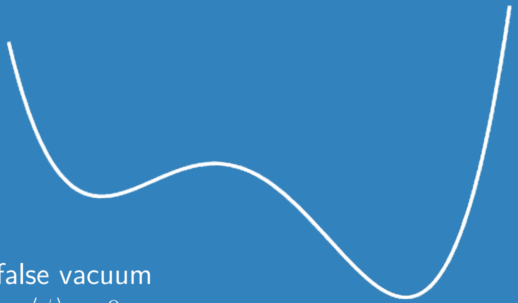
Space-time constant $\phi = v$ with v corresponding to extrema of the potential are solutions of the eom.



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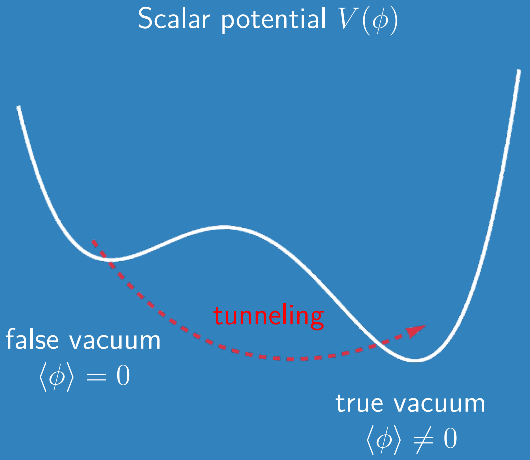

Tunnelling bubbles

Nucleation rate:

$$\Gamma(T) = A(T) \cdot \exp\left(-S\right)$$

For tunnelling in finite temperatures:

$$S = \frac{S_3}{T} \qquad A(T) = T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}}$$

where S_3 is an action of $\mathrm{O}(3)$-symmetric solution of the eom.

Nucleation condition:

$$\frac{\Gamma(T_n)}{H^4} \approx 1$$

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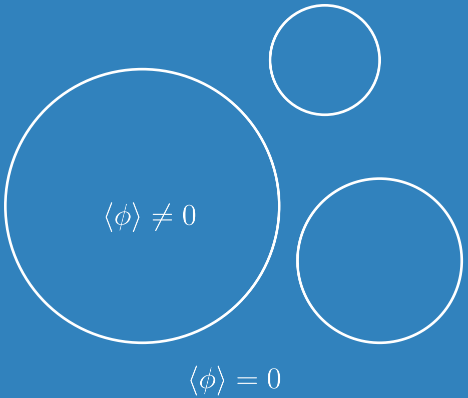
Nucleation of bubbles

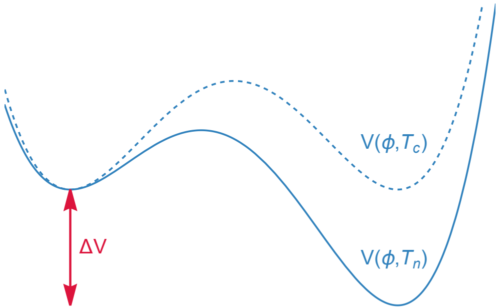


Phase transition parameters

- Critical and nucleation temperatures: T_c, T_n
- Level of supercooling: T_n/T_c
- Transition strength: $\alpha \sim \Delta V/\rho_r$
  In this work:

$$\alpha_{\bar{\theta}} = \frac{\Delta\bar{\theta}}{3w_s}\,, \qquad \text{with} \qquad \bar{\theta} = \epsilon - \frac{p}{c_b^2}$$

  with the speed of sound in the broken phase c_b and model-dependent energy e, pressure p and enthalpy w.
- Bubble-wall velocity: v_w



Dynamics of the steady state expansion

Integrated eom of the growing bubble:

$$\int \mathrm{d}z \frac{\mathrm{d}\phi}{\mathrm{d}z}\left(\Box\phi + \frac{\partial V_{\mathrm{eff}}}{\partial\phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3p}{(2\pi)^3 2E_i}\delta f_i(p,x)\right) = 0$$

$$\Big\downarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}z}\frac{\partial V_{\mathrm{eff}}}{\partial\phi} = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}z} - \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}$$

$$\Delta V_{\mathrm{eff}} = \boxed{\int \mathrm{d}z \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}} - \boxed{\sum_i \int \mathrm{d}\phi \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3p}{(2\pi)^3 2E_i}\delta f_i(p,x)}$$

driving force = hydrodynamic backreaction + non-equilibrium friction

- Boltzmann eq. + eom (different approaches: e.g fluid ansatz)
- LTE approximation (only hydrodynamic backreaction)
- Numerical simulations with effective friction η parametrizing δf

Bag model

Cosmic plasma coexist in two phases:

- Symmetric phase outside the bubble
- Broken phase inside the bubble

Equation of state

$$\epsilon_s = 3a_sT_s^4 + \theta_s$$
$$p_s = a_sT_s^4 - \theta_s$$

$$\epsilon_b = 3a_bT_b^4 + \theta_b$$
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Strength of the transition is defined as

$$\alpha = \left.\frac{\theta_s - \theta_b}{\epsilon_r}\right|_{T=T_n} .$$

Hydrodynamics of bag model

Energy-momentum tensor for the plasma is given by

$$T^{\mu\nu} = w u^\mu u^\nu + g^{\mu\nu} p$$

Conservation of $T^{\mu\nu}$ along the flow leads to

$$\partial_\mu (u^\mu w) - u_\mu \partial^\mu p = 0,$$

while its projection orthogonal to the flow (with $\bar{u}_\mu u^\mu = 0$ and $\bar{u}^2 = 1$) gives

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Hydrodynamic equation

$$2\frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1 \right] \partial_\xi v,$$

with Lorentz-transformed fluid velocity $\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$ and $\xi = r/t$.

Analytic methods for stationary profiles

Hydrodynamic equation

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Matching equations

1 $\omega_-\gamma_-^2 v_- = \omega_+\gamma_+^2 v_+$

2 $\omega_-\gamma_-^2 v_-^2 + p_- = \omega_+\gamma_+^2 v_+^2 + p_+$

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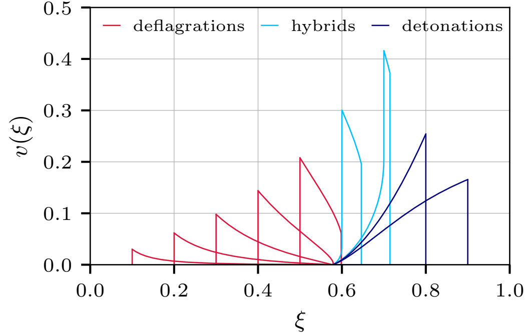
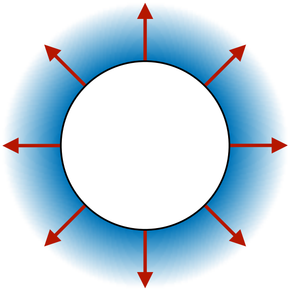
Solving hydrodynamic equation with proper boundary conditions and matching conditions (1 and 2), we get profiles $v(\xi)$ depending on ξ_w, α.



Bubble profiles

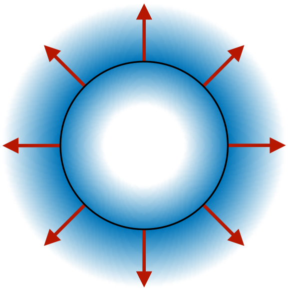
deflagration

$\xi_w < c_s$



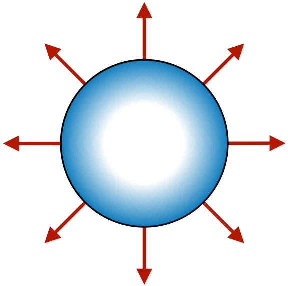
hybrid

$c_J > \xi_w > c_s$



detonation

$c_J < \xi_w$



Where Jouget velocity is $c_J = \frac{1}{\sqrt{3}} \frac{1+\sqrt{1+3\alpha^2+2\alpha}}{1+\alpha}$.

Scalar field coupled to perfect fluid

The system consists of

- relativistic perfect fluid
- real scalar field ϕ.

The field acquires temperature dependent effective potential V.

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Equation of state

$$\begin{aligned} \epsilon(\phi, T) &= 3aT^4 + V(\phi, T) - T\frac{\partial V}{\partial T} \\ p(\phi, T) &= aT^4 - V(\phi, T) \end{aligned}$$

with $a = (\pi^2/90)g_*$

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\end{aligned}
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Energy-momentum tensor

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}_{\text{field}} + T^{\mu\nu}_{\text{fluid}} \\
T^{\mu\nu}_{\text{field}} &= \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi\right) \\
T^{\mu\nu}_{\text{fluid}} &= wu^\mu u^\nu + g^{\mu\nu}p
\end{aligned}
$$

Equations of motion

Total energy-momentum tensor in conserved, but both contributions are not, due to the extra coupling term parametrized by effective friction η

$$\nabla_\mu T^{\mu\nu}_{\text{field}} = \frac{\partial V}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi = -\nabla_\mu T^{\mu\nu}_{\text{fluid}}.$$

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Equation of motion of scalar field

$$-\partial_t^2 \phi + \frac{1}{r^2}\partial_r(r^2 \partial_r \phi) - \frac{\partial V}{\partial \phi} = \eta\gamma(\partial_t \phi + v\partial_r \phi)$$

Equations of motion of plasma

$$\partial_t \tau + \frac{1}{r^2}\partial_r(r^2(\tau + p)v) = \frac{\partial V}{\partial \phi}\partial_t \phi + \eta\gamma(\partial_t \phi + v\partial_r \phi)\partial_t \phi,$$

$$\partial_t Z + \frac{1}{r^2}\partial_r\left(r^2 Z v\right) + \partial_r p = -\frac{\partial V}{\partial \phi}\partial_r \phi - \eta(\partial_t \phi + v\partial_r \phi)\partial_r \phi.$$

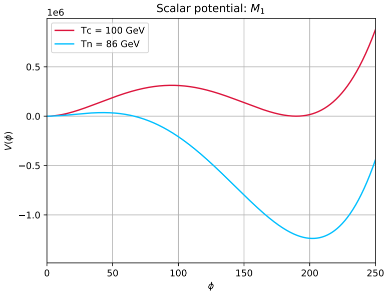
where $Z := w\gamma^2 v$ and $\tau := w\gamma^2 - p$

Benchmark potential

For the effective potential $V(\phi, T)$ we use a simple polynomial potential augmented with high temperature corrections.

Effective potential

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}\delta T\phi^3 + \frac{1}{4}\lambda\phi^4,$$



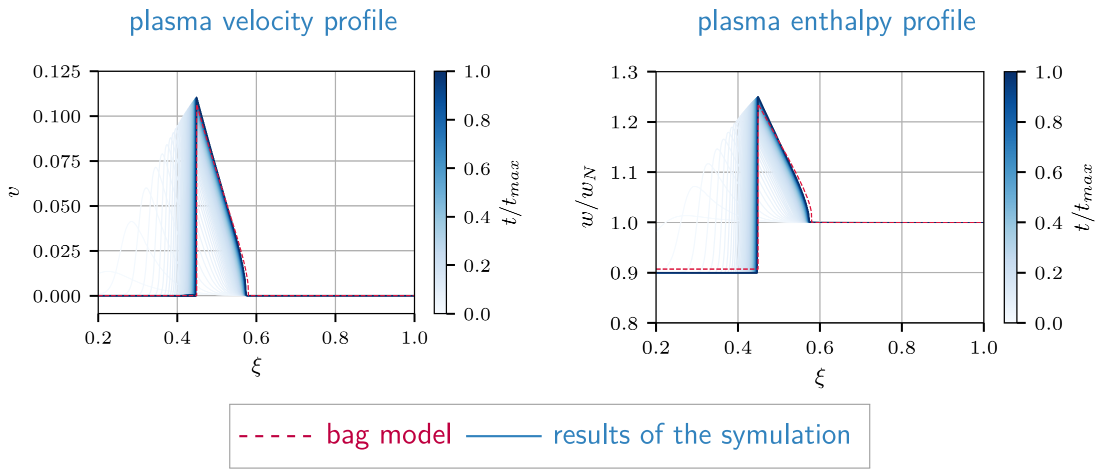
Model	T_0	γ	δ	λ	T_n	α
M_1	$\frac{100}{\sqrt{2}}$	$\frac{1}{18}$	$\frac{\sqrt{10}}{72}$	$\frac{10}{648}$	86	0.005
M_2	$\frac{100}{\sqrt{2}}$	$\frac{2}{18}$	$\frac{\sqrt{10}}{72}$	$\frac{5}{648}$	80	0.05

For each model we perform a scan over η, logarithmicly varying the friction in range:

$$\eta/T_c \in [0.01, 1]$$

Stationary states

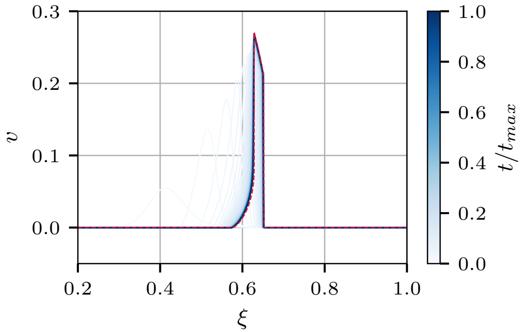
1. Deflagration $(\xi_w = 0.45)$



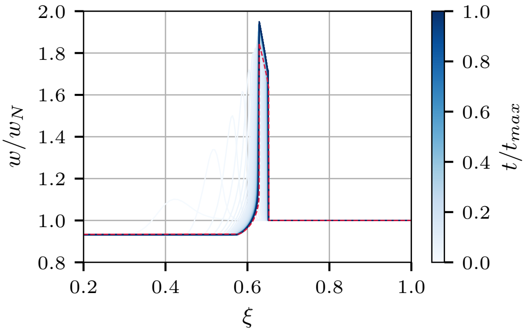
Stationary states

2. Hybrid $(\xi_w = 0.63)$

plasma velocity profile



plasma enthalpy profile

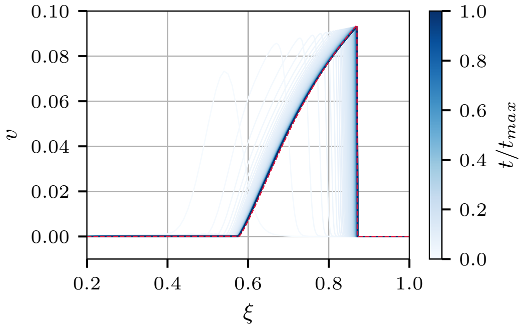


- - - - - bag model —— results of the symulation

Stationary states

3. Detonation ($\xi_w = 0.87$)

plasma velocity profile



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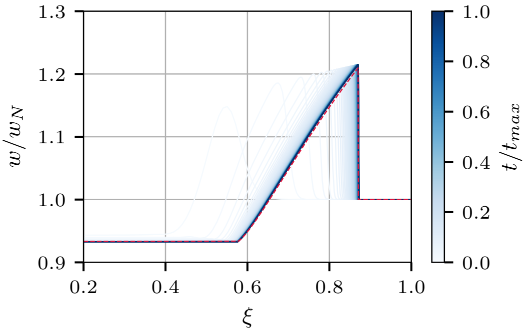
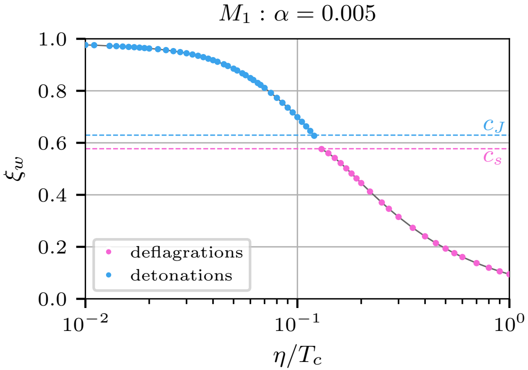


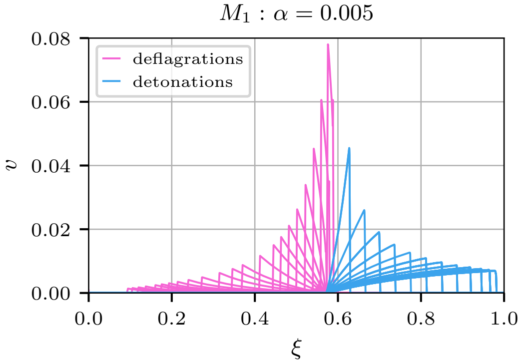
- - - - - bag model ——— results of the symulation

Scan over friction η

Model 1: $\alpha = 0.005$, $c_J \approx 0.63$

- Only deflagrations and detonations (no hybrids)
- There is a velocity gap for $\xi_w \in (0.57, 0.63)$

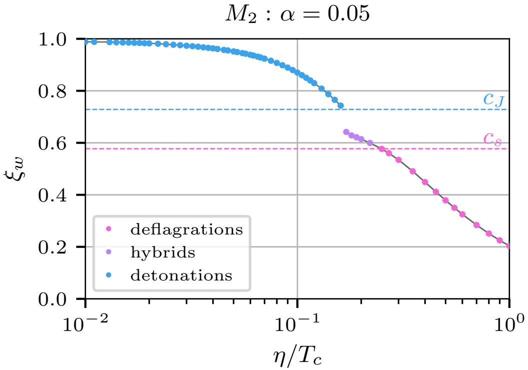


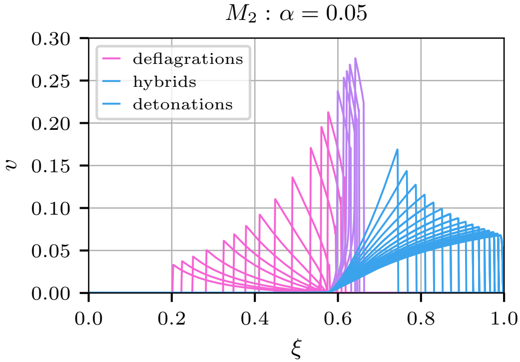


Scan over friction η

Model 2: $\alpha = 0.05,\ c_J \approx 0.73$

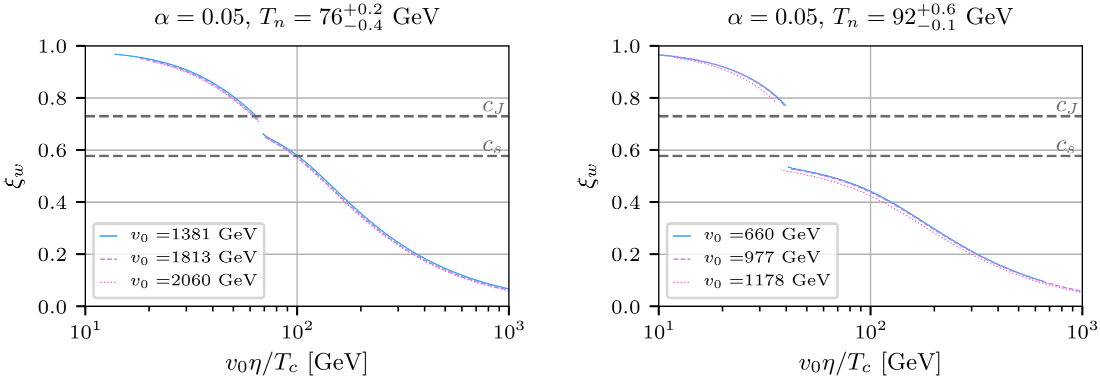
- All three kinds of solution are possible
- There is a velocity gap for $\xi_w \in (0.63, 0.74)$





Dependence on the vacuum expectation value

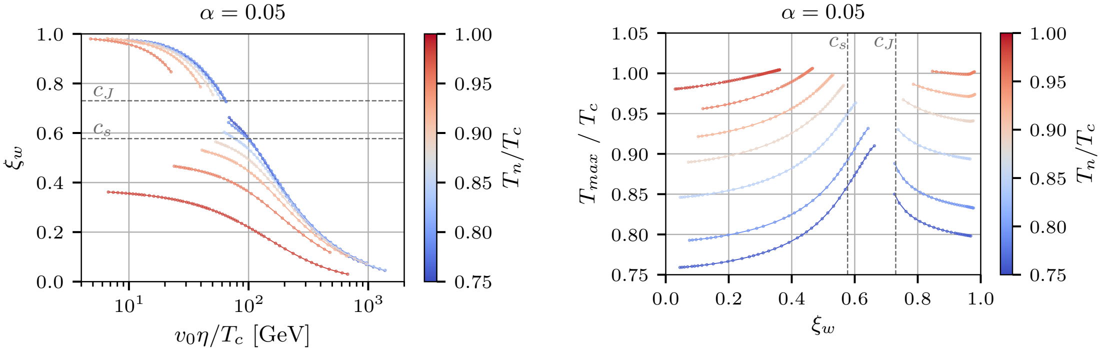
We randomly sample parameters of the potential and compare different models resulting with the same T_n/T_c and α.



Field value in the true vacuum v_0 fully determines position of the gap in terms of friction parameter η.

Dependence on the nucleation temperature

We randomly sample parameters of the potential and compare cases with the same α.



Higher T_n/T_c leads to wider velocity gap.[1]

1. Krajewski, T., Lewicki, M. & Zych, M. *Phys. Rev. D* **108,** 103523. arXiv: 2303.18216 [astro-ph.CO].

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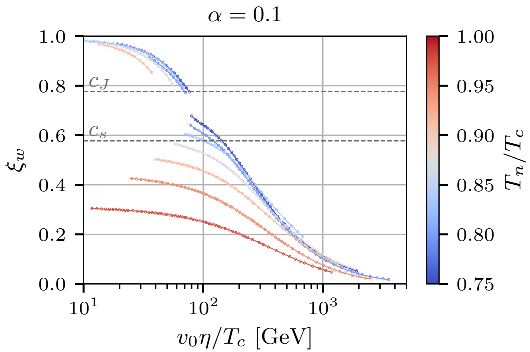
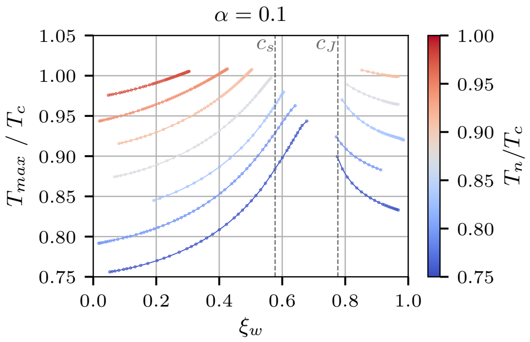


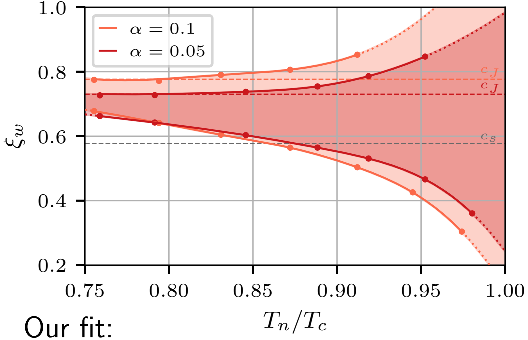


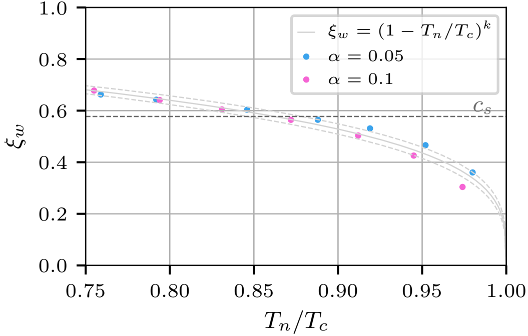
Results do not strongly depend on α.[1]

1. Krajewski, T., Lewicki, M. & Zych, M. *Phys. Rev. D* **108,** 103523. arXiv: 2303.18216 [astro-ph.CO].

Constraints on the wall velocity

Possible explanation (in low velocity limit): Hydrodynamical obstruction resulting from the heating of the plasma in front of the phase transition boundary.[2]





Our fit:

$$\xi_w^{max} = \left(1 - \frac{T_n}{T_c}\right)^k \quad \text{with} \quad k = 0.2768 \pm 0.0055$$

2. Konstandin, T. & No, J. M. *JCAP* **02**, 008. arXiv: 1011.3735 [hep-ph].

LTE means conservation of entropy

Entropy can be defined using thermodynamical relations

$$w = \rho + p = Ts, \qquad s = \frac{\partial p}{\partial T}.$$

One can compute

$$u_\nu \nabla_\mu T_f^{\mu\nu} = T\nabla_\mu (s u^\mu) + u^\mu \nabla_\mu T \overbrace{(w/T - \partial_T p)}^{=0} - u^\mu \nabla_\mu \phi \partial_\phi p \,,$$

where we used $u_\nu \nabla_\mu u^\nu = 0$, $u_\mu u^\mu = 1$.
The observation that $u_\nu \nabla_\mu T_\phi^{\mu\nu} = -u^\mu \nabla_\mu \phi \partial_\phi V = u^\mu \nabla_\mu \phi \partial_\phi p$ leads us to

$$\partial_\mu (s u^\mu) = 0,$$

and the third matching condition[3]

$$s_- \gamma_- v_- = s_+ \gamma_+ v_+ \,.$$

3. Ai, W.-Y., Laurent, B. & van de Vis, J. *JCAP* **07**, 002. arXiv: 2303.10171 [astro-ph.CO].

Analytic methods for stationary profiles

Hydrodynamic equation

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v,$$

Matching equations

1 $\omega_-\gamma_-^2 v_- = \omega_+\gamma_+^2 v_+$

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Bag equation of state

$\epsilon_s = 3a_s T_s^4 + \theta_s$ $\qquad$ $\epsilon_b = 3a_b T_b^4 + \theta_b$

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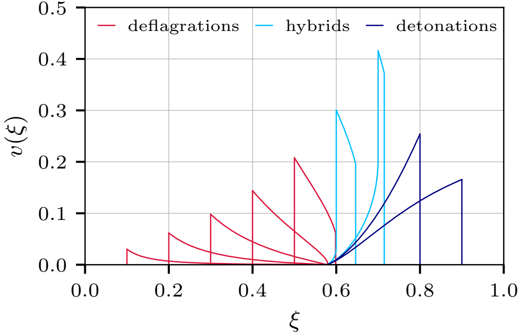
Solving hydrodynamic equation with proper boundary conditions and matching conditions (1 and 2), we get profiles $v(\xi)$ depending on ξ_w, α.



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2 $\omega_-\gamma_-^2 v_-^2 + p_- = \omega_+\gamma_+^2 v_+^2 + p_+$

3 $s_-\gamma_- v_- = s_+\gamma_+ v_+$ (if $\delta f = 0$)

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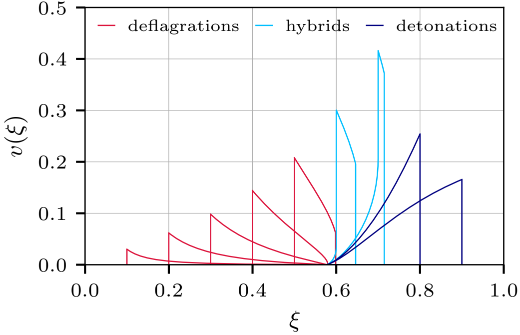


Adding 3 we can determine the velocity of the wall v_w.

Scalar singlet extension

Model: SM Higgs dublet H and Z_2-symmetric real singlet s.
Tree-level potential (unitary gauge):

$$V_0(h,s) = \frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{4}\lambda_{hs} h^2 s^2 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4$$

$$\lambda_h = \frac{m_h^2}{2v^2} \qquad \text{and} \qquad \mu_h^2 = -\lambda_h v^2,$$

with $m_h = 125.09$ GeV and $v = 246.2$ GeV.

$$\boxed{\text{free parametres: } m_s, \lambda_s, \lambda_{hs}}$$

Effective potential:

$$V_{\text{eff}}(h,s,T) = V_0(h,s) + V_{\text{CW}}(h,s,T) + V_{\text{T}}(h,s,T)$$

- $V_{\text{CW}}(h,s,T)$ - Coleman-Weinberg potential (here neglected)
- $V_{\text{T}}(h,s,T)$ - thermal potential

Thermal potential

Thermal functions

$$J_\sigma(x) = -\sigma^{-1}\int_0^\infty \mathrm{d}y y^2 \log\left(1 - \sigma \exp\left(-\sqrt{y^2 + x^2}\right)\right)$$

High-temperature expansion: $(x \ll 1)$:

$$J_{+1}(x) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 + \mathcal{O}(x^3) \qquad J_{-1}(x) \approx -\frac{7}{8}\frac{\pi^4}{45} + \frac{\pi^2}{24}x^2 + \mathcal{O}(x^4 \log x^2)$$

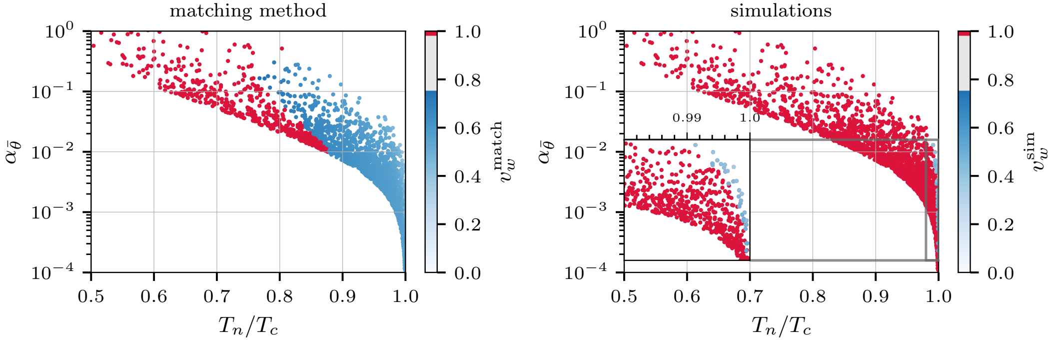
$$V_T = \sum_i \frac{n_i T^4}{2\pi^2} J_{\sigma_i}\left(\frac{m_i(h,s)}{T}\right) \overset{m_i \ll T}{\approx} -\frac{g_* \pi^2}{90}T^4 + \sum_i \frac{c_i n_i}{24} m_i^2(h,s) T^2$$

Effectively tree-level potential with temperature-dependent mass terms

$$\mu_h^2(T) \coloneqq \mu_h^2 + c_h^2 T^2 \quad \text{and} \quad \mu_s^2(T) \coloneqq \mu_s^2 + c_s^2 T^2,$$

$$c_h^2 = \frac{1}{48}\left(9g^2 + 3g'^2 + 12y_t^2 + 24\lambda_h + 2\lambda_{hs}\right) \quad \text{and} \quad c_s^2 = \frac{1}{12}\left(2\lambda_{hs} + 3\lambda_s\right)$$

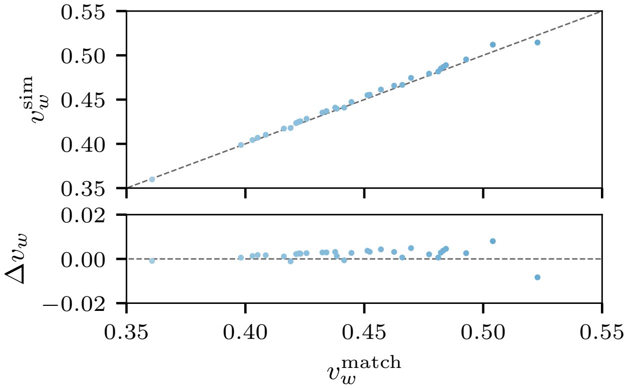
Analytical treatment vs real-time simulations in LTE



While matching equations predict significant number of stationary deflagrations and hybrids, in real-time simulations only few indeed evolve towards stationary state.[4]

4. Krajewski, T., Lewicki, M. & Zych, M. *JHEP* **05**, 011. arXiv: 2402.15408 [astro-ph.CO].

Precision of analytical treatment in LTE

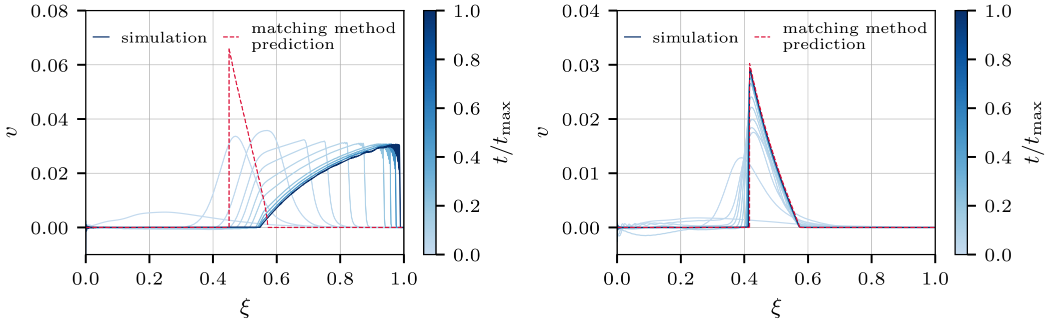


If the stationary state is achieved for a given model, bubble-wall velocity is very accurately predicted by the matching equations.[4]

4. Krajewski, T., Lewicki, M. & Zych, M. *JHEP* **05**, 011. arXiv: 2402.15408 [astro-ph.CO].

Evolution of bubbles in LTE

Self-similar profiles: $\xi = r/t$



Two possible scenarios[4] for the growing bubble in LTE:

- rapid expansion beyond Chapman-Jouguet velocity leading to a runaway scenario,
- evolution toward a stationary state predicted by matching conditions.

4. Krajewski, T., Lewicki, M. & Zych, M. *JHEP* **05**, 011. arXiv: 2402.15408 [astro-ph.CO].

Approaches to non-equilibrium friction

LTE in the entire system

Assumption of $\delta f = 0$ leads to $\partial_\mu (s u^\mu) = 0$ and the matching condition:

$$s_- \gamma_- v_- = s_+ \gamma_+ v_+ .$$

Ballistic approximations

The released laten heat is balanced by the work against the pressure generated by particles scattered by the wall:

$$\Delta V_0 = \Delta P.$$

Entropy production

When the entropy is produced $\partial_\mu (s u^\mu) = f_s\,(v, \phi, T)$, the generalized matching condition can be introduced:

$$\frac{T_+}{T_-} = \frac{\gamma_-}{\gamma_+} \left(1 + \frac{T_+ \Delta S}{w_+ \gamma_+ v_+} \right) .$$

Ballistic approximations

LTE only away of the wall, but ballistic motion inside the wall[5]

$$\Delta P = \int \frac{d^3p}{(2\pi)^3} \sum_{j\in\pm 1} f_j(p) \frac{(n\cdot p)^2}{E_i} \theta(-j n\cdot p) \left[\mathcal{T}_j(n\cdot p) \left(1 - \sqrt{1 - j\frac{\Delta m^2}{(n\cdot p)^2}} \right) + 2\mathcal{R}_j(n\cdot p) \right],$$

where $\mathcal{R}$ and $\mathcal{T} = 1 - \mathcal{R}$ are reflection and transmission coefficients respectively.

Fully ballistic fluid[6]

$$\Delta P(T, v_w) = \frac{\rho}{3} \frac{(1+v_w)^2}{1-v_w} \left[1 - G_{\Delta P}\left(\frac{\Delta m}{T} \sqrt{\frac{1-v_w}{1+v_w}} \right) \right],$$

where $G_{\Delta P}(x) \equiv (1/4)\left(e^{-x}(2+2x+x^2) + x^2 K_2(x)\right)$ with K_2 being the modified Bessel function.

5. Lewicki, M., Vaskonen, V. & Veermäe, H. *Phys. Rev. D* **106,** 103501. arXiv: 2205.05667 [astro-ph.CO].
6. Lewicki, M. *et al.* *Phys. Rev. D* **108,** 036023. arXiv: 2305.07702 [hep-ph].

Toy model

We consider general quadratic potential

$$V_0(\phi) = \lambda\left(\frac{\eta}{2}v^2\phi^2 - \frac{1+\eta}{3}v\phi^3 + \frac{1}{4}\phi^4\right),$$

so that the local maximum is at $\phi = \eta v$ and local minimum at $\phi = v$.
The potential energy difference between the vacua is

$$\Delta V_0 = \frac{\lambda v^4}{12}(1-2\eta)\,.$$

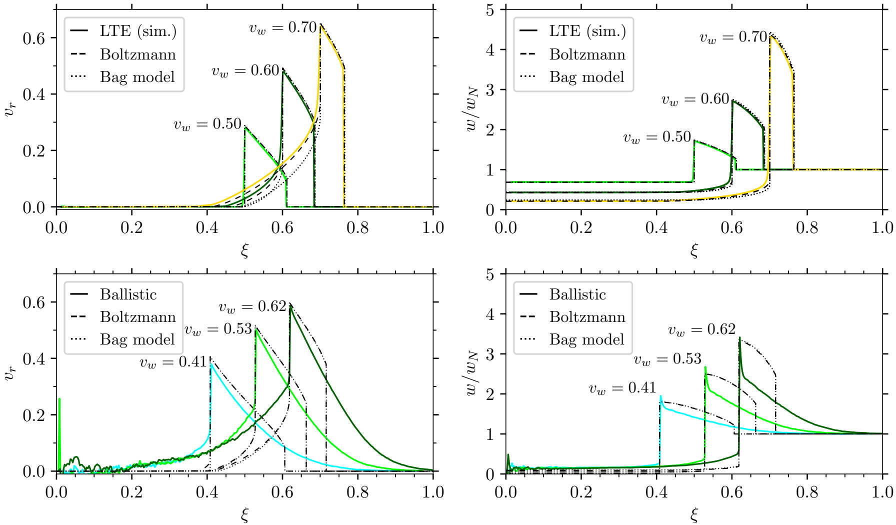
As an illustrative example, we include an additional fermionic field with a field-dependent mass

$$m_\psi^2 = y^2\phi^2$$

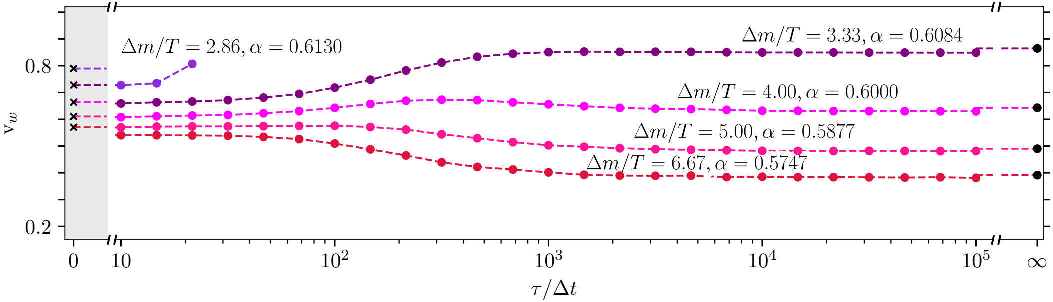
which generate the thermal correction to the potential

$$V_T = \frac{T^4}{2\pi^2}J_0\left(\frac{y\phi}{T}\right).$$

Plasma profiles in ballistic simulations



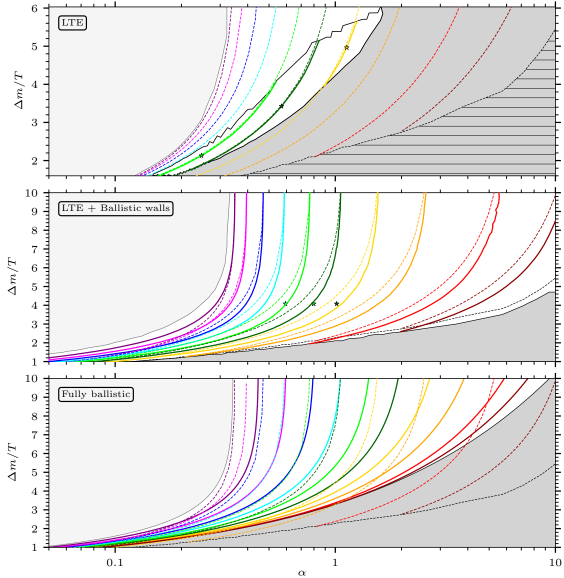
Thermalization in N-body simulations



N-body simulations correctly reproduce the free streaming limit for long mean free time τ, but due to limitations of the algorithm are not fully consistent with LTE limit ($\tau \to 0$).[7]

7. Krajewski, T. *et al.* arXiv: 2411.15094 [hep-ph].

Comparison of ballistic method with LTE



Effective friction term

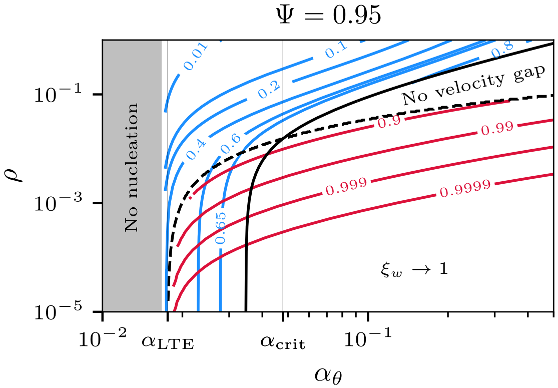
Entropy production at the bubble front:

$$\partial_\mu (u^\mu s) = \frac{\eta}{T} (u^\mu \partial_\mu \phi)^2$$

Integrating over the field profile, we get generalized 3rd matching equation:[8]

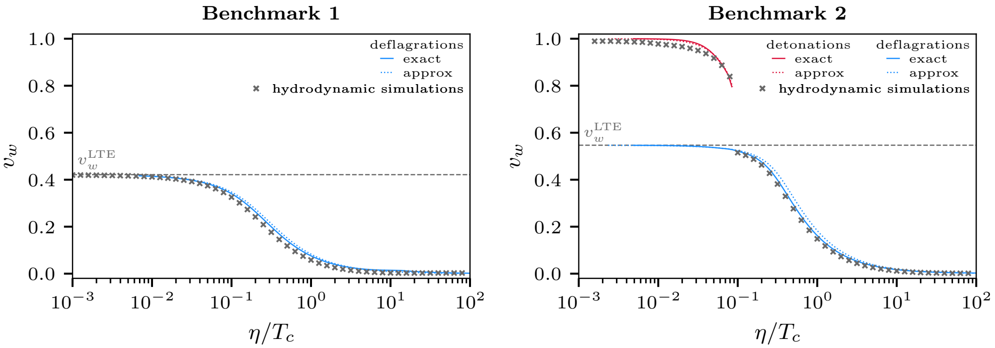
$$\frac{T_-}{T_+} = \frac{\gamma_+}{\gamma_-} \frac{1}{1 + \rho\, \gamma_+ \, v_+},$$

with $\rho \equiv \eta v_0^2 / (3 w_+ L_w)$.



8. Krajewski, T., Lewicki, M., Nałęcz, I. & Zych, M. arXiv: 2411.16580 [astro-ph.CO].

Analytical treatment vs real-time simulations $(\eta \neq 0)$



New third matching equation allows determining bubble-wall velocity as a function of η. Detonation branch explains the runaway behaviour in the LTE limit.[8]

8. Krajewski, T., Lewicki, M., Nałęcz, I. & Zych, M. arXiv: 2411.16580 [astro-ph.CO].

Summary

- We found good agreement between the analytical profiles and our hydrodynamical numerical results whenever the latter exist.
- The hydrodynamical obstruction preventing the realisation of fast hybrids is very generic.
- We always find some solutions to be excluded and the gap in solutions becomes wider as the nucleation temperature predicted by the potential is closer to the critical one.
- Depending on the non-equilibrium contribution to the friction, walls can be slower (particles ballistic only inside walls and thermalizing outside) or even faster (free streaming case) than LTE predictions.
- In order to calculate the terminal velocity, one needs to understand the production of the entropy on the wall.

Summary

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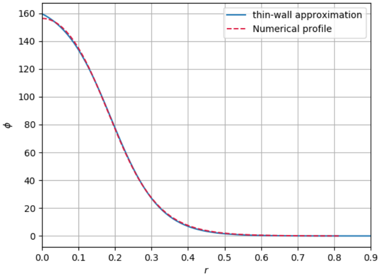
Thank you for your attention!

Initial conditions

1 scalar field ϕ: critical bubble



Fit

$$\phi_0(r) = \frac{v_0}{2}\left[1 - \tanh\left(\frac{r - r_0}{L}\right)\right]$$

free parameters:

v_0 - initial field amplitude

r_0 - bubble radius

L - bubble-wall size

2 plasma temperature T: nucleation temperature T_n

3 plasma velocity v: plasma at rest ($v = 0$)

Lattice:

$$\delta r = 0.01\ \text{GeV}^{-1} \quad \delta t = 0.001\ \text{GeV}^{-1} \quad t_{max} = 120\ \text{GeV}^{-1} \quad r_{max} = ct_{max}$$