

# Feynman rules for the Boltzmann equation

Peter Maták

In collaboration with T. Blažek



COMENIUS  
UNIVERSITY  
BRATISLAVA

National Centre for Nuclear Research in Warsaw, Poland

5 November 2024

# Outline of this seminar

## Part I.

- From unitarity to quantum statistics [Eur. Phys. J. C 81 (2021) 1050]
- Anomalous thresholds and thermal masses [Eur. Phys. J. C 82 (2022) 214]
- On-shell intermediate states [Phys. Rev. D 109 (2024) 043008]

## Part II.

- $CP$  asymmetries from vacuum diagrams [Phys. Rev. D 103 (2021) L091302]
- Leptogenesis from asymmetric scatterings of massless particles [Phys. Rev. D 110 (2024) 055042]

# Part I.

## Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 \quad (1)$$

Which processes contribute to  $\phi_1$  evolution in the expanding universe?

# Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 \quad (1)$$

$$\dot{n}_{\phi_1} + 3Hn_{\phi_1} = -\langle \Gamma_\Phi \rangle \left( \frac{n_{\phi_1}}{n_{\phi_1}^{\text{eq}}} - 1 \right) n_\Phi^{\text{eq}} \quad \langle \Gamma_\Phi \rangle = \gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} / n_\Phi^{\text{eq}} \quad (2)$$

$$\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \int [dk_1][dk_2] (2\pi)^4 \delta^{(4)}(p_\Phi - k_1 - k_2) |M_{\Phi \rightarrow \phi_1 \phi_2}|^2 \quad (3)$$

$$[dk] = \frac{d^3k}{(2\pi)^3 2E_k} \quad S_{fi} = \mathbb{1}_{fi} + i T_{fi} = \mathbb{1}_{fi} + i(2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi}$$

# Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 \quad (1)$$

$$\dot{n}_{\phi_1} + 3Hn_{\phi_1} = -\langle \Gamma_\Phi \rangle \left( \frac{n_{\phi_1}}{n_{\phi_1}^{\text{eq}}} - 1 \right) n_\Phi^{\text{eq}} \quad \langle \Gamma_\Phi \rangle = \gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} / n_\Phi^{\text{eq}} \quad (2)$$

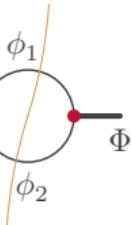
$$\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \int [dk_1][dk_2] \frac{1}{V_4} |T_{\Phi \rightarrow \phi_1 \phi_2}|^2 \quad (3)$$

$$[dk] = \frac{d^3 k}{(2\pi)^3 2E_k} \quad S_{fi} = \mathbb{1}_{fi} + i T_{fi} = \mathbb{1}_{fi} + i(2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi}$$

# Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 \quad (1)$$

$$\dot{n}_{\phi_1} + 3Hn_{\phi_1} = -\langle\Gamma_\Phi\rangle \left( \frac{n_{\phi_1}}{n_{\phi_1}^{\text{eq}}} - 1 \right) n_\Phi^{\text{eq}} \quad \langle\Gamma_\Phi\rangle = \gamma_{\Phi \rightarrow \phi_1\phi_2}^{\text{eq}} / n_\Phi^{\text{eq}} \quad (2)$$

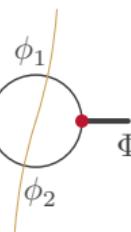
$$\gamma_{\Phi \rightarrow \phi_1\phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \times - \begin{array}{c} \phi_1 \\ \phi_2 \\ \hline \Phi \end{array} \quad (3)$$


$$[dk] = \frac{d^3k}{(2\pi)^3 2E_k} \quad |T_{fi}|^2 = -i T_{if}^\dagger i T_{fi}$$

## Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 \quad (1)$$

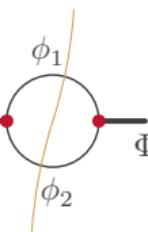
Which processes contribute to  $\phi_1$  evolution in the expanding universe?

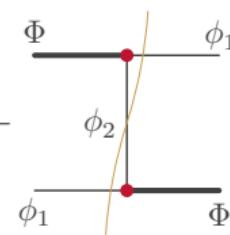
$$\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \times -\Phi \circlearrowleft \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \Phi \quad (3)$$


# Boltzmann equation: unitary completion

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 \quad (1)$$

Which processes contribute to  $\phi_1$  evolution in the expanding universe?

$$\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \times - \begin{array}{c} \phi_1 \\ \circ \\ \phi_2 \end{array} \quad (3)$$


$$\gamma_{\Phi \phi_1 \rightarrow \phi_1 \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \int [dk_1] e^{-E_1/T} \times - \begin{array}{c} \Phi & \phi_1 \\ & \circ \\ & \phi_2 \\ \phi_1 & \Phi \end{array} ? \quad (4)$$


# Boltzmann equation: unitary completion

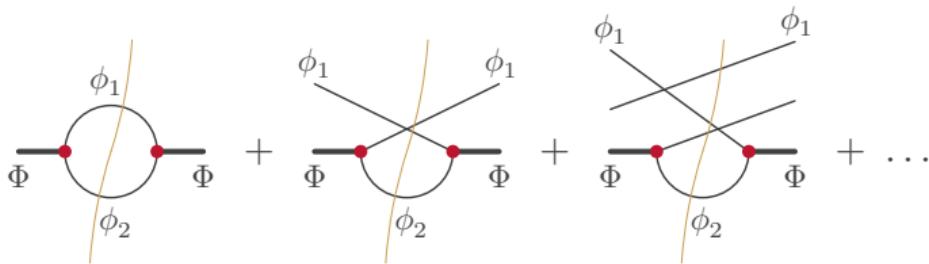
$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 \quad (1)$$

Which processes contribute to  $\phi_1$  evolution in the expanding universe?

$$\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \times - \begin{array}{c} \phi_1 \\ \Phi \end{array} \circlearrowleft \begin{array}{c} \phi_2 \\ \Phi \end{array} \quad (3)$$

$$\gamma_{\Phi \phi_1 \rightarrow \phi_1 \phi_1 \phi_2}^{\text{eq}} = \int [dp_\Phi] e^{-E_\Phi/T} \int [dk_1] e^{-E_1/T} \times - \begin{array}{c} \phi_1 \\ \Phi \end{array} \circlearrowleft \begin{array}{c} \phi_1 \\ \Phi \end{array} \quad ? \quad (4)$$

# Boltzmann equation: unitary completion



$$\int [dp_\Phi] e^{-E_\Phi/T} \int [dk_1][dk_2] \left[ 1 + \frac{1}{e^{E_1/T} - 1} \right] (2\pi)^4 \delta^{(4)}(p_\Phi - k_1 - k_2) |M_{\Phi \rightarrow \phi_1 \phi_2}|^2 \quad (5)$$

[Blažek, Maták '21b]

$T_{fi}^* \neq T_{if}$  and holomorphic cuts

$$S = \mathbb{1} + iT \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$T_{fi}^* \neq T_{if}$  and holomorphic cuts

$$S = \mathbb{1} + iT \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$$S^\dagger S = \mathbb{1} \quad \rightarrow \quad i T^\dagger = i T - i T i T^\dagger \quad (7)$$

$T_{fi}^* \neq T_{if}$  and holomorphic cuts

$$S = \mathbb{1} + iT \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$$S^\dagger S = \mathbb{1} \quad \rightarrow \quad i T^\dagger = i T - i T i T^\dagger \quad (7)$$

$$|T_{fi}|^2 = -i T_{if}^\dagger i T_{fi} = -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_{n,k} i T_{in} i T_{nk} i T_{kf} i T_{fi} + \dots \quad (8)$$

[Coster, Stapp '70, Bourjaily, Hannesdottir, *et al.* '21, Hannesdottir, Mizera '22, Blažek, Maták '21a]

$T_{fi}^* \neq T_{if}$  and holomorphic cuts

$$S = \mathbb{1} + iT \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$$S^\dagger S = \mathbb{1} \quad \rightarrow \quad i T^\dagger = i T - i T i T^\dagger \quad (7)$$

$$\gamma_{fi} = \frac{1}{V_4} \int \prod_{k=1}^p [d\mathbf{p}_k] f_{ik}(\mathbf{p}_k) \int \prod_{l=1}^q [d\mathbf{p}_l] \left( -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} + \dots \right) \quad (9)$$

[Blažek, Maták '21a]

# Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$

$$\gamma_{\Phi\phi_1 \rightarrow \phi_1\phi_1\phi_2}^{\text{eq}} \sim - \begin{array}{c} \text{Diagram 1: } \text{Two horizontal lines labeled } \phi_1 \text{ and } \Phi. \text{ A black line segment connects the left } \Phi \text{ point to a red dot on the } \phi_1 \text{ line. A yellow curve starts from the right } \Phi \text{ point, goes up to a red dot on the } \phi_1 \text{ line, and then down to another red dot on the } \phi_2 \text{ line. Labels: } \phi_1, \phi_1, \phi_1, \Phi, \Phi, \phi_2. \\ \text{Diagram 2: } \text{Two horizontal lines labeled } \phi_1 \text{ and } \Phi. \text{ A black line segment connects the left } \Phi \text{ point to a red dot on the } \phi_1 \text{ line. A yellow curve starts from the right } \Phi \text{ point, goes up to a red dot on the } \phi_1 \text{ line, and then down to another red dot on the } \phi_2 \text{ line. Labels: } \phi_1, \phi_1, \phi_1, \Phi, \Phi, \phi_2. \\ \text{Diagram 3: } \text{Two horizontal lines labeled } \phi_1 \text{ and } \Phi. \text{ A black line segment connects the left } \Phi \text{ point to a red dot on the } \phi_1 \text{ line. A yellow curve starts from the right } \Phi \text{ point, goes up to a red dot on the } \phi_1 \text{ line, and then down to another red dot on the } \phi_2 \text{ line. Labels: } \phi_1, \phi_1, \phi_1, \Phi, \Phi, \phi_2. \end{array} - + \quad (11)$$

# Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$

$$\gamma_{\Phi\phi_1 \rightarrow \phi_1\phi_1\phi_2}^{\text{eq}} \sim - \begin{array}{c} \text{Diagram 1: } \text{Two } \phi_1 \text{ lines meeting at a vertex connected to a } \Phi \text{ line.} \\ \text{Diagram 2: } \text{Two } \phi_1 \text{ lines meeting at a vertex connected to a } \Phi \text{ line.} \\ \text{Diagram 3: } \text{Two } \phi_1 \text{ lines meeting at a vertex connected to a } \Phi \text{ line.} \end{array} - + \quad (11)$$

$$\frac{1}{k^2 + i\epsilon} = \text{P.V.} \cdot \frac{1}{k^2} - i\pi\delta(k^2) \quad \begin{array}{c} \text{Diagram 1: } \text{A } \phi_1 \text{ line meeting a } \Phi \text{ line at a vertex, with a vertical } \phi_1 \text{ line attached to the } \Phi \text{ line.} \\ \text{Diagram 2: } \text{A } \phi_1 \text{ line meeting a } \Phi \text{ line at a vertex, with a vertical } \phi_1 \text{ line attached to the } \Phi \text{ line.} \\ \text{Diagram 3: } \text{A } \phi_1 \text{ line meeting a } \Phi \text{ line at a vertex, with a vertical } \phi_1 \text{ line attached to the } \Phi \text{ line.} \end{array} = + \frac{1}{2} \quad (12)$$

# Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$

$$\gamma_{\Phi\phi_1 \rightarrow \phi_1\phi_1\phi_2}^{\text{eq}} \sim -2 \quad \begin{array}{c} \text{Diagram showing a triangle loop with vertices } \phi_1, \phi_2, \Phi \text{ and internal lines } \phi_1, \phi_1, \phi_1. \\ \text{The left vertex is at } \Phi, \text{ the right vertex is at } \Phi, \text{ and the top vertex is at } \phi_1. \end{array} = \dot{m}_{\phi_1}^2(T) \times \frac{\partial \gamma_{\Phi \rightarrow \phi_1\phi_2}^{\text{eq}}}{\partial m_{\phi_1}^2} \quad (13)$$

[Blažek, Maták '22]

$$2\theta(k^0)\delta(k^2)\text{P.V.}\frac{1}{k^2} = -\frac{1}{(k^0 + |\mathbf{k}|)^2} \frac{\partial \delta(k^0 - |\mathbf{k}|)}{\partial k^0} \quad (14)$$

[Frye, *et al.* '19, Racker '19]

# Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$

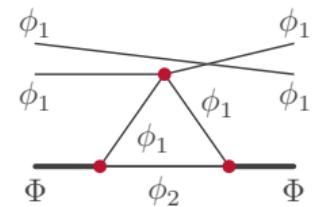
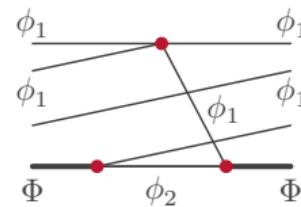
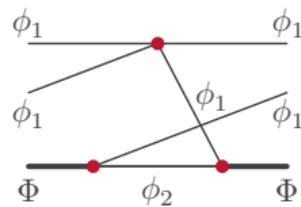
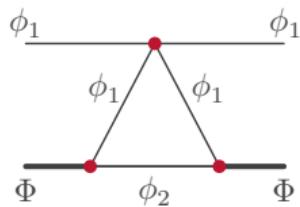
$$\gamma_{\Phi\phi_1 \rightarrow \phi_1\phi_1\phi_2}^{\text{eq}} \sim -2 \quad \begin{array}{c} \text{Diagram showing a triangle loop with vertices } \Phi, \phi_1, \phi_2. \text{ The top edge is labeled } \phi_1, \text{ the left edge is labeled } \phi_1, \text{ and the right edge is labeled } \phi_1. \text{ The bottom horizontal axis is labeled } \Phi, \text{ and the left vertical axis is labeled } \phi_1. \text{ Red dots mark the vertices. Orange lines connect the vertices.} \\ \text{The expression to the right is: } = \mathring{m}_{\phi_1}^2(T) \times \frac{\partial \gamma_{\Phi \rightarrow \phi_1\phi_2}^{\text{eq}}}{\partial m_{\phi_1}^2} \end{array} \quad (13)$$

[Blažek, Maták '22]

$$\mathring{m}_{\phi_1}^2(T) = \lambda \int [d\mathbf{k}_1] e^{-E_1/T} = \frac{\lambda}{4\pi^2} T^2 \quad (15)$$

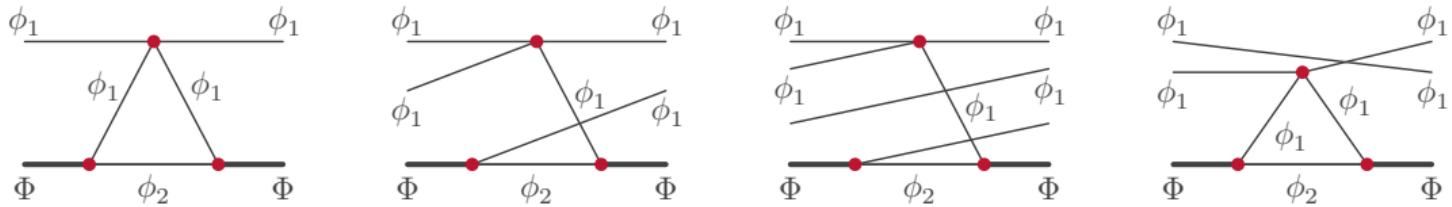
# Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$



# Anomalous thresholds and thermal corrections

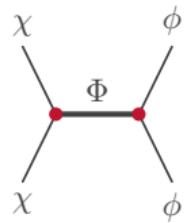
$$\mathcal{L}_{\text{int.}} = -\mu\Phi\phi_1\phi_2 - \frac{1}{4!}\lambda\phi_1^4 \quad (10)$$



$m_{\phi_1}^2(T) \times \frac{\partial \gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}}}{\partial m_{\phi_1}^2}$  with  $m_{\phi_1}^2(T) = \frac{\lambda}{24} T^2$  and quantum statistics in  $\gamma_{\Phi \rightarrow \phi_1 \phi_2}^{\text{eq}}$

## On-shell intermediate states

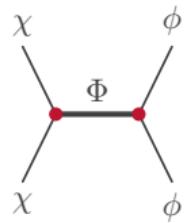
$$\mathcal{L}_{\text{int.}} = -\frac{1}{2}\lambda_{\text{DM}}\Phi\chi^2 - \frac{1}{2}\lambda_{\text{SM}}\Phi\phi^2 \quad (16)$$



$$\left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \left| \frac{1}{s - M^2 + iM\Gamma} \right|^2 \quad (17)$$

## On-shell intermediate states

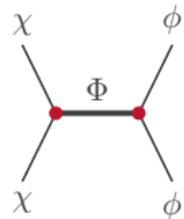
$$\mathcal{L}_{\text{int.}} = -\frac{1}{2}\lambda_{\text{DM}}\Phi\chi^2 - \frac{1}{2}\lambda_{\text{SM}}\Phi\phi^2 \quad (16)$$



$$\left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \left| \frac{1}{s - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(s - M^2) \quad (17)$$

## On-shell intermediate states

$$\mathcal{L}_{\text{int.}} = -\frac{1}{2}\lambda_{\text{DM}}\Phi\chi^2 - \frac{1}{2}\lambda_{\text{SM}}\Phi\phi^2 \quad (16)$$



$$\left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \left| \frac{1}{s - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(s - M^2) \quad (17)$$

$$\gamma_{\chi\chi \rightarrow \phi\phi}^{\text{LO}} \stackrel{\text{def.}}{=} \gamma_{\chi\chi \rightarrow \Phi} \times \text{Br}(\Phi \rightarrow \phi\phi) \quad (18)$$

Resonance in  $\chi\chi \rightarrow \phi\phi$  already included in  $\chi\chi \rightarrow \Phi$  and  $\Phi \rightarrow \phi\phi$ .  
**The double-counting has to be removed!**

[Kolb, Wolfram '80]

## On-shell intermediate states

$$\gamma_{\chi\chi \rightarrow \phi\phi}^{\text{NLO}} \sim - \text{Diagram} ? \quad (19)$$

The diagram illustrates a next-to-leading order (NLO) correction to the annihilation of two scalar particles ( $\chi$ ) into two scalar particles ( $\phi$ ). The process is shown as follows:

- External Lines:** Two incoming  $\chi$  particles enter from the left and right respectively, represented by black lines.
- Intermediate States:** The annihilation is mediated by two  $\Phi$  particles, which are exchange bosons. They are shown as red dots connected by a horizontal black line.
- Final State:** The two  $\phi$  particles are emitted from the vertices of the  $\Phi$  exchange, represented by black lines.
- Question Mark:** A question mark is placed to the right of the final state lines, indicating that the final state is not fully specified or is a ghost state.

## On-shell intermediate states

$$\gamma_{\chi\chi \rightarrow \phi\phi}^{\text{NLO}} \sim - \begin{array}{c} \text{Diagram 1: } \chi \text{ (left)} \xrightarrow{\Phi} \phi \text{ (top loop)} \xrightarrow{\Phi} \chi \text{ (right)} \\ \text{Diagram 2: } \chi \text{ (left)} \xrightarrow{\Phi} \phi \text{ (middle loop)} \xrightarrow{\Phi} \chi \text{ (right)} \\ \text{Diagram 3: } \chi \text{ (left)} \xrightarrow{\Phi} \phi \text{ (bottom loop)} \xrightarrow{\Phi} \chi \text{ (right)} \end{array} - + + - \quad (19)$$

Diagram 1: A Feynman diagram showing two incoming fermion lines labeled  $\chi$  meeting at a vertex labeled  $\Phi$ . The outgoing lines are also  $\chi$ . Between these vertices is a circular loop containing a vertical orange line labeled  $\phi$ . The loop is closed by two vertices labeled  $\Phi$ .

Diagram 2: Similar to Diagram 1, but the vertical orange line labeled  $\phi$  is positioned in the middle of the loop, between the two  $\Phi$  vertices.

Diagram 3: Similar to Diagram 1, but the vertical orange line labeled  $\phi$  is positioned in the bottom part of the loop.

Diagram 4: Similar to Diagram 1, but the vertical orange line labeled  $\phi$  is positioned in the top part of the loop.

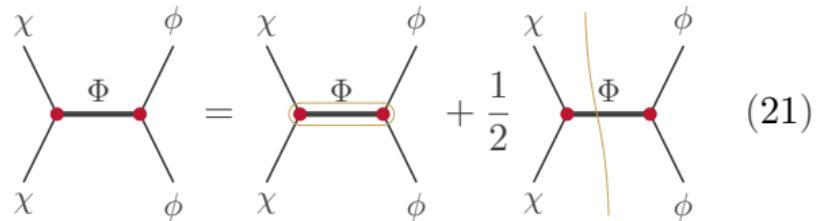
Diagram 5: Similar to Diagram 2, but the vertical orange line labeled  $\phi$  is positioned in the middle part of the loop.

Diagram 6: Similar to Diagram 3, but the vertical orange line labeled  $\phi$  is positioned in the bottom part of the loop.

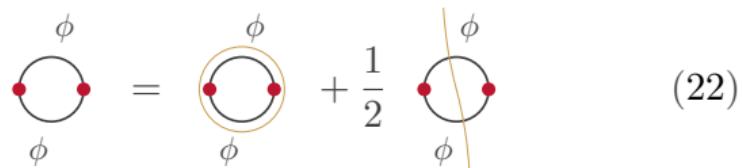
Diagram 7: Similar to Diagram 1, but the vertical orange line labeled  $\phi$  is positioned in the bottom part of the loop.

## On-shell intermediate states

$$\frac{1}{s - M^2 + i\epsilon} = \mathcal{P} \frac{1}{s - M^2} - i\pi\delta(s - M^2)$$

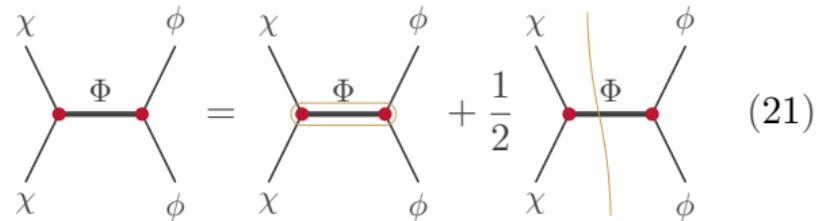


$$\Sigma(s) = \Sigma_R(s) + i\Sigma_I(s)$$



# On-shell intermediate states

$$\frac{1}{s - M^2 + i\epsilon} = \mathcal{P} \frac{1}{s - M^2} - i\pi\delta(s - M^2) \quad (21)$$



$$\Sigma(s) = \Sigma_R(s) + i\Sigma_I(s)$$

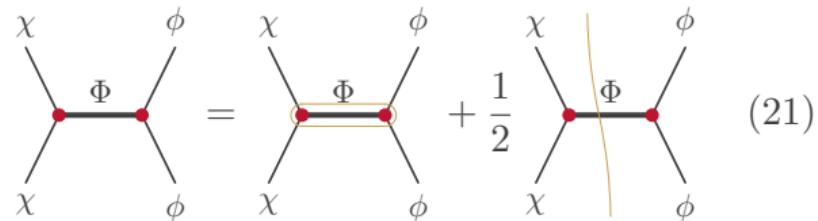
$$= \quad (22)$$

$$-\quad - \quad \propto -\frac{\partial}{\partial s}\pi\delta(s - M^2) \quad (20)$$

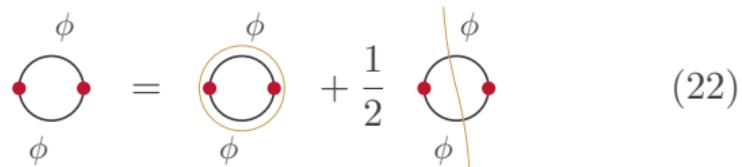
[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

# On-shell intermediate states

$$\frac{1}{s - M^2 + i\epsilon} = \mathcal{P} \frac{1}{s - M^2} - i\pi\delta(s - M^2)$$



$$\Sigma(s) = \Sigma_R(s) + i\Sigma_I(s)$$



Feynman diagram identity (23): The difference between two loop corrections is proportional to the derivative of the propagator with respect to  $s$ . The left term is a bare propagator with a self-energy insertion (orange line) and a bare propagator with a self-energy insertion (orange line). The right term is  $-\frac{1}{4}$  times a bare propagator with a self-energy insertion (orange line) and a bare propagator with a self-energy insertion (orange line) and a vertical gluon line (orange line).

[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

# On-shell intermediate states

$$\frac{\epsilon}{M\Gamma} \left| \frac{1}{s - M^2 + i\epsilon} \right|^2 \rightarrow \frac{\pi}{M\Gamma} \delta(s - M^2) \quad (24)$$

$$\frac{\epsilon}{M\Gamma} \left[ \frac{1}{(s - M^2)^2 - \epsilon^2} - \frac{2\epsilon^2}{[(s - M^2)^2 - \epsilon^2]^2} \right] \rightarrow 0 \quad (25)$$

$$-\frac{1}{4} \left[ \text{Diagram with loop and two ϕ lines} \right] \propto -\frac{\partial}{\partial s} \mathcal{P} \frac{1}{s - M^2} \quad (23)$$

[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

# On-shell intermediate states

$$\gamma_{\chi\chi \rightarrow \phi\phi}^{\text{NLO}} \sim - \text{Diagram 1} - \frac{1}{4} \text{Diagram 2} \quad (26)$$

$$- \text{Diagram 3} - \text{Diagram 4}$$

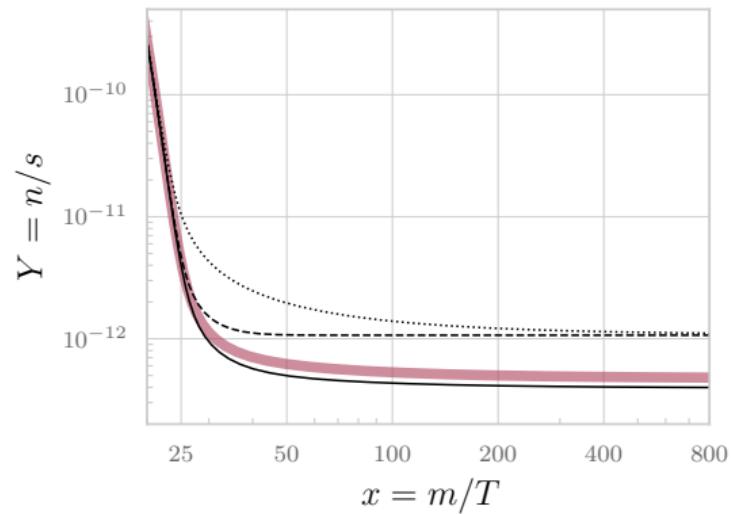
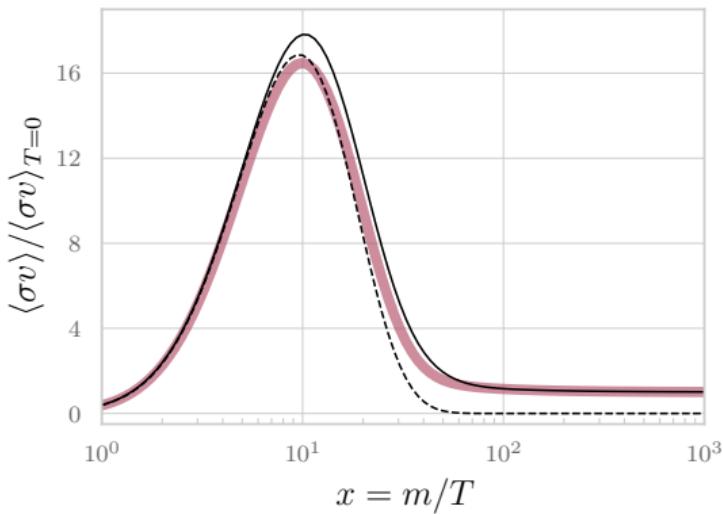
$$\sim 2\lambda_{\text{DM}}^2 \left[ \Sigma_I(s) \frac{\partial}{\partial s} \mathcal{P} \frac{1}{s - M^2} - \Sigma_R(s) \frac{\partial}{\partial s} \pi \delta(s - M^2) \right] \times \text{Br}(\Phi \rightarrow \phi\phi) \quad (27)$$

[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

# Resonant dark matter annihilation

$$\frac{dY}{dx} = - \left( \frac{45}{\pi} G \right)^{-1/2} \frac{g_*^{1/2} m}{x^2} \langle \sigma v \rangle (Y^2 - Y_{\text{eq}}^2) \quad (28)$$

[Bernstein '88; Gondolo, Gelmini '91]



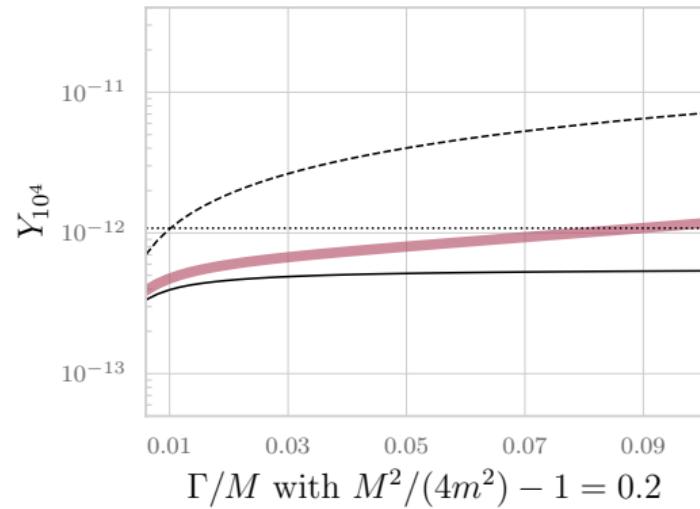
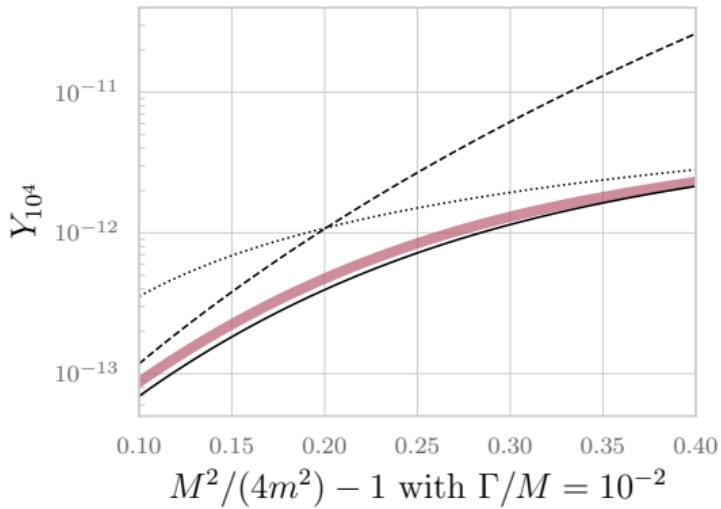
$$M = 100 \text{ TeV}, m = 45 \text{ TeV}, \Gamma/M = 10^{-2}$$

[Maták '24]

# Resonant dark matter annihilation

$$\frac{dY}{dx} = - \left( \frac{45}{\pi} G \right)^{-1/2} \frac{g_*^{1/2} m}{x^2} \langle \sigma v \rangle (Y^2 - Y_{\text{eq}}^2) \quad (28)$$

[Bernstein '88; Gondolo, Gelmini '91]



[Maták '24]

## Where it all comes from?

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp \left\{ - \sum_p F_p a_p^\dagger a_p \right\} \quad \leftarrow \quad Z = \prod_p Z_p = \prod_p \frac{\exp F_p}{\exp F_p - 1} \quad (29)$$

$$\exp\{-E_p/T\} \quad \rightarrow \quad \exp\{-F_p\} = \frac{f_p}{1 \pm f_p} \quad f_p = \text{Tr} [a_p^\dagger a_p \rho] \quad (30)$$

[Wagner '91]

## Where it all comes from?

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp \left\{ - \sum_p F_p a_p^\dagger a_p \right\} \quad \leftarrow \quad Z = \prod_p Z_p = \prod_p \frac{\exp F_p}{\exp F_p - 1} \quad (29)$$

$$\exp\{-E_p/T\} \quad \rightarrow \quad \exp\{-F_p\} = \frac{f_p}{1 \pm f_p} \quad f_p = \text{Tr} [a_p^\dagger a_p \rho] \quad (30)$$

[Wagner '91]

$$\rho' = S \rho S^\dagger \quad \rightarrow \quad (1 + i T) \rho (1 - i T + i T i T - \dots) \quad (31)$$

The collision term for the Boltzmann equation is obtained as  $\text{Tr} [a_p^\dagger a_p (\rho - \rho')] / V_4$ .

[McKellar, Thomson '94, Blažek, Maták '21b]

## Summary I.

- Unitarity may help in calculating reaction rates for the Boltzmann equation.

$$\gamma_{fi}^{\text{eq}} = \frac{1}{V_4} \int \prod_{k=1}^p [d\mathbf{p}_k] f_{i_k}^{\text{eq}}(\mathbf{p}_k) \int \prod_{l=1}^q [d\mathbf{p}_l] \left( -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} + \dots \right)$$

- Completing diagrams by all possible winding numbers accounts for quantum statistics.
- Anomalous thresholds approximate thermal-mass effects in lower-order process kinematics.
- There is no double-counting of on-shell intermediate states in fixed-order results.

## Part II.

## Imaginary kinematics in Feynman diagrams

$$i T_{if}^* - i T_{fi} = \sum_n T_{nf}^* T_{ni} \quad T_{fi} = \sum_{\text{diagrams}} C_{fi} K_{fi} \quad C_{fi} = C_{if}^*, K_{fi} = K_{if} \quad (32)$$

# Imaginary kinematics in Feynman diagrams

$$i T_{if}^* - i T_{fi} = \sum_n T_{nf}^* T_{ni} \quad T_{fi} = \sum_{\text{diagrams}} C_{fi} K_{fi} \quad C_{fi} = C_{if}^*, K_{fi} = K_{if} \quad (32)$$

$$2 \operatorname{Im} K_{fi} = \sum_n K_{fn}^* K_{ni} \quad (33)$$

[Cutkosky '60; Veltman '63]

## CP asymmetries and unitarity constraints

$$S^\dagger S = SS^\dagger \quad \rightarrow \quad \sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \text{for } i T = S - \mathbb{1} \quad (34)$$

$$CPT \text{ symmetry} \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{\bar{f}\bar{i}}|^2 = |T_{fi}|^2 - |T_{if}|^2 \quad (35)$$

## CP asymmetries and unitarity constraints

$$S^\dagger S = SS^\dagger \quad \rightarrow \quad \sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \text{for} \quad i T = S - \mathbb{1} \quad (34)$$

$$CPT \text{ symmetry} \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{\bar{f}\bar{i}}|^2 = |T_{fi}|^2 - |T_{if}|^2 \quad (35)$$

$$\sum_f \Delta |T_{fi}|^2 = \sum_i \Delta |T_{if}|^2 = 0 \quad (36)$$

[Dolgov '79; Kolb, Wolfram '80, See also Hook '11, Baldes, Bell, Petraki, Volkas '14]

## CP asymmetries and unitarity constraints

$$S^\dagger S = SS^\dagger \quad \rightarrow \quad \sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \text{for} \quad i T = S - \mathbb{1} \quad (34)$$

$$CPT \text{ symmetry} \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{\bar{f}\bar{i}}|^2 = |T_{fi}|^2 - |T_{if}|^2 \quad (35)$$

$$\sum_f \Delta |T_{fi}|^2 = \sum_i \Delta |T_{fi}|^2 = 0 \quad (36)$$

[Dolgov '79; Kolb, Wolfram '80, See also Hook '11, Baldes, Bell, Petraki, Volkas '14]

$$\left. \begin{array}{l} T_{fi} = C_{\text{tree}} K_{\text{tree}} + C_{\text{loop}} K_{\text{loop}} \\ T_{if} = C_{\text{tree}}^* K_{\text{tree}} + C_{\text{loop}}^* K_{\text{loop}} \end{array} \right\} \quad \Delta |T_{fi}|^2 = -4 \operatorname{Im}[C_{\text{tree}} C_{\text{loop}}^*] \operatorname{Im}[K_{\text{tree}} K_{\text{loop}}^*] \quad (37)$$

## CP asymmetries and unitarity constraints

$$S^\dagger S \rightarrow T = T^\dagger + i T^\dagger T \quad (38)$$

$$\Delta |T_{fi}|^2 = \left| T_{if}^* + i \sum_n T_{fn}^\dagger T_{ni} \right|^2 - |T_{if}|^2 = -2 \operatorname{Im} \left[ \sum_n T_{if} T_{fn}^\dagger T_{ni} \right] + \left| \sum_n T_{fn}^\dagger T_{ni} \right|^2 \quad (39)$$

[Kolb, Wolfram '80]

## CP asymmetries and unitarity constraints

$$S^\dagger S \rightarrow T = T^\dagger + i T^\dagger T \quad (38)$$

$$\Delta |T_{fi}|^2 = \left| T_{if}^* + i \sum_n T_{fn}^\dagger T_{ni} \right|^2 - |T_{if}|^2 = -2 \operatorname{Im} \left[ \sum_n T_{if} T_{fn}^\dagger T_{ni} \right] + \left| \sum_n T_{fn}^\dagger T_{ni} \right|^2 \quad (39)$$

[Kolb, Wolfram '80]

No further on-shell cuts means  $T_{if}^* = T_{fi}$   $\rightarrow \Delta |T_{fi}|^2 = -2 \operatorname{Im} \left[ \sum_n T_{if} T_{fn} T_{ni} \right]$  (40)

# CP asymmetries and unitarity constraints

$$S^\dagger S \rightarrow T = T^\dagger + i T^\dagger T \quad (38)$$

$$\Delta |T_{fi}|^2 = \left| T_{if}^* + i \sum_n T_{fn}^\dagger T_{ni} \right|^2 - |T_{if}|^2 = -2 \operatorname{Im} \left[ \sum_n T_{if} T_{fn}^\dagger T_{ni} \right] + \left| \sum_n T_{fn}^\dagger T_{ni} \right|^2 \quad (39)$$

[Kolb, Wolfram '80]

No further on-shell cuts means  $T_{if}^* = T_{fi}$   $\rightarrow \Delta |T_{fi}|^2 = -2 \operatorname{Im} \left[ \sum_n T_{if} T_{fn} T_{ni} \right]$  (40)

$$\Delta |T_{fi}|^2 = \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_n i T_{if} i T_{fn} i T_{ni} \quad (41)$$

[Covi, Roulet, Vissani '98]

# CP asymmetries and unitarity constraints



2 April 1998

PHYSICS LETTERS B

Physics Letters B 424 (1998) 101–105

E. Roulet et al. / Physics Letters B 424 (1998) 101–105

103

## On the CP asymmetries in Majorana neutrino decays

Esteban Roulet <sup>a</sup>, Laura Covi <sup>b</sup>, Francesco Vissani <sup>c</sup>

<sup>a</sup> Depto. de Física Teórica, Universidad de Valencia, E-46100 Burjasot, Valencia, Spain

<sup>b</sup> School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YL, United Kingdom

<sup>c</sup> Deutsches Elektronen Synchrotron, DESY, 22603 Hamburg, Germany

Received 12 January 1998

Editor: R. Gatto

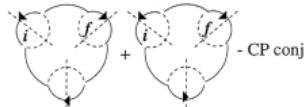


Fig. 3. Pictorial representation of the self-energy contribution to the cross section asymmetry  $\epsilon_\sigma$ .

Therefore, one can pictorially represent the self-energy contributions to  $\sigma(\ell^+ H^+ \rightarrow \ell^- H)$  as in Fig. 2, where the cut blobs are the initial ( $i$ ) and final ( $f$ ) states, while the remaining blob stands for the one-loop self-energy. This last is actually the sum of two contributions, one with a lepton and one with an antilepton.

Now, in the computation of  $\epsilon_\sigma$ , only the absorptive part of the loop will contribute, and the Cutkoski

Turning now to the vertex contributions, to see the cancellations we need to add the three contributions shown in Fig. 4 (including the tree level  $u$ -channel interfering with the one-loop self-energy diagram). In terms of cut diagrams, this can be expressed as in Fig. 5, where CP-conjugate stands for the same four diagrams with all the arrows reversed. Hence, the CP-conjugate contribution will exactly cancel the four diagrams, since changing the directions of the arrows just exchanges among themselves the first and fourth diagrams, as well as the second and third ones. We then see explicitly that the absorptive part of the self-energies are also playing here a crucial role, enforcing the cancellation of the CP violation produced by the vertex diagrams.

The only remaining diagrams to be considered are the interference of the one-loop vertex diagrams with the tree level  $u$ -channel. Pictorially, they are represented in Fig. 6, and they again cancel since the two

$$\Delta |T_{fi}|^2 = \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_n i T_{if} i T_{fn} i T_{ni} \quad (41)$$

[Covi, Roulet, Vissani '98]

## Asymmetries with holomorphic cuts

$$S = 1 + i T \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$$S^\dagger S = 1 \quad \rightarrow \quad i T^\dagger = i T - i T i T^\dagger \quad (7)$$

$$|T_{fi}|^2 = -i T_{if}^\dagger i T_{fi} = -i T_{if} i T_{fi} + \sum_n i T_{in} i T_{nf} i T_{fi} - \sum_{n,k} i T_{in} i T_{nk} i T_{kf} i T_{fi} + \dots \quad (8)$$

[Coster, Stapp '70, Bourjaily, Hannesdottir, *et al.* '21, Hannesdottir, Mizera '22, Blažek, Maták '21a]

## Asymmetries with holomorphic cuts

$$S = 1 + i T \quad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \quad (6)$$

$$\begin{aligned} \Delta |T_{fi}|^2 &= |T_{fi}|^2 - |T_{if}|^2 = \sum_n \left( i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) \\ &\quad - \sum_{n,k} \left( i T_{in} i T_{nk} i T_{kf} i T_{fi} - i T_{if} i T_{fk} i T_{kn} i T_{ni} \right) \\ &\quad + \dots \end{aligned} \quad (42)$$

[Blažek, Maták '21a, see also Roulet, Covi, Vissani '98]

## Consequences for the asymmetry generation

$$\Delta \dot{n}_{f_1} + 3H\Delta n_{f_1} = \sum_i \sum_{f \ni f_1} \left( \frac{n_{i_1}}{n_{i_1}^{\text{eq}}} \frac{n_{i_2}}{n_{i_2}^{\text{eq}}} \cdots \frac{n_{i_p}}{n_{i_p}^{\text{eq}}} - 1 \right) \times \Delta \gamma_{fi}^{\text{eq}} + \text{wash-out terms} \quad (43)$$

$f_1$  in the final state of the contributing processes  
out-of-equilibrium initial state

$\Delta n_{f_1}$  source term

[detailed derivation in Racker '19]

## Example: Leptogenesis with Dirac neutrinos

- Introduced in Phys. Rev. Lett. **84** (2000) 4039 [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons

$$Y_B = \frac{28}{79} Y_{B-L_{\text{SM}}} = \frac{28}{79} \Delta_{\nu_R} \quad (44)$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

## Example: Leptogenesis with Dirac neutrinos

- Introduced in Phys. Rev. Lett. **84** (2000) 4039 [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons

$$Y_B = \frac{28}{79} Y_{B-L_{\text{SM}}} = \frac{28}{79} \Delta_{\nu_R} \quad (44)$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L X_i^\dagger + \bar{e}_R^c G_i \nu_R X_i^\dagger + \text{H.c.} \quad (45)$$

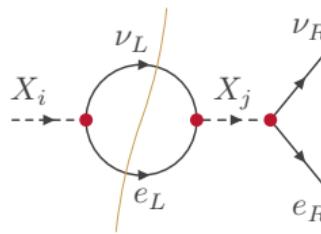
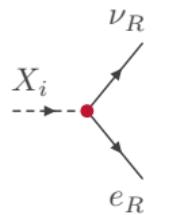
[Heeck, Heisig, Thapa '23a]

# Example: Leptogenesis with Dirac neutrinos

- Introduced in Phys. Rev. Lett. **84** (2000) 4039 [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons

$$Y_B = \frac{28}{79} Y_{B-L_{\text{SM}}} = \frac{28}{79} \Delta_{\nu_R} \quad (44)$$

[Kuzmin, Rubakov, Shaposhnikov '85, Harvey, Turner '90]



[Heeck, Heisig, Thapa '23a]

## Example: Leptogenesis with Dirac neutrinos

- Introduced in Phys. Rev. Lett. **84** (2000) 4039 [Dick, Lindner, Ratz, and Wright 2000]
- Lepton-number conserving decays of heavy particles
- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons

$$Y_B = \frac{28}{79} Y_{B-L_{\text{SM}}} = \frac{28}{79} \Delta_{\nu_R} \quad (44)$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

$$\Delta |T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta |T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (46)$$

## Example: Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 = \text{---} \circlearrowleft \nu_R \circlearrowright X_j \circlearrowleft X_i \text{---} - \text{---} \circlearrowleft \nu_L \circlearrowright X_j \circlearrowleft X_i \text{---} \quad (47)$$

$$\Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = \text{---} \circlearrowleft \nu_L \circlearrowright X_j \circlearrowleft X_i \text{---} - \text{---} \circlearrowleft \nu_R \circlearrowright X_j \circlearrowleft X_i \text{---} \quad (48)$$

$$\Delta|T_{X_i \rightarrow \nu_R e_R}|^2 + \Delta|T_{X_i \rightarrow \nu_L e_L}|^2 = 0 \quad (46)$$

## Example: Leptogenesis with Dirac neutrinos

$$\Delta|T_{\nu_R e_R \rightarrow X_i}|^2 = \begin{array}{c} \text{Diagram 1: } \nu_R \text{ enters } X_i, \text{ then } X_i \text{ enters } e_L \text{ (clockwise), } e_L \text{ enters } X_j, \text{ then } X_j \text{ enters } e_R \\ \text{Diagram 2: } \nu_R \text{ enters } X_j, \text{ then } X_j \text{ enters } e_L \text{ (clockwise), } e_L \text{ enters } X_i, \text{ then } X_i \text{ enters } e_R \end{array} - \quad (49)$$

$$\Delta|T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = \begin{array}{c} \text{Diagram 1: } \nu_R \text{ enters } X_j, \text{ then } X_j \text{ enters } e_L \text{ (clockwise), } e_L \text{ enters } X_i, \text{ then } X_i \text{ enters } e_R \\ \text{Diagram 2: } \nu_R \text{ enters } X_i, \text{ then } X_i \text{ enters } e_L \text{ (clockwise), } e_L \text{ enters } X_j, \text{ then } X_j \text{ enters } e_R \end{array} - \quad (50)$$

$$\Delta|T_{\nu_R e_R \rightarrow X_i}|^2 + \Delta|T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = 0 \quad (51)$$

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L X_i^\dagger + \bar{e}_R^c G_i \nu_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (52)$$

[Heeck, Heisig, Thapa '23b]

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L X_i^\dagger + \bar{e}_R^c G_i \nu_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (52)$$

[Heeck, Heisig, Thapa '23b]

~~$$\Delta |T_{\nu_R e_R \rightarrow X_i}|^2 + \Delta |T_{\nu_R e_R \rightarrow \nu_L e_L}|^2 = 0 \quad (53)$$~~

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2} \bar{L}^c F_i L X_i^\dagger + \bar{e}_R^c G_i \nu_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (52)$$

[Heeck, Heisig, Thapa '23b]

$SU(3) \times SU(2) \times U(1)$	spin	$(B - L)(X)$	asymmetry-generating operators
$(\mathbf{1}, \mathbf{1}, -1)$	0	-2	$\nu_R e_R X^\dagger, LLX^\dagger$
$(\mathbf{1}, \mathbf{2}, 1/2)$	0	0	$\bar{H}X, \bar{\nu}_R LX, \bar{L}e_R X, \bar{Q}d_R X, \bar{u}_R QX, X^\dagger H^\dagger HH$
$(\mathbf{3}, \mathbf{1}, -1/3)$	0	-2/3	$d_R \nu_R X^\dagger, u_R e_R X^\dagger, QLX^\dagger, u_R d_R X, QQX$
$(\mathbf{3}, \mathbf{1}, 2/3)$	0	-2/3	$u_R \nu_R X^\dagger, d_R d_R X$
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	4/3	$\bar{Q}\nu_R X, \bar{d}_R LX$
$(\mathbf{1}, \mathbf{2}, -1/2)$	1/2	-1	$\bar{X}L, \bar{\nu}_R XH, \bar{X}e_R H$

[Heeck, Heisig, Thapa '23a]

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (54)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

$SU(3) \times SU(2) \times U(1)$	spin	$(B - L)(X)$	asymmetry-generating operators
$(\mathbf{1}, \mathbf{1}, -1)$	0	-2	$\nu_R e_R X^\dagger, LLX^\dagger$
$(\mathbf{1}, \mathbf{2}, 1/2)$	0	0	$\bar{H}X, \bar{\nu}_R LX, \bar{L}e_R X, \bar{Q}d_R X, \bar{u}_R QX, X^\dagger H^\dagger HH$
$(\mathbf{3}, \mathbf{1}, -1/3)$	0	-2/3	$d_R \nu_R X^\dagger, u_R e_R X^\dagger, QLX^\dagger, u_R d_R X, QQX$
$(\mathbf{3}, \mathbf{1}, 2/3)$	0	-2/3	$u_R \nu_R X^\dagger, d_R d_R X$
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	4/3	$\bar{Q}\nu_R X, \bar{d}_R LX$
$(\mathbf{1}, \mathbf{2}, -1/2)$	1/2	-1	$\bar{X}L, \bar{\nu}_R XH, \bar{X}e_R H$

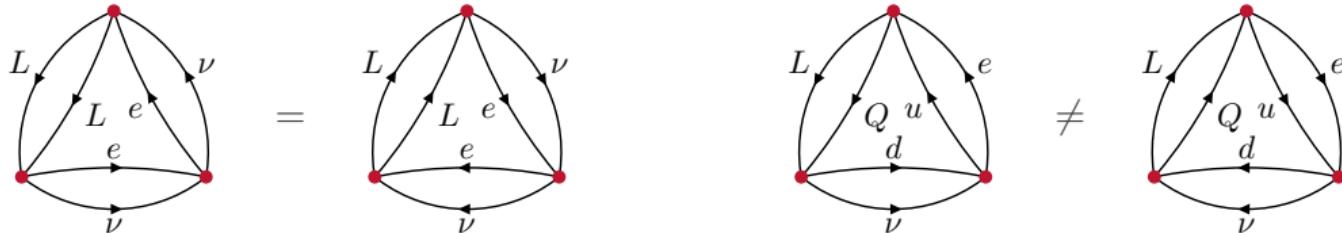
[Heeck, Heisig, Thapa '23a]

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (54)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

$$\Delta |T_{fi}|^2 = \sum_n \left( i T_{in} i T_{nf} i T_{fi} - i T_{if} i T_{fn} i T_{ni} \right) - \dots$$



[see also Roulet, Covi, Vissani '98, Botella, Nebot, Vives '06]

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (54)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

- $B$  and  $L$  individually conserved
- first generation only, ignoring SM interactions at  $T_{\text{reh}} > 3 \times 10^{13}$  GeV

[Bento '03, Garbrecht, Schwaller '14]

# Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \quad M_X \gg T_{\text{reh}} \quad (54)$$

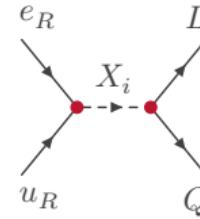
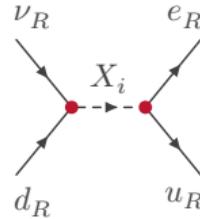
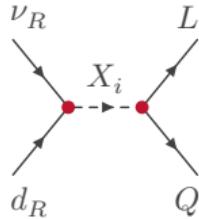
[Blažek, Heeck, Heisig, Maták, Zaujec '24]

- $B$  and  $L$  individually conserved
- first generation only, ignoring SM interactions at  $T_{\text{reh}} > 3 \times 10^{13}$  GeV

[Bento '03, Garbrecht, Schwaller '14]

$$\left. \begin{array}{lcl} \Delta_{d_R} + \Delta_{u_R} + \Delta_Q = 0 & \Delta_{\nu_R} + \Delta_{e_R} + \Delta_L = 0 \\ \Delta_{\nu_R} = \Delta_{d_R} & \Delta_L = \Delta_Q & \Delta_{e_R} = \Delta_{u_R} \end{array} \right\} \quad \Delta_a \equiv \frac{n_a - n_{\bar{a}}}{s} \quad (55)$$

# Dirac leptogenesis without heavy particles?



$$\langle \sigma_1 v \rangle = \frac{16}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{F_i^* F_j G_j^* G_i}{M_i^2 M_j^2} \equiv \frac{16}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_1}{\zeta(3)^2} \approx \frac{T^2}{T_{\text{reh}}^4} \alpha_1 \quad (56)$$

$$\langle \sigma_2 v \rangle = \frac{8}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{K_i^* K_j G_j^* G_i}{M_i^2 M_j^2} \equiv \frac{8}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_2}{\zeta(3)^2} \approx \frac{1}{2} \frac{T^2}{T_{\text{reh}}^4} \alpha_2 \quad (57)$$

$$\langle \sigma_3 v \rangle = \frac{16}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{F_i^* F_j K_j^* K_i}{M_i^2 M_j^2} \equiv \frac{16}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_3}{\zeta(3)^2} \approx \frac{T^2}{T_{\text{reh}}^4} \alpha_3 \quad (58)$$

## Dirac leptogenesis without heavy particles?

$$\frac{d Y_{\nu_R}}{dx} = - \left. \frac{1}{x^4} \frac{\Gamma}{H} \right|_{T_{\text{reh}}} \left( Y_{\nu_R} - Y_{\nu_R}^{\text{eq}} \right) \quad \Gamma = \frac{5}{9} s Y_{\nu_R}^{\text{eq}} \left( \langle \sigma_1 v \rangle + \langle \sigma_2 v \rangle \right) \quad (59)$$

## Dirac leptogenesis without heavy particles?

$$\frac{d Y_{\nu_R}}{dx} = - \frac{1}{x^4} \frac{\Gamma}{H} \Big|_{T_{\text{reh}}} \left( Y_{\nu_R} - Y_{\nu_R}^{\text{eq}} \right) \quad \Gamma = \frac{5}{9} s Y_{\nu_R}^{\text{eq}} \left( \langle \sigma_1 v \rangle + \langle \sigma_2 v \rangle \right) \quad (59)$$

$$Y_{\nu_R}(x) = \frac{135\zeta(3)}{8\pi^4 h_*} \left( 1 - \exp \left[ - \frac{\Gamma}{\mathcal{H}} \Big|_{T_{\text{reh}}} \frac{x^3 - 1}{3x^3} \right] \right) \quad (60)$$

# Dirac leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \rightarrow LQ}|^2 = \text{Diagram } 1 - \text{Diagram } 2 \quad (61)$$

$$\Delta|T_{\nu_R d_R \rightarrow e_R u_R}|^2 = \text{Diagram } 3 - \text{Diagram } 4 \quad (62)$$

$$\Delta|T_{\nu_R d_R \rightarrow LQ}|^2 + \Delta|T_{\nu_R d_R \rightarrow e_R u_R}|^2 = 0 \quad (63)$$

# Dirac leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \rightarrow LQ}|^2 = \text{Diagram } 1 - \text{Diagram } 2 \quad (61)$$

$$\Delta|T_{\nu_R d_R \rightarrow e_R u_R}|^2 = \text{Diagram } 3 - \text{Diagram } 4 \quad (64)$$

$$\Delta\langle\sigma_1 v\rangle = -\Delta\langle\sigma_2 v\rangle \equiv \frac{64}{\pi^2} \frac{T^4}{T_{\text{reh}}^6} \frac{\epsilon}{\zeta(3)^2} \approx \frac{T^4}{T_{\text{reh}}^6} \epsilon \quad (65)$$

## Freeze-in and wash-in

$$\left( \frac{d\Delta_L}{dx} \right)_{\text{source}} = - \left( \frac{d\Delta_{e_R}}{dx} \right)_{\text{source}} \rightarrow \left( \frac{d\Delta_{\nu_R}}{dx} \right)_{\text{source}} = 0 \quad (66)$$

$$\left( \frac{d\Delta_L}{dx} \right)_{\text{wash-out}} \neq - \left( \frac{d\Delta_{e_R}}{dx} \right)_{\text{wash-out}} \rightarrow \left( \frac{d\Delta_{\nu_R}}{dx} \right)_{\text{wash-in}} \neq 0 \quad (67)$$

[see also Domcke, Kamada, Mukaida, Schmitz, Yamada '21, Aristizabal, Nardi, Muñoz '09]

## Freeze-in and wash-in

$$\frac{d\Delta_L}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{H} \frac{ds}{dx} \left\{ \Delta \langle \sigma_1 v \rangle \left( Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) + \frac{10}{9} \langle \sigma_3 v \rangle \left( \Delta_L - 2\Delta_{e_R} \right) + \frac{8}{9} \langle \sigma_1 v \rangle \left[ \Delta_L - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left( \Delta_L - \frac{3}{2} \Delta_{\nu_R} \right) \right] \right\} \quad (68)$$

$$\frac{d\Delta_{e_R}}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{H} \frac{ds}{dx} \left\{ \Delta \langle \sigma_2 v \rangle \left( Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) - \frac{10}{9} \langle \sigma_3 v \rangle \left( \Delta_L - 2\Delta_{e_R} \right) + \frac{8}{9} \langle \sigma_2 v \rangle \left[ 2\Delta_{e_R} - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left( 2\Delta_{e_R} - \frac{3}{2} \Delta_{\nu_R} \right) \right] \right\} \quad (69)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

# Freeze-in and wash-in

$$\frac{d\Delta_L}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{H} \frac{ds}{dx} \left\{ \Delta \langle \sigma_1 v \rangle \left( Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) + \frac{10}{9} \langle \sigma_3 v \rangle \left( \Delta_L - 2\Delta_{e_R} \right) \right. \\ \left. + \frac{8}{9} \langle \sigma_1 v \rangle \left[ \Delta_L - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left( \Delta_L - \frac{3}{2} \Delta_{\nu_R} \right) \right] \right\} \quad (68)$$

$$\frac{d\Delta_{e_R}}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{H} \frac{ds}{dx} \left\{ \Delta \langle \sigma_2 v \rangle \left( Y_{\nu_R}^{\text{eq}} - Y_{\nu_R} \right) - \frac{10}{9} \langle \sigma_3 v \rangle \left( \Delta_L - 2\Delta_{e_R} \right) \right. \\ \left. + \frac{8}{9} \langle \sigma_2 v \rangle \left[ 2\Delta_{e_R} - \frac{17}{8} \Delta_{\nu_R} + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \left( 2\Delta_{e_R} - \frac{3}{2} \Delta_{\nu_R} \right) \right] \right\} \quad (69)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

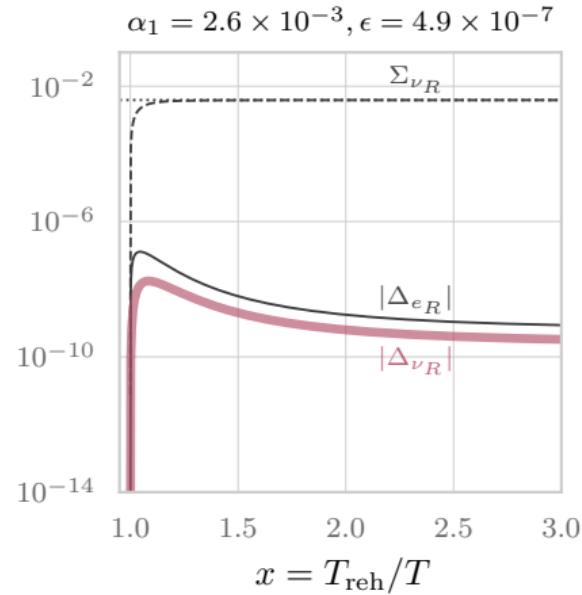
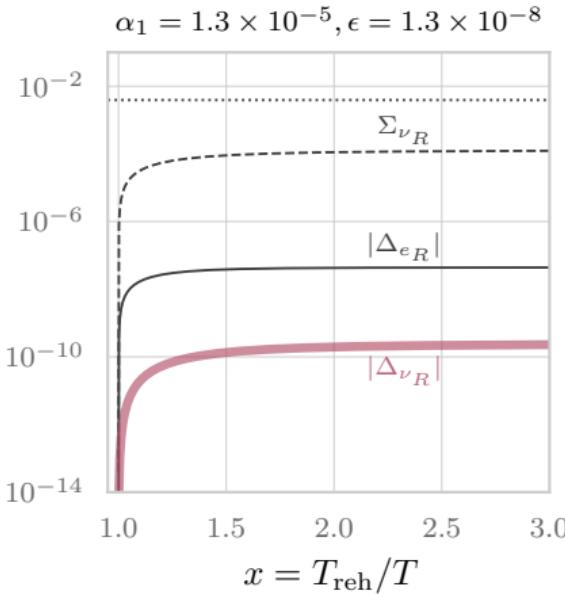
## Freeze-in and wash-in

$$\frac{d\Delta_{\nu_R}}{dx} = \frac{Y_{\nu_R}^{\text{eq}}}{H} \frac{ds}{dx} \left\{ \frac{5}{9} \langle \sigma_1 v \rangle \left( 5 + \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \right) \Delta_{\nu_R} + \frac{1}{9} \langle \sigma_2 v \rangle \left( 17 + 3 \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \right) \Delta_{\nu_R} \right. \\ \left. + \frac{8}{9} (\langle \sigma_1 v \rangle - 2 \langle \sigma_2 v \rangle) \left( 1 + \frac{1}{4} \frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} \right) \Delta_{e_R} \right\} \quad (70)$$

$$\frac{Y_{\nu_R}}{Y_{\nu_R}^{\text{eq}}} = 1 - \exp \left[ - \frac{\Gamma}{\mathcal{H}} \Big|_{T_{\text{reh}}} \frac{x^3 - 1}{3x^3} \right] \quad (71)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

# Numerical solution for $T_{\text{reh}} = 10^{14} \text{ GeV}$



$$\langle \sigma_1 v \rangle = 1.5 \times 10^{-33} \text{ GeV}^{-2}/x^2$$

$$|\Delta \langle \sigma_1 v \rangle| = 6.0 \times 10^{-36} \text{ GeV}^{-2}/x^4$$

$$\langle \sigma_1 v \rangle = 3.1 \times 10^{-31} \text{ GeV}^{-2}/x^2$$

$$|\Delta \langle \sigma_1 v \rangle| = 2.2 \times 10^{-34} \text{ GeV}^{-2}/x^4$$

## Summary II.

- Holomorphic cutting rules allow for easy tracking of asymmetry cancellations due to the *CPT* and unitarity constraints.
- Leptogenesis with  $\nu_R$  as the only out-of-equilibrium particles is possible. Their asymmetry is washed in, although the source term vanishes.

## Summary II.

- Holomorphic cutting rules allow for easy tracking of asymmetry cancellations due to the *CPT* and unitarity constraints.
- Leptogenesis with  $\nu_R$  as the only out-of-equilibrium particles is possible. Their asymmetry is washed in, although the source term vanishes.

Thank you for your attention!