ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

DEPARTMENT OF

PHYSICS AND ASTRONOMY “AUGUSTO RIGHI” – DIFA

QUANTUM PROPERTIES OF H $\rightarrow$ VV: PRECISE PREDICTIONS IN THE SM AND SENSITIVITY TO NEW PHYSICS

(WORK IN PROGRESS)

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HECA Seminar

NCBJ and IFT, University of Warsaw

Motivation:

It is challenging to see entanglement at High Energy Colliders(HEC) and it is interesting to check the sensitivity of HEC to probe quantum correlations

We saw it at LHC!!!!!

Observation of quantum entanglement in top-quark pairs using the ATLAS detector

ATLAS Collaboration

We report the highest-energy observation of entanglement, in top−antitop quark events produced at the Large Hadron Collider, using a proton−proton collision data set with a center-of-mass energy of $\sqrt{s} = 13$ TeV and an integrated luminosity of 140 fb^{-1} recorded with the ATLAS experiment. Spin entanglement is detected from the measurement of a single observable D, inferred from the angle between the charged leptons in their parent top- and antitop-quark rest frames. The observable is measured in a narrow interval around the top−antitop quark production threshold, where the entanglement detection is expected to be significant. It is reported in a fiducial phase space defined with stable particles to minimize the uncertainties that stem from limitations of the Monte Carlo event generators and the parton shower model in modelling top-quark pair production. The entanglement marker is measured to be $D = -0.547 \pm 0.002\,(\mathrm{stat.}) \pm 0.021\,(\mathrm{syst.})$ for $340 < m_{t\bar{t}} < 380$ GeV. The observed result is more than five standard deviations from a scenario without entanglement and hence constitutes both the first observation of entanglement in a pair of quarks and the highest-energy observation of entanglement to date.

Observation of quantum entanglement in top quark pair production in proton-proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration

6 June 2024

Submitted to Reports on Progress in Physics

Abstract: Entanglement is an intrinsic property of quantum mechanics and is predicted to be exhibited in the particles produced at the Large Hadron Collider. A measurement of the extent of entanglement in top quark-antiquark ($\mathrm{t\bar{t}}$) events produced in proton-proton collisions at a center-of-mass energy of 13 TeV is performed with the data recorded by the CMS experiment at the CERN LHC in 2016, and corresponding to an integrated luminosity of 36.3 fb^{-1}. The events are selected based on the presence of two leptons with opposite charges and high transverse momentum. An entanglement-sensitive observable D is derived from the top quark spin-dependent parts of the $\mathrm{t\bar{t}}$ production density matrix and measured in the region of the $\mathrm{t\bar{t}}$ production threshold. Values of $D < -1/3$ are evidence of entanglement and D is observed (expected) to be $-0.480^{+0.026}_{-0.029}$ ($-0.467^{+0.026}_{-0.029}$) at the parton level. With an observed significance of 5.1 standard deviations with respect to the non-entangled hypothesis, this provides observation of quantum mechanical entanglement within $\mathrm{t\bar{t}}$ pairs in this phase space. This measurement provides a new probe of quantum mechanics at the highest energies ever produced.

Motivation:

- As we already have signal of sensitivity of high energy colliders for quantum observables, it motivates us to study the impact of higher order EW correction and of new physics on the quantum observables(QO).

Why $H \rightarrow VV$?

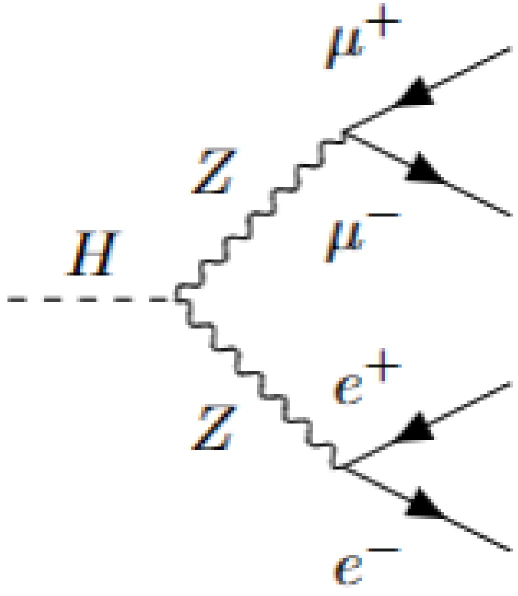
- Quantum information of $H \rightarrow ZZ^*$ is highly studied in several paper at LHC at LO and also puts constraints on New physics.

- Due to the scalar nature of Higgs, the ZZ^* is highly entangled state on the whole phase space and it is shown:
  1. Violation of Bell's inequality
  2. Decaying particles of Z bosons keep information of Z polarization due to chiral decay.

- Experimental advantage:
  1. Pure signal
  2. Fully re-constructable final state (no neutrinos)
  3. Disadvantage: small statistics

List of work on VV spin correlation at colliders

- Testing entanglement and Bell inequalities in $H \to ZZ$ by J. A. Aguilar-Saavedra , A. Bernal , J. A. Casas , and J. M. Moreno
- Entanglement and Bell inequalities violation in $H \to ZZ$ with anomalous coupling by Alexander Bernal, Pawel Caban and Jakub Rembielinski
- Quantum state tomography, entanglement detection and Bell violation prospects in weak decays of massive particles: Rachel Ashby-Pickering, Alan J. Barr, Agnieszka Wierzchucka
- Bell inequalities and quantum entanglement in weak gauge boson production at the LHC and future colliders by Marco Fabbrichesi, Roberto Floreanini, Emidio Gabrielli, Luca Marzola
- Spin Correlations in Decay Chains Involving W Bosons⋆ by Jennifer M. Smillie
- Stringent bounds on HWW and HZZ anomalous couplings with quantum tomography at the LHC by M. Fabbrichesia, R. Floreaninia, E. Gabriellib,a,c,d and L. Marzolad
- Bell-type inequalities for systems of relativistic vector bosons by Alan J. Barr, Paweł Caban, and Jakub Rembieliński
- Breaking down the entire W boson spin observables from its decay by J. A. Aguilar-Saavedra, J. Bernabéu
- Testing Bell inequalities in Higgs boson decays by Alan J. Barr
- The Z boson spin observables as messengers of new physics by J. A. Aguilar-Saavedra, J. Bernabéu, V. A. Mitsou, A. Segarra



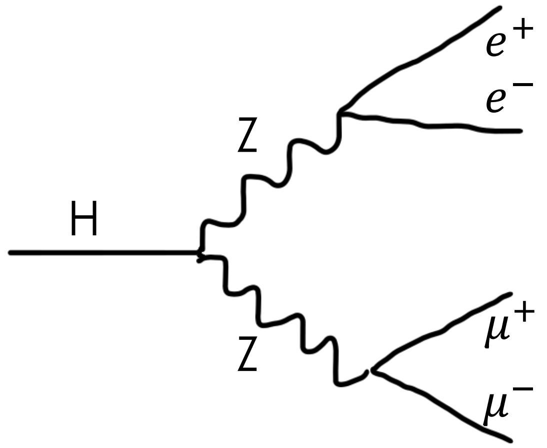
Questions : What do we want to study?

- Quantum observables related to spin correlation between bi-partite $(Z_a Z_b)$. Z-boson has spin-1 and spin can have three polarizations, which also called “qutrit”. We need the spin density matrix of bi-partite qutrit system.

At colliders we can’t measure the quantum state of the bi-partite qutrit system directly

- we can measure “Direction and momentum of decaying particles”

Outline



Reconstruct angular distribution and get an ensemble of all decays

Quantum tomography technique

Spin density matrix for $Z_a Z_b$

Define QO

Study effect of NLO EW corrections on quantum observables

Compare NLO sensitivity of these observables with New physics effect

Outline of talk:

- Definitions of quantum observables.
- Define irreducible tensor operator parameterization for the spin density matrix.
- Quantum state tomography.
- Observables and density matrix at LO for SM for $H \to 4f$.
- NLO effect on density matrix and on observables.
- Effect of new intermediate states and EFT on quantum observables.
- Conclusion.

Entanglement is hallmark of quantum mechanics

- Lets take a bi-partite (particle 'a' and particle 'b') quantum system. If we can write pure state of bi-partite system as follows:

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \qquad \text{Separable state}$$

$$|\psi_{AB}\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \qquad \text{Entangle state}$$

- For given $\{p_i, |\psi_i\rangle\}$ an ensemble of pure state, the density operator/matrix for the quantum system is defined as

$$\rho \equiv \sum_i p_i |\psi_i\rangle\langle\psi_i|. \quad \text{Where } \sum_i p_i = 1$$

with the characteristics

1. Tr(ρ)=1
2. For any arbitrary vector $|\varphi\rangle$, It satisfies the positivity condition $\langle\varphi|\rho|\varphi\rangle \geq 0$

Entanglement Measures

- Peres-Horodecki condition for Entanglement

$$\rho \equiv \sum_i p_i \rho_i^a \otimes \rho_i^b \longrightarrow \rho^{T_b} \equiv \sum_i p_i \rho_i^a \otimes (\rho_i^b)^T$$

If ρ^{T_b} has Trace one and it is positive definite → State is separable

For non-positive definite → State is entangled

- Von Neumann entropy: For pure state

$$S(\rho) = -Tr(\rho \; Log_2(\rho))$$

For mix state

$$S\left(\sum_i p_i \rho_i\right) = H(p_i) + \sum_i p_i S(\rho_i)$$

where $0 \le S(\sum_i p_i \rho_i) \le Log(d)$, *d=3 for qutrit and* $H_{\text{bin}}(p) \equiv -p \log p - (1-p)\log(1-p)$,

$S(\rho)$ *=0 (separable state),* $S(\rho) = Log(d)$, *(Maximally entangled)*

Above entanglement measures are good for pure state, not for mixed states.

Entanglement witness: quantities that give conditions sufficient to establish the presence of entanglement in the system.

❖ Concurrence

1. For ***pure state***, it is defined analytically as follow

$$\mathcal{C}(|\psi\rangle) = \sqrt{2(1 - \operatorname{tr}(\rho_r)^2)}, \; r = a\,(or)\,b,$$

P. Rungta,V. Buzek, C.M. Caves,M.Hillery,G.J.MilburnPhys.Rev. A **64**, 042315 (2001),

where $\rho_a = Tr_b(\rho)$ is the reduced density matrix.

2. For ***mix state***, It is defined using optimization process

$$\mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i\, \mathcal{C}[|\Psi_i\rangle]\,,$$

where the infimum is taken over all the possible decompositions of ρ into pure states.

$\mathcal{C}(\rho) > 0$ Entanglement condition

No analytical formula for concurrence for mix state bi-partite qutrit system.

Concurrence

- The analytical form of upper and lower bounds on the concurrence for qutrits

$$(\mathcal{C}(\rho))^2 \geq 2\max(\operatorname{Tr}\rho^2 - \operatorname{Tr}\rho_a^2, \operatorname{Tr}\rho^2 - \operatorname{Tr}\rho_b^2), \qquad (\mathcal{C}(\rho))^2 \leq 2\min(1 - \operatorname{Tr}\rho_a^2, 1 - \operatorname{Tr}\rho_b^2)$$

C.-J. Zhang, Y.-X. Gong, Y.-S. Zhang, and G.-C. Guo, Phys. Rev. A, vol. 78, p. 042308,Oct 2008
F. Mintert, A. Buchleitner, Phys. Rev. Lett. **98**, 140505 (2007)

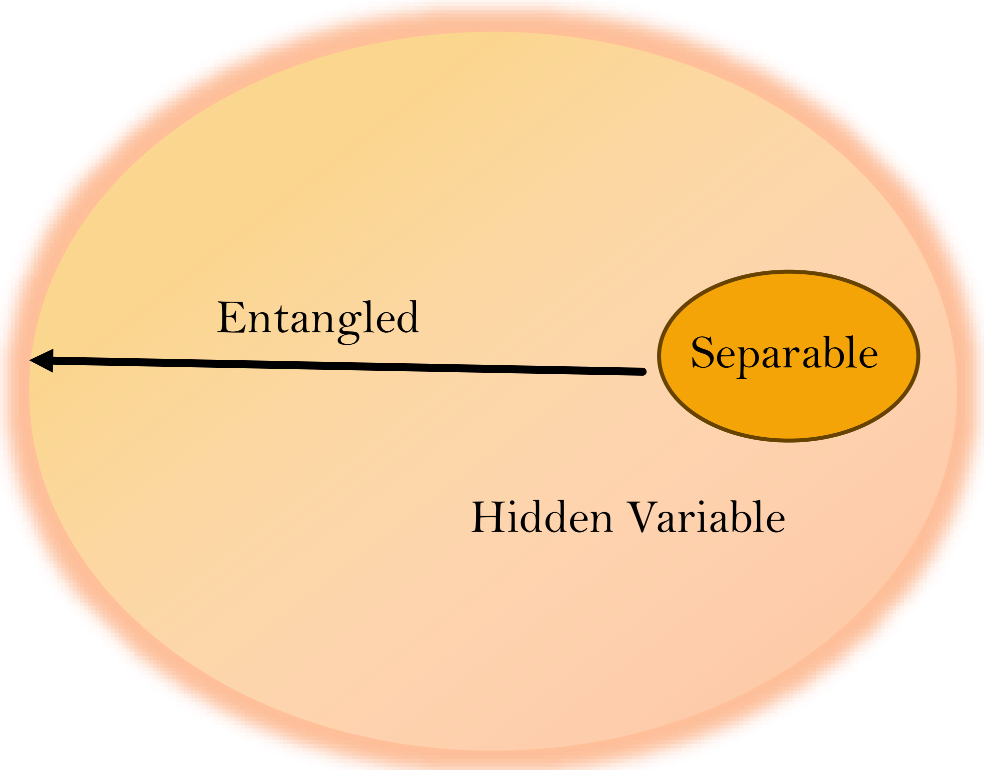
If lower bound of concurrence is greater than zero then the quantum state of system is entangled.

Purity

- Criterion to decide if a state is mixed or pure

Pure state : $\operatorname{Tr}(\rho^2)=1$

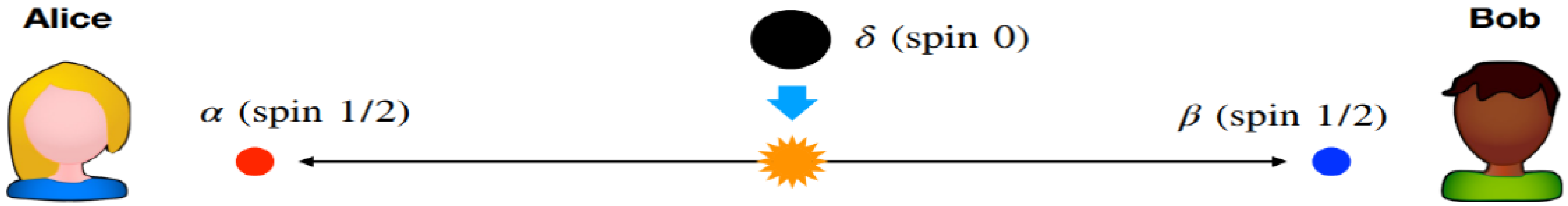
Mixed state: $\operatorname{Tr}(\rho^2)<1$



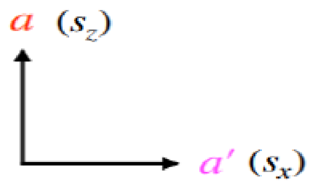
Bell type inequalities : are inequalities which can discriminate QM from any local-real hidden variable theories.

Let’s write CHSH inequality for a bi-particle qubit system

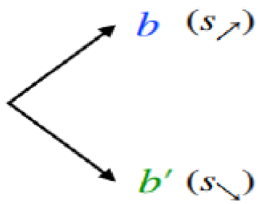
[Clauser, Horne, Shimony, Holt, 1969]



From Kazuki slides



$$R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|$$



For LHV theories

$$\boxed{\langle ab\rangle = \int a(\lambda)b(\lambda)P(\lambda)d\lambda}$$

$$\boxed{\int P(\lambda)d\lambda = 1}$$

For QM

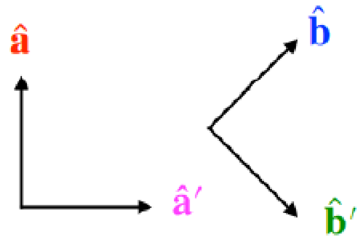
$$\boxed{\langle s_a s_b\rangle = \langle \Psi^{(0,0)} | s_a s_b | \Psi^{(0,0)}\rangle = (\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})}$$

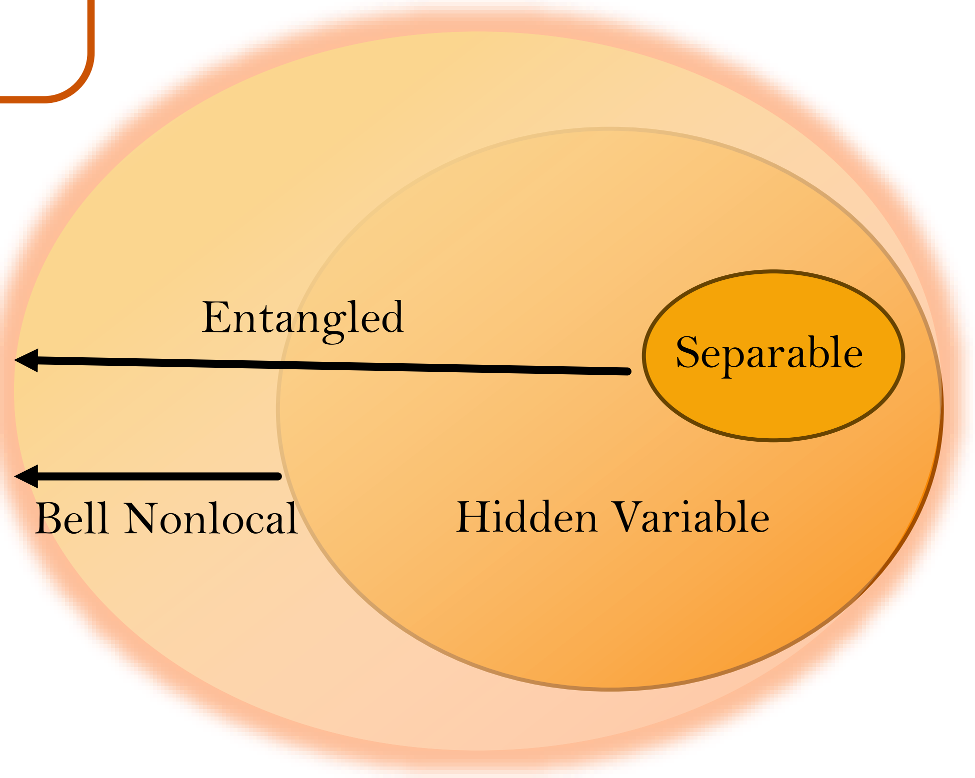
Bell type inequalities

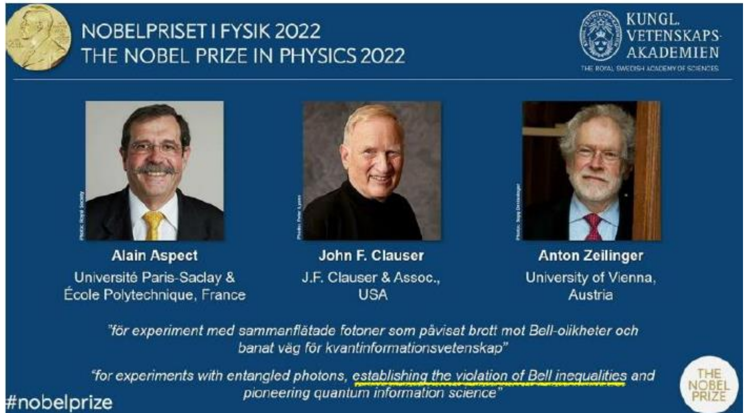
$$R_{\mathrm{CHSH}} \leq \begin{cases} 1 & \text{Local-hidden Variable(LHV) theories} \\ \sqrt{2} & \text{QM} \end{cases}$$

$$R_{\mathrm{CHSH}} = \frac{1}{2}\left|\langle s_a s_b\rangle - \langle s_a s_{b'}\rangle + \langle s_{a'} s_b\rangle + \langle s_{a'} s_{b'}\rangle\right|$$

$$= \frac{1}{2}\left|\underbrace{(\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} - \underbrace{(\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}')}_{-\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}'\cdot\hat{\mathbf{b}})}_{\frac{1}{\sqrt{2}}} + \underbrace{(\hat{\mathbf{a}}'\cdot\hat{\mathbf{b}}')}_{\frac{1}{\sqrt{2}}}\right| = \sqrt{2}$$







Bell nonlocality for the qutrit system

CGLMP inequality

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)$$
$$- P(A_1 = B_1 - 1) - P(B_1 = A_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \leq 2.$$

D. Collins, N. Gisin, N. Linden, S. Massar and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

P$(A_i = B_j + k)$ are the probability that the outcomes for party A and B, measuring A_i *and* B_j, differ by k modulo 3 .

Upper value in QM $\qquad I_3 \approx 2.9149$

A. Ac´ın, T. Durt, N. Gisin, and J. I. Latorre, "Quantum nonlocality in two three-level systems,"Phys. Rev. A, vol. 65, p. 052325, May 2002

$$\begin{aligned} B &= \frac{4}{3\sqrt{3}}(T_1^1 \otimes T_1^1 + T_{-1}^1 \otimes T_{-1}^1) + \frac{2}{3}(T_2^2 \otimes T_2^2 + T_{-2}^2 \otimes T_{-2}^2) \\ &= \frac{2}{\sqrt{3}}(S_x^T \otimes S_x + S_y^T \otimes S_y) + \lambda_4^T \otimes \lambda_4 + \lambda_5^T \otimes \lambda_5 \end{aligned}$$

How can we measure it?

As we know we can compute expectation value of any operator in QM if we know density matrix

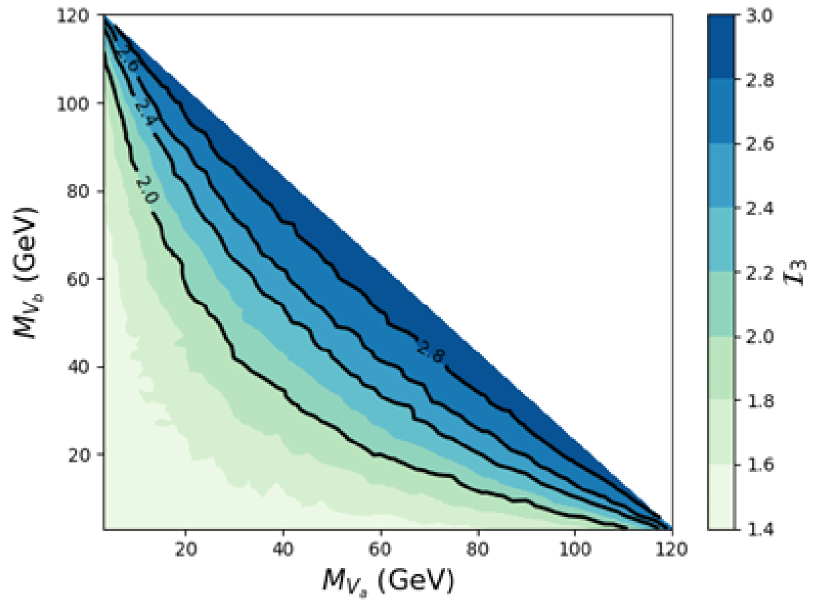
$$B' = (V \otimes U)^T B (V \otimes U)$$

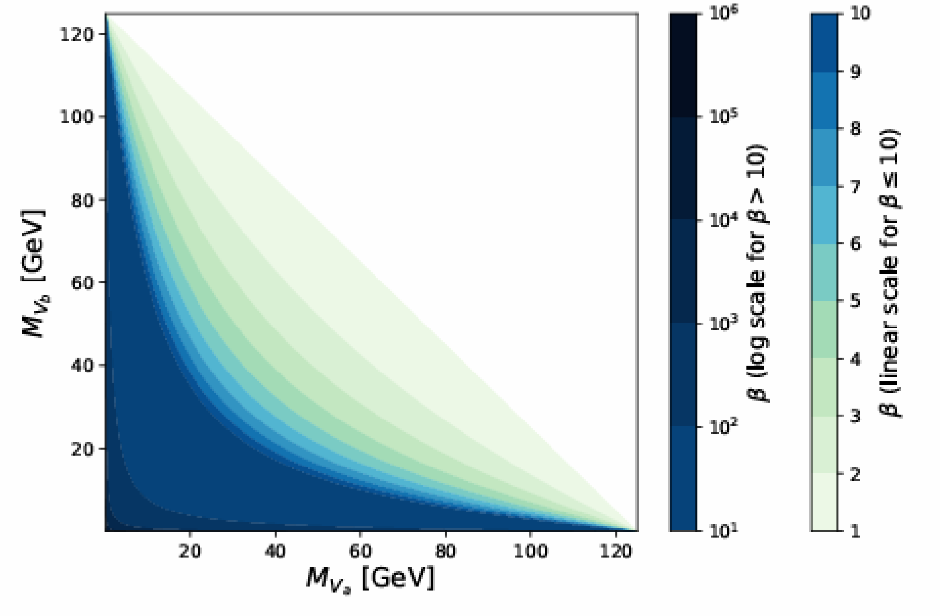
$$I_3 = Tr[\rho B']$$

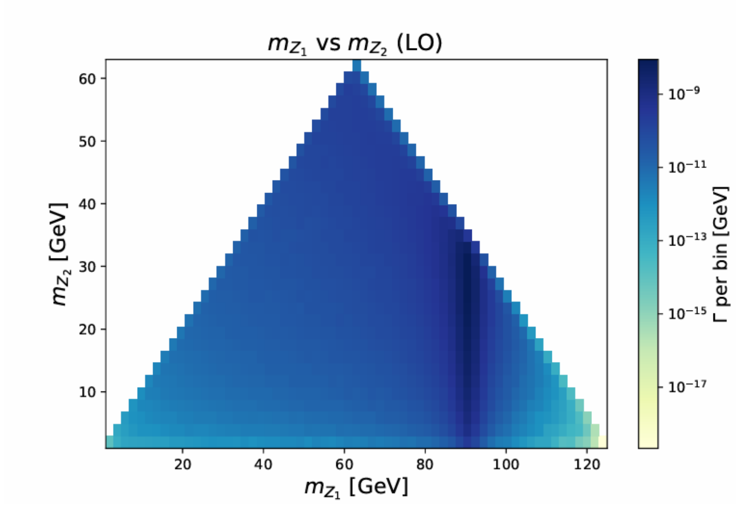
Where B' is bell operator.

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Bell nonlocality for the qutrit system







Quantum tomography

The Polarization operator basis parameterization/ irreducible tensor parameterization

$$\rho = \frac{1}{9}[\mathbf{1}_3 \otimes \mathbf{1}_3 + A^a_{LM}\hat{T}^{LM} \otimes \mathbf{1}_3 + A^b_{LM}\mathbf{1}_3 \otimes \hat{T}^{LM} + C_{L_1,M_1,L_2,M_2}\hat{T}^{L_1M_1} \otimes \hat{T}^{L_2M_2}]$$

For spin-1, the spin operator S and polarization operator are relates as

$$T_{00} = \frac{1}{\sqrt{3}}\hat{I}, \qquad T_{1M} = \frac{1}{\sqrt{2}}\hat{S}_M, \qquad T_{2M} = \sum_{\mu\nu} C^{2M}_{1\mu 1\nu}\hat{S}_\mu\hat{S}_\nu$$

Constraint on A and C coefficient in spherical basis

$$\begin{aligned} (A^j_{L,M})^* &= (-1)^M A^j_{L,-M}, \qquad j = 1,2 \\ C_{L_1,M_1,L_2,M_2} &= (-1)^{M_1+M_2}(C_{L_1,-M_1,L_2,-M_2})^* \end{aligned}$$

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We know how the angular differential cross section is related to density matrix:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{2S_a+1}{4\pi}\frac{2S_b+1}{4\pi}\sum_{\lambda_a,\lambda_a',\lambda_b,\lambda_b'}\rho(\lambda_a,\lambda_a',\lambda_b,\lambda_b')\Gamma_a(\lambda_a,\lambda_a')\Gamma_b(\lambda_b,\lambda_b')$$
$$= \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho(\Gamma_a \otimes \Gamma_b)^T\right],$$

The traces of decay density matrix can be written in term of spherical harmonics as

$$\mathrm{Tr}\left[\mathbf{1}_3\Gamma^T\right] = 2\sqrt{\pi}Y_0^0(\theta,\phi), \quad \mathrm{Tr}\left[T^1_M\Gamma^T\right] = B_1 Y_1^M(\theta,\phi), \quad \mathrm{Tr}\left[T^2_M\Gamma^T\right] = B_2 Y_2^M(\theta,\phi)$$

These traces of decay matrix is same for all spin-1 particle decay except B_1 coefficient, which depends on decay products

$B_1 = \sqrt{2\pi}\alpha$ and $B_2 = \sqrt{\frac{2\pi}{5}}$

Spin analyzing power

Quantum tomography

Now we have normalized joint angular distribution in term of spherical harmonics and function

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}[1 + A^a_{LM} B^a_L Y^M_L(\theta_a,\phi_a) + A^b_{LM} B^b_L Y^M_L(\theta_b,\phi_b) + C_{L_1M_1L_2M_2} B^a_{L_1} B^b_{L_2} Y^{M_1}_{L_1}(\theta_a,\phi_a) Y^{M_2}_{L_2}(\theta_b,\phi_b)]$$

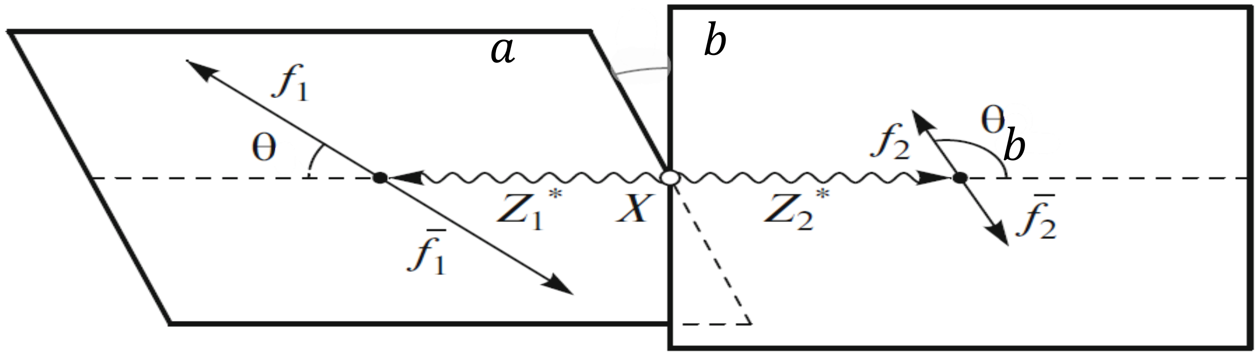
We can compute full spin density matrix using experimental data by using following tomographic reconstruction:

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^{*M}_L(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B^j_L}{4\pi} A^j_{LM} \quad j = a, b$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^{*M_1}_{L_1}(\Omega_a) Y^{*M_2}_{L_2}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_1} B^b_{L_2}}{(4\pi)^2} C_{L_1M_1L_2M_2}$$

Total 80 parameters

$$d\Omega_a = \sin\theta_a d\theta_a d\phi_a$$



The spin density matrix and quantum observables at LO

Observables and density matrix

Lets compute amplitude square for generic vector and pseudo-vector currents

$$i\mathcal{M} = \frac{ic_V\ \bar{u}_1\gamma_\mu(c_LP_L + c_RP_R)v_2\ \bar{u}_3\gamma^\mu(d_LP_L + d_RP_R)v_4}{(m_a^2 - M_{V_a}^2 + iM_{V_a}\Gamma_{V_a})(m_b^2 - M_{V_b}^2 + iM_{V_b}\Gamma_{V_b})}\,,$$

$$\begin{aligned} B_1^a &= \sqrt{2\pi}\frac{c_R^2 - c_L^2}{c_R^2 + c_L^2} \\ B_1^b &= \sqrt{2\pi}\frac{d_R^2 - d_L^2}{d_R^2 + d_L^2} \end{aligned}$$

Non-zero A and C coefficients for vector and vector-axial couplings

$$A_{2,0}^a = A_{2,0}^b \neq 0$$

$$\boxed{\frac{A_{2,0}^a}{\sqrt{2}} + 1 = C_{2,2,2,-2}} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

All A and C coefficient are real in this case

$$\rho = \frac{1}{9}[\mathbf{1}_3 \otimes \mathbf{1}_3 + A_{LM}^a \hat{T}^{LM} \otimes \mathbf{1}_3 + A_{LM}^b \mathbf{1}_3 \otimes \hat{T}^{LM} + C_{L_1,M_1,L_2,M_2}\hat{T}^{L_1M_1} \otimes \hat{T}^{L_2M_2}]$$

Observables and density matrix

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$$B_1^a = \sqrt{2\pi}\frac{c_R^2 - c_L^2}{c_R^2 + c_L^2}$$

$$B_1^b = \sqrt{2\pi}\frac{d_R^2 - d_L^2}{d_R^2 + d_L^2}$$

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$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

All A and C coefficient are real in this case

$$\rho = \frac{1}{3}\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 + \frac{1}{\sqrt{2}}A_{2,0}^1 & 0 & C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{2,-1,2,1} & 0 & 1 - \sqrt{2}A_{2,0}^1 & 0 & C_{2,1,2,-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,2,1} & 0 & 1 + \frac{1}{\sqrt{2}}A_{2,0}^1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Observables and density matrix

First task: reconstruct the quantum state: easy in this case but not always

$$\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&x&0&y&0&x&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&y&0&z&0&y&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&x&0&y&0&x&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$\xleftarrow{a_+ = a_-}$$

$$\boxed{\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}}$$

$$\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&a_+a_+^*&0&a_+a_0^*&0&a_+a_-^*&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&a_0a_+^*&0&a_0a_0^*&0&a_0a_-^*&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&a_-a_+^*&0&a_-a_0^*&0&a_-a_-^*&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$z = 1 - 2x$

By looking this we can directly write the helicity state.

$$|\psi> = a_+|+-> + a_0|0\,0> + a_-|-+>$$

State is entangled, if any 2 'a's' are non-zero.

$\boxed{a_+ = a_-}$ Also CP conserving condition

Observables and density matrix

1. Entanglement

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-1,2,1} & 0 & 1-\sqrt{2}A^1_{2,0} & 0 & C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,2,1} & 0 & 1+\frac{1}{\sqrt{2}}A^1_{2,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Sufficient condition for Entanglement

$\boxed{C_{2,2,2,-2} \neq 0 \text{ or } C_{2,1,2,-1} \neq 0}$

$$|\psi> = a_+|+-> + a_0|0\,0> + a_-|-+>$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Numerical Results

- Generate event for $H \rightarrow e^+e^-\mu^+\mu^-$ with Madgraph5 aMC@NLO at NLO EW accuracy. In the analysis we label large invariant mass is $Z_{1/a}$ and other one is $Z_{2/b}$.

- Define Helicity basis, $\hat{z}$-axis is taken in the direction of the Z_1 three-momentum in the H rest frame.

$$\hat{x} = sign(\cos\theta)(\hat{p} - \cos\theta\hat{z})/\sin\theta,\ \hat{y} = \hat{z} \times \hat{x}$$

- The angles $(\theta_{1/a}, \phi_{1/a})$ are the polar coordinates of the 3-momentum of negatively charge lepton from the $Z_{1/a}$, in the $Z_{1/a}$ rest frame.

Observables at LO level

Sufficient condition for Entanglement

$C_{2,2,2,-2} \neq 0$ or $C_{2,1,2,-1} \neq 0$

	inclusive	$m_{Z_2} > 10$ GeV	$m_{Z_2} > 20$ GeV	$m_{Z_2} > 30$ GeV
$C_{2,2,2,-2}$	0.581 ± 0.005	0.619 ± 0.004	0.713 ± 0.004	0.775 ± 0.003
$C_{2,1,2,-1}$	-0.938 ± 0.004	-0.975 ± 0.003	-1.017 ± 0.003	-1.014 ± 0.003
$\mathcal{I}_3$	2.601 ± 0.006	2.672 ± 0.004	2.772 ± 0.004	2.794 ± 0.005

Bell nonlocality condition $I_3 > 2$

For maximal entangled state $I_3 \approx 2.8729$

Observables at LO level

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

$$\boxed{\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1}$$

$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

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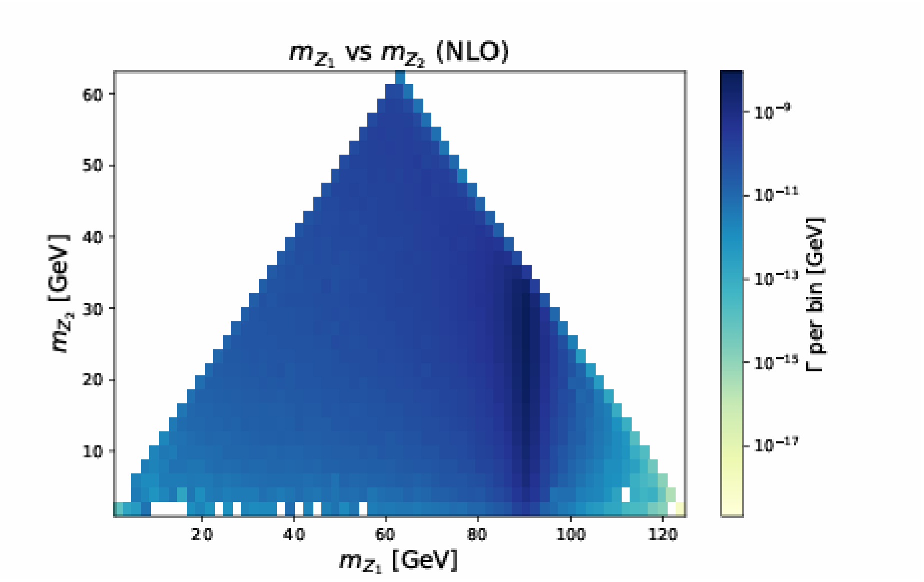
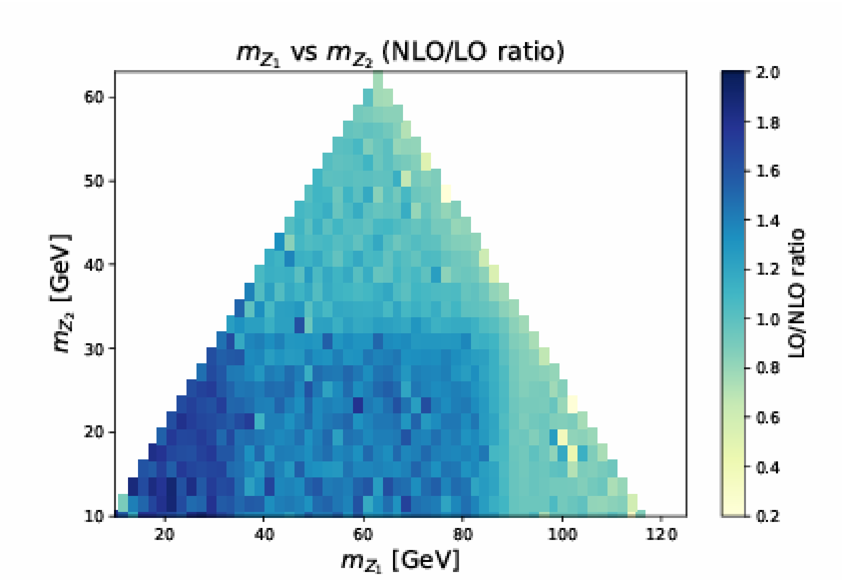
$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 & z & 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

No cuts

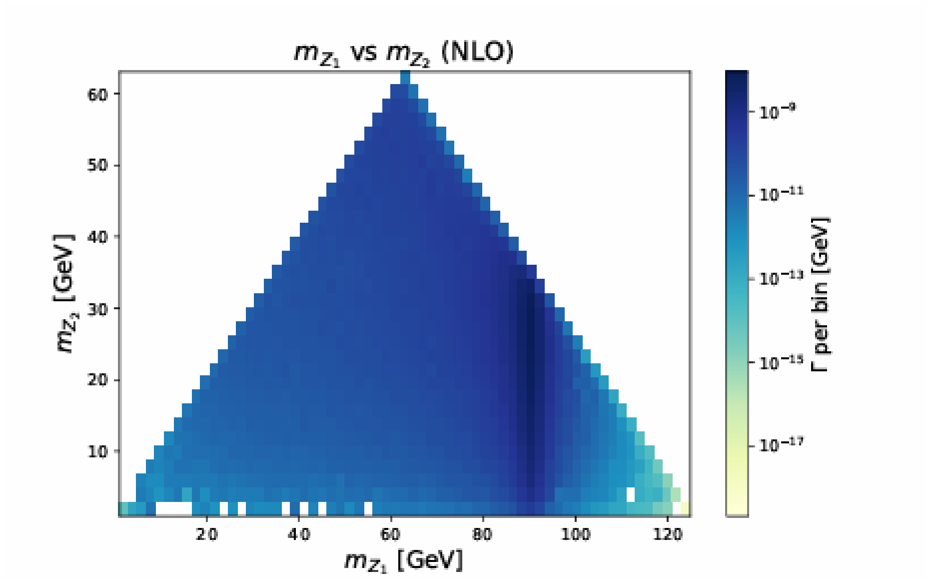
$$\rho^{\text{tomo}}_{\text{LO}} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.195 \pm 0.002 & \cdot & -0.313 \pm 0.003 & \cdot & 0.194 \pm 0.001 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -0.313 \pm 0.003 & \cdot & 0.612 \pm 0.001 & \cdot & -0.313 \pm 0.003 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.194 \pm 0.001 & \cdot & -0.313 \pm 0.003 & \cdot & 0.195 \pm 0.003 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

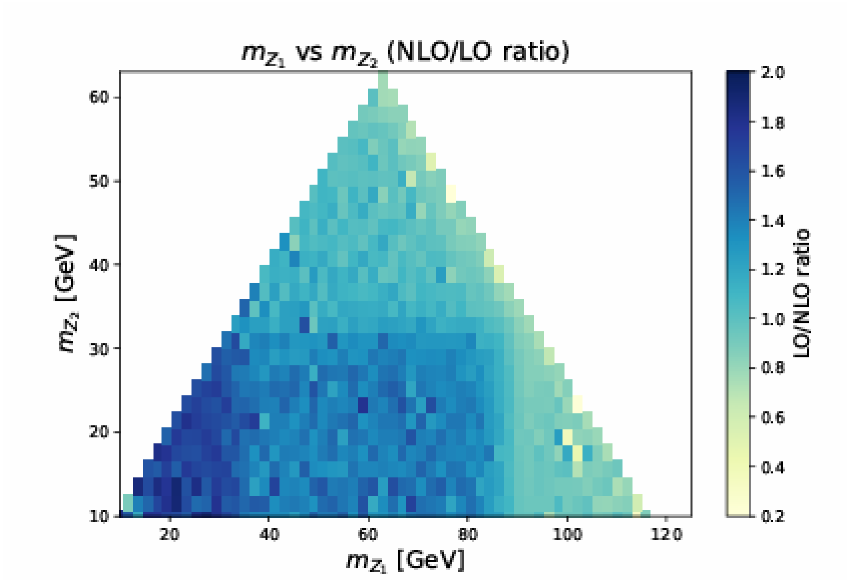
NLO EW Results





NLO EW Results





We studied the NLO effects for 3 different cuts on boson masses

1. No cuts
2. $M_{Z_2} > 30$ GeV (small NLO correction, small beta reason)
3. $85 < M_{Z_1} < 95$ GeV (Large phase space)

NLO EW Results

$$A^{a}_{2,0} = A^{b}_{2,0} \neq 0$$

$$\boxed{\frac{A^{a}_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}} \neq 0$$

$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$

$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

No cuts

	LO	NLO	NLO / LO
$A^{1}_{2,0}$	-0.592 $\pm$ 0.001	-0.509 $\pm$ 0.002	0.860 $\pm$ 0.002
$A^{2}_{2,0}$	-0.591 $\pm$ 0.001	-0.565 $\pm$ 0.002	0.956 $\pm$ 0.002
$C_{2,1,2,-1}$	-0.937 $\pm$ 0.002	-0.943 $\pm$ 0.004	1.006 $\pm$ 0.003
$-C_{1,1,1,-1}$	-0.94 $\pm$ 0.01	-0.16 $\pm$ 0.02	0.17 $\pm$ 0.02
$A^{1}_{2,0}/\sqrt{2}+1$	0.5817 $\pm$ 0.0007	0.640 $\pm$ 0.001	1.101 $\pm$ 0.002
$C_{2,2,2,-2}$	0.581 $\pm$ 0.003	0.568 $\pm$ 0.004	0.977 $\pm$ 0.006
$-C_{1,0,1,0}$	0.59 $\pm$ 0.01	0.03 $\pm$ 0.02	0.06 $\pm$ 0.04
$C_{2,0,2,0}$	1.418 $\pm$ 0.003	1.400 $\pm$ 0.005	0.987 $\pm$ 0.003
$C_{1,0,1,0}+2$	1.41 $\pm$ 0.01	1.97 $\pm$ 0.02	1.39 $\pm$ 0.01

All relation between the coefficients are broken and coefficients are getting from 1 to 90 % corrections at NLO EW.

NLO EW Results

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

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All relation between the coefficients are broken and coefficients are getting from 1 to 90 % corrections at NLO EW.

$$\rho_{\rm NLO} =$$

$$\begin{pmatrix}
0.099\pm0.004 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.004\pm0.002 & 0 & 0.131\pm0.004 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.111\pm0.004 & 0 & -0.183\pm0.004 & 0 & 0.189\pm0.001 & 0 & 0 \\
0 & 0.131\pm0.004 & 0 & -0.009\pm0.002 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.183\pm0.004 & 0 & 0.591\pm0.001 & 0 & -0.183\pm0.004 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.009\pm0.002 & 0 & 0.131\pm0.004 & 0 \\
0 & 0 & 0.189\pm0.001 & 0 & -0.183\pm0.004 & 0 & 0.110\pm0.003 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.131\pm0.004 & 0 & 0.004\pm0.002 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.099\pm0.003
\end{pmatrix}$$

$$\frac{\rho_{\rm NLO}}{\rho_{\rm LO}} = \begin{pmatrix}
0.85\pm0.01 & 0.84\pm0.01 & 0.988\pm0.005 \\
0.84\pm0.01 & 0.978\pm0.002 & 0.84\pm0.01 \\
0.988\pm0.005 & 0.84\pm0.01 & 0.85\pm0.02
\end{pmatrix}$$

NLO EW Results

$M_{Z_2} > 30$ GeV

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

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$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

	LO	NLO	NLO / LO
$A^1_{2,0}$	-0.318 ± 0.001	-0.241 ± 0.002	0.758 ± 0.005
$A^2_{2,0}$	-0.318 ± 0.001	-0.306 ± 0.002	0.962 ± 0.004
$C_{2,1,2,-1}$	-1.014 ± 0.003	-1.021 ± 0.005	1.007 ± 0.004
$-C_{1,1,1,-1}$	-1.01 ± 0.02	-0.68 ± 0.02	0.67 ± 0.02
$A^1_{2,0}/\sqrt{2}+1$	0.775 ± 0.001	0.830 ± 0.001	1.070 ± 0.001
$C_{2,2,2,-2}$	0.775 ± 0.003	0.766 ± 0.004	0.987 ± 0.004
$-C_{1,0,1,0}$	0.77 ± 0.02	0.49 ± 0.03	0.63 ± 0.02
$C_{2,0,2,0}$	1.225 ± 0.004	1.240 ± 0.006	1.012 ± 0.004
$C_{1,0,1,0}+2$	1.23 ± 0.02	1.51 ± 0.03	1.23 ± 0.02

All relation between the coefficients are broken and coefficients are getting from 1 to 37 % corrections at NLO EW.

NLO EW Results

$M_{Z_2} > 30$ GeV

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$C_{2,1,2,-1}$	-1.014 ± 0.003	-1.021 ± 0.005	1.007 ± 0.004
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$A^1_{2,0}/\sqrt{2}+1$	0.775 ± 0.001	0.830 ± 0.001	1.070 ± 0.001
$C_{2,2,2,-2}$	0.775 ± 0.003	0.766 ± 0.004	0.987 ± 0.004
$-C_{1,0,1,0}$	0.77 ± 0.02	0.49 ± 0.03	0.63 ± 0.02
$C_{2,0,2,0}$	1.225 ± 0.004	1.240 ± 0.006	1.012 ± 0.004
$C_{1,0,1,0}+2$	1.23 ± 0.02	1.51 ± 0.03	1.23 ± 0.02

All relation between the coefficients are broken and coefficients are getting from 1 to 37 % corrections at NLO EW.

$$\frac{\rho_{\text{NLO}}\left(m_{Z_2} > 30\,\text{GeV}\right)}{\rho_{\text{NLO}}(\text{no cuts})} =$$

$$\begin{pmatrix}
0.56\pm0.05 & & & & & & & & \\
& & & & 0.44\pm0.04 & & & & \\
& & 1.98\pm0.08 & & 1.54\pm0.04 & & 1.35\pm0.01 & & \\
& 0.44\pm0.04 & & & & & & & \\
& & 1.54\pm0.04 & & 0.800\pm0.002 & & 1.54\pm0.04 & & \\
& & & & & & & 0.44\pm0.04 & \\
& & 1.34\pm0.01 & & 1.54\pm0.04 & & 1.98\pm0.08 & & \\
& & & & & 0.44\pm0.04 & & & \\
& & & & & & & & 0.56\pm0.05
\end{pmatrix}$$

NLO EW Results

- The NLO EW corrections changed whole structure of the spin density matrix. Although at LO the quantum state of $H \to 4l$, is still hold the pure state of $Z_a Z_b$.

	LO	NLO
no cuts	0.917 ± 0.007	0.669 ± 0.003
$m_{Z_2} > 30$ GeV	0.956 ± 0.008	0.789 ± 0.008
$85 < m_{Z_1} < 95$ GeV	0.924 ± 0.005	0.696 ± 0.004

Value of purity $\mathrm{Tr}(\rho^2)$

- For mixed state, we can't use same entanglement definition as LO. The lower bound of concurrence

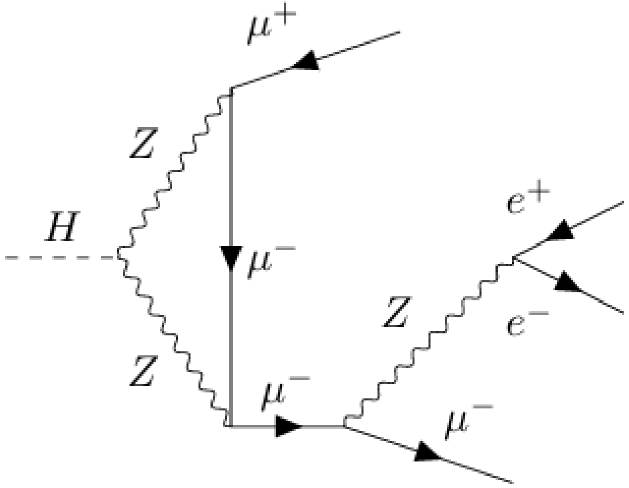
$(\mathcal{C}(\rho))^2$	no cuts	$m_{Z_2} > 30$ GeV	$85 < m_{Z_1} < 95$ GeV
LO	0.93 ± 0.01	1.18 ± 0.02	0.98 ± 0.01
NLO	0.498 ± 0.006	0.87 ± 0.02	0.524 ± 0.007

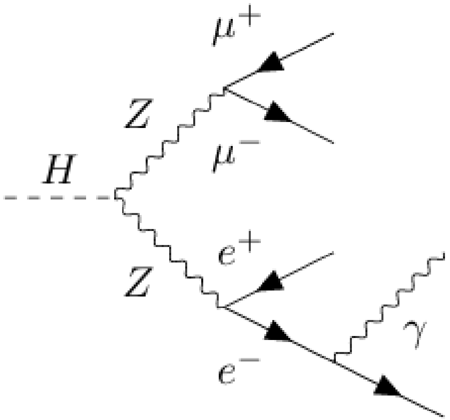
Value of the lower bound of the concurrence squared

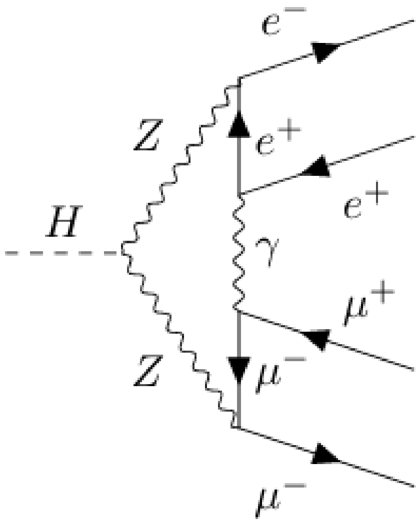
- CGLMP inequality

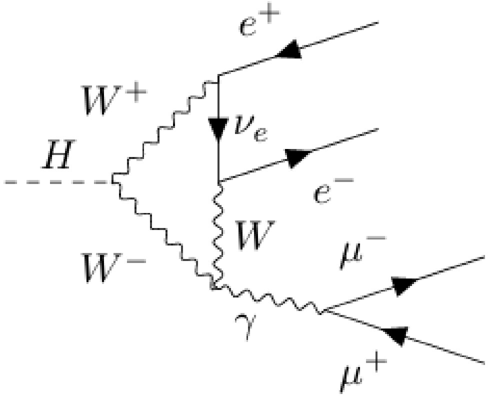
	no cuts	$m_{Z_2} > 30$ GeV	$85 < m_{Z_1} < 95$ GeV
$\mathcal{I}_3$, LO O^ρ_B	2.62 ± 0.02	2.79 ± 0.01	2.64 ± 0.01
$\mathcal{I}_3$, NLO O^ρ_B	2.548 ± 0.009	2.76 ± 0.01	2.597 ± 0.008

NLO EW Results

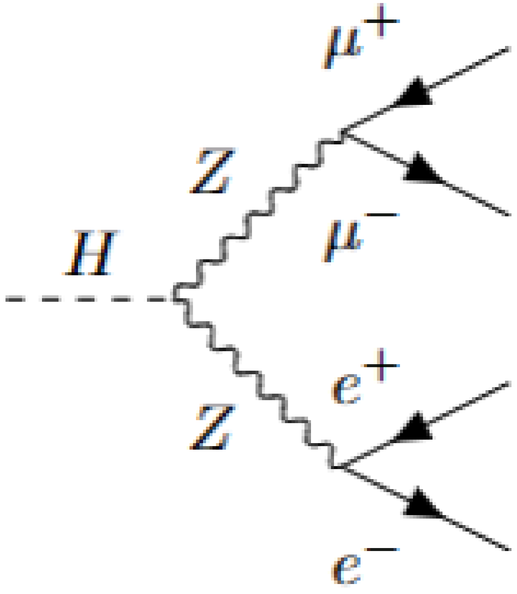








LO diagram



Conclusions of NLO EW corrections part:

- As we see for the top pair, LHC is sensitive to probe quantum observables. So it is time we do computation with precision.
- It is important to construct full spin density matrix using experiment data instead of just computing 2 parameters using LO approximation.
- NLO EW corrections are modifying whole shape of the spin density matrix and also getting contribution from different states instead of only a pure state like LO. This demands the careful use of entanglement observables to measure entanglement.
- It is still possible to look for new physics but we have to do detailed study of the spin density matrix and have to find the quantities which are not strongly affected by NLO EW corrections.
- And we can also look for the parameter space where we can reduce NLO corrections e.g $M_{Z_2} > 30$ GeV although it will also reduce the number of events.
- Stay tuned for final paper: you will find more information with more details.

New Physics contribution in $H \to 4l$

- Through Effective Field Theory (EFT) operators that modify the HVV vertex.
- Through the projection of various intermediate states onto the spin density matrix of bipartite qutrit system.

How NLO correction can be misinterpreted as new physics?

1. What we are measuring at collider? -> four fermion angular momentum distribution generated from Higgs decay. Lets write a generic current for H-> 4f

$$\mathcal{L}^7_{\rm EFT} = \frac{h}{\Lambda^3}\sum_i a_i \bar{\psi}_1 \Gamma^i \psi_2 \, \bar{\psi}_3 \Gamma^i \psi_4,$$

With
$$\Gamma^i = \{1, \gamma_5, \sigma_{\mu\nu}, \gamma_\mu, \gamma_\mu\gamma_5\},$$
$$a_i = \{a_S, a_5, a_T, a_V, a_A\}$$

This is similar to using simplified models with resonant intermediate states.

$$\sum_s \mathcal{M}_S^* \mathcal{M}_S = \frac{1}{\Lambda^6}\Big(16|c_S|^2 \Pi_0 \left(a^2+b^2\right)\left(a'^2+b'^2\right)\Big),$$
$$\sum_s \mathcal{M}_T^* \mathcal{M}_T = \frac{1}{\Lambda^6}\left(128|c_T|^2(2\Pi_1+2\Pi_2-\Pi_0)\right),$$
$$\sum_s (\mathcal{M}_S^* \mathcal{M}_T + \mathcal{M}_S \mathcal{M}_T^*) = -\frac{1}{\Lambda^6}\left(64 \mathrm{Re}(c_S c_T^*)\left[(ab'+a'b)\Pi_\epsilon + (aa'-bb')(\Pi_1-\Pi_2)\right]\right),$$

As we know the A and C coefficient are proportional to amplitude square due to following equation

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_L^{*M}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B_L^j}{4\pi} A_{LM}^j \quad j=a,b$$
$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y_{L_1}^{*M_1}(\Omega_a) Y_{L_2}^{*M_2}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B_{L_1}^a B_{L_2}^b}{(4\pi)^2} C_{L_1 M_1 L_2 M_2}$$

V V

$$A^a_{2,0} = A^b_{2,0} \neq 0$$

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$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

$$\rho_{\text{NLO}} = \begin{pmatrix} 0.08 & 0.00 & 0.00 & 0.01 & -0.01 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.12 & 0.01 & -0.02 & 0.00 & 0.00 & 0.01 \\ 0.00 & 0.00 & 0.12 & 0.01 & -0.18 & 0.00 & 0.19 & 0.00 & 0.00 \\ 0.00 & 0.12 & 0.01 & 0.00 & 0.01 & 0.00 & -0.01 & 0.01 & 0.01 \\ 0.01 & 0.00 & -0.18 & 0.01 & 0.59 & 0.00 & -0.18 & 0.00 & -0.01 \\ 0.00 & 0.02 & 0.00 & 0.00 & 0.00 & -0.01 & 0.01 & 0.14 & 0.00 \\ 0.00 & 0.00 & 0.19 & 0.01 & -0.18 & 0.01 & 0.12 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.01 & 0.00 & 0.14 & 0.01 & 0.01 & 0.00 \\ 0.00 & 0.01 & 0.00 & 0.01 & 0.02 & 0.00 & 0.00 & 0.01 & 0.09 \end{pmatrix}$$

S T

$$\begin{aligned} A^1_{2,0} &= A^2_{2,0} \neq 0, & C_{2,0,2,0} &\neq 0 \\ C_{2,-1,2,1} &= C_{2,1,2,-1} \neq 0 & C_{2,-2,2,2} &= C_{2,2,2,-2} \neq 0 \\ C_{1,-1,1,1} &= C^*_{1,1,1,-1} \neq 0 & C_{1,0,1,0} &\neq 0 \end{aligned}$$

$$\rho = \begin{pmatrix} X_+ + q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_- & 0 & -Y - r^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_+ - q & 0 & Y - r^* & 0 & Z_1 & 0 & 0 \\ 0 & -Y - r & 0 & X_- & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y - r & 0 & Z & 0 & Y - r^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_- & 0 & -Y - r^* & 0 \\ 0 & 0 & Z_1 & 0 & Y - r & 0 & X_+ - q & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Y - r & 0 & X_- & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_+ + q \end{pmatrix}$$

Effect of modified H to ZZ vertex on spin density matrix

$$A_V(EFT) = \frac{1}{v}\left(a_1 g_{\mu\nu} m_V^2 + a_2(g_{\mu\nu} p_a.p_b - p_{a\nu}p_{b\mu}) + a_3 \epsilon_{\mu\nu\alpha\beta} p_a^\alpha p_b^\beta\right) *$$
$$\bar{u}(p_1)\gamma_\mu(c_L P_L + c_R P_R)v(p_2)\ \bar{u}(p_3)\gamma_\nu(d_L P_L + d_R P_R)v(p_4)$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_+a_+^* & 0 & a_+a_0^* & 0 & a_+a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0a_+^* & 0 & a_0a_0^* & 0 & a_0a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_-a_+^* & 0 & a_-a_0^* & 0 & a_-a_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Higher Dim Operators

$$\mathcal{L}_{hZZ} = \frac{M_Z^2}{v} a_1 Z_\mu Z^\mu h + \frac{a_2}{4v} h Z_{\mu\nu} Z^{\mu\nu} + \frac{a_3}{4v} h Z_{\mu\nu} \tilde{Z}^{\mu\nu}$$

Again we got pure density matrix of pure state
$|\psi> = a_+|+-> + a_0|0\,0> + a_-|-+>$

$\boxed{a_+ \neq a_-}$

$$a_\pm = \frac{4}{3}\pi m_a m_b \sqrt{(c_L^2 + c_R^2)(d_L^2 + d_R^2)}\left(-2a_1^* m_V^2 + a_2^*(m_a^2 + m_b^2 - m_h^2) \mp i a_3^* \lambda^{1/2}(m_h^2, m_a^2, m_b^2)\right)$$
$$a_0 = \frac{4}{3}\pi \sqrt{(c_L^2 + c_R^2)(d_L^2 + d_R^2)}\left(2a_2^* m_a^2 m_b^2 - a_1^* m_V^2(m_a^2 + m_b^2 - m_h^2)\right)$$

EFT

Complex numbers

$$A^a_{1,0} = -A^b_{1,0}$$
$$A^a_{2,0} = A^b_{2,0}$$
$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = -C_{1,0,1,0}$$
$$C_{1,-1,1,1} = C^*_{1,1,1,-1} = -C_{2,-1,2,1} = -C^*_{2,1,2,-1}$$
$$C_{2,2,2,-2} = C^*_{2,-2,2,2}$$

New non-zero coefficiients

$$C_{1,0,2,0} = -C_{2,0,1,0}$$
$$C_{2,0,2,0} = 2 + C_{1,0,1,0}$$
$$\frac{A^b_{1,0}}{\sqrt{2}} = C_{2,0,1,0}$$
$$C_{1,-1,2,1} = C^*_{1,1,2,-1} = -C_{2,-1,1,1} = -C^*_{2,1,1,-1}$$

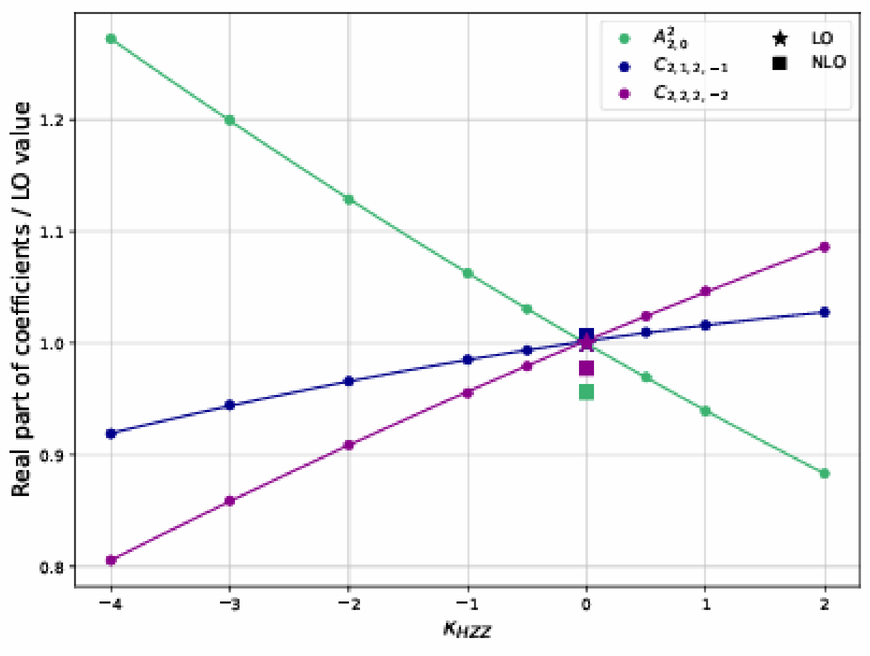
$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{1}{\sqrt{2}}A^2_{2,0}-\sqrt{\frac{3}{2}}A^2_{1,0} & 0 & -C_{2,1,1,-1}+C_{2,1,2,-1} & 0 & C_{2,2,2,-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{2,-1,1,1}+C_{2,-1,2,1} & 0 & 1-\sqrt{2}A^2_{2,0} & 0 & C_{2,1,1,-1}+C_{2,1,2,-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{2,-2,2,2} & 0 & C_{2,-1,1,1}+C_{2,-1,2,1} & 0 & 1+\frac{1}{\sqrt{2}}A^2_{2,0}+\sqrt{\frac{3}{2}}A^2_{1,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

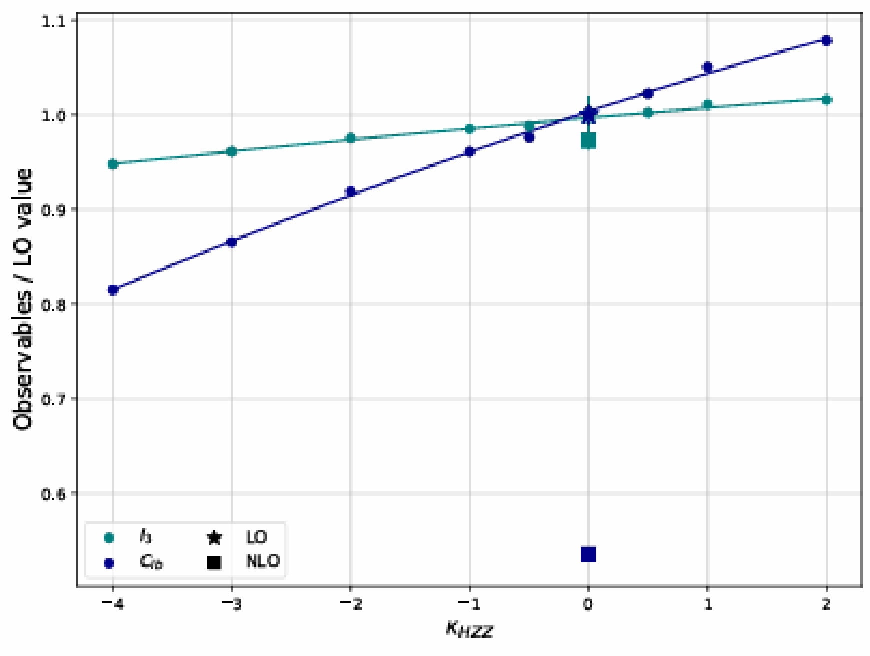
All 9 entries are different from non-zero.

SM

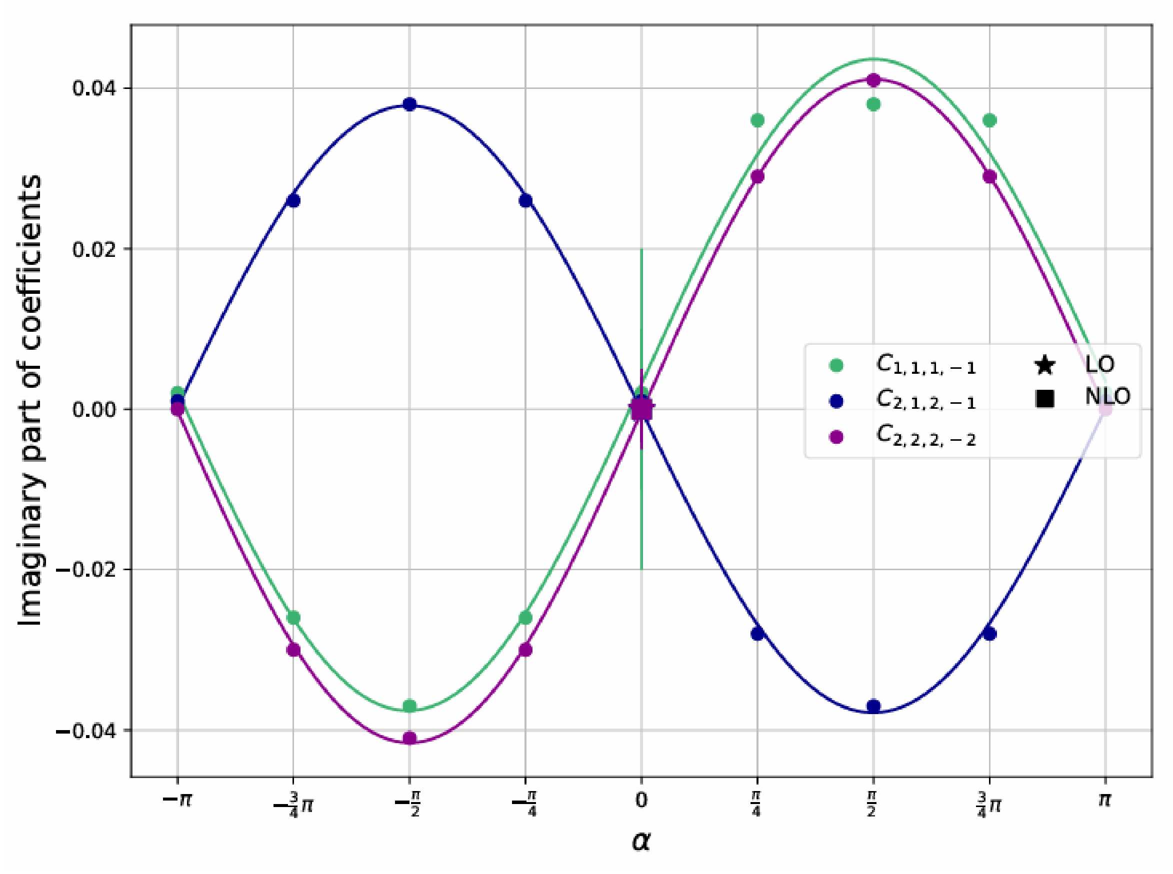
$$A^a_{2,0} = A^b_{2,0} \neq 0$$
$$\boxed{\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}} \neq 0$$
$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0$$
$$\boxed{C_{2,2,2,-2} = -C_{1,0,1,0}} = 2 - C_{2,0,2,0} \neq 0$$

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y & 0 & z & 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & y & 0 & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$





$$\mathcal{L}_{hZZ} = \frac{M_Z^2}{v} a_1 Z_\mu Z^\mu h + \frac{a_2}{4v} h Z_{\mu\nu} Z^{\mu\nu} + \frac{a_3}{4v} h Z_{\mu\nu} \tilde{Z}^{\mu\nu}$$



Imaginary part of coefficients
0.04
0.02
0.00
−0.02
−0.04
$-\pi$
$-\frac{3}{4}\pi$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3}{4}\pi$
π
α
$C_{1,1,1,-1}$
$C_{2,1,2,-1}$
$C_{2,2,2,-2}$
LO
NLO

Thank you for the attention!