Feynman rules for the Boltzmann equation

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Outline of this seminar

Part I.

- From unitarity to quantum statistics [Eur. Phys. J. C 81 (2021) 1050]
- Anomalous thresholds and thermal masses [Eur. Phys. J. C 82 (2022) 214]
- On-shell intermediate states [Phys. Rev. D 109 (2024) 043008]

Part II.

- CP asymmetries from vacuum diagrams [Phys. Rev. D 103 (2021) L091302]
- Leptogenesis from asymmetric scatterings of massless particles [Phys. Rev. D 110 (2024) 055042]

Part I.

Boltzmann equation: the simplest example

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 \tag{1}$$

Which processes contribute to ϕ_1 evolution in the expanding universe?

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$$\dot{n}_{\phi_1} + 3Hn_{\phi_1} = -\langle \Gamma_\Phi \rangle \left(\frac{n_{\phi_1}}{n^{\text{eq}}_{\phi_1}} - 1 \right) n^{\text{eq}}_\Phi \qquad \langle \Gamma_\Phi \rangle = \gamma^{\text{eq}}_{\Phi \to \phi_1 \phi_2} / n^{\text{eq}}_\Phi \tag{2}$$

$$\gamma^{\text{eq}}_{\Phi \to \phi_1 \phi_2} = \int [\mathrm{d}\boldsymbol{p}_\Phi] \, \mathrm{e}^{-E_\Phi / T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2](2\pi)^4 \delta^{(4)}(p_\Phi - k_1 - k_2) |M_{\Phi \to \phi_1 \phi_2}|^2 \tag{3}$$

$$[\mathrm{d}\boldsymbol{k}] = \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} \qquad S_{fi} = \mathbb{1}_{fi} + \mathrm{i}\, T_{fi} = \mathbb{1}_{fi} + \mathrm{i}(2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi}$$

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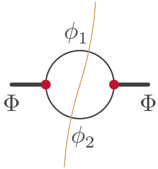

$$[\mathrm{d}\boldsymbol{k}] = \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} \qquad |T_{fi}|^2 = -\mathrm{i}\, T^\dagger_{if} \mathrm{i}\, T_{fi}$$

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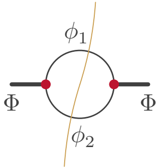

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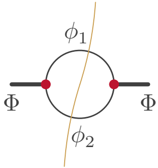
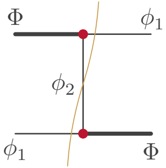


$$\gamma^{\text{eq}}_{\Phi\phi_1\to\phi_1\phi_1\phi_2} = \int [\mathrm{d}\boldsymbol{p}_\Phi]\, \mathrm{e}^{-E_\Phi/T} \int [\mathrm{d}\boldsymbol{k}_1]\, \mathrm{e}^{-E_1/T} \times - \quad ? \tag{4}$$



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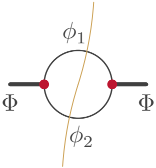

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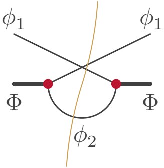

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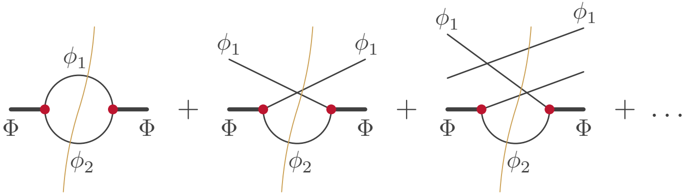

$$\int [\mathrm{d}\boldsymbol{p}_\Phi] \mathrm{e}^{-E_\Phi/T} \int [\mathrm{d}\boldsymbol{k}_1][\mathrm{d}\boldsymbol{k}_2] \left[1 + \frac{1}{\mathrm{e}^{E_1/T} - 1}\right] (2\pi)^4 \delta^{(4)}(p_\Phi - k_1 - k_2) |M_{\Phi\to\phi_1\phi_2}|^2 \quad (5)$$

[Blažek, Maták '21b]

$T^*_{fi} \neq T_{if}$ and holomorphic cuts

$$S = \mathbb{1} + \mathrm{i}\,T \qquad\qquad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \tag{6}$$

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$$|T_{fi}|^2 = -\mathrm{i}\,T^\dagger_{if} \mathrm{i}\,T_{fi} = -\mathrm{i}\,T_{if} \mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in} \mathrm{i}\,T_{nf} \mathrm{i}\,T_{fi} - \sum_{n,k} \mathrm{i}\,T_{in} \mathrm{i}\,T_{nk} \mathrm{i}\,T_{kf} \mathrm{i}\,T_{fi} + \dots \tag{8}$$

[Coster, Stapp '70, Bourjaily, Hannesdottir, *et al.* '21, Hannesdottir, Mizera '22, Blažek, Maták '21a]

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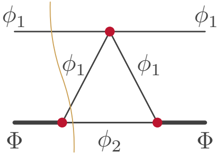
$$\gamma_{fi} = \frac{1}{V_4} \int \prod_{k=1}^{p} [\mathrm{d}\boldsymbol{p}_k]\, f_{i_k}(\boldsymbol{p}_k) \int \prod_{l=1}^{q} [\mathrm{d}\boldsymbol{p}_l] \Big(-\mathrm{i}\, T_{if} \mathrm{i}\, T_{fi} + \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} + \ldots \Big) \tag{9}$$

[Blažek, Maták '21a]

Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 - \frac{1}{4!} \lambda \phi_1^4 \tag{10}$$

$$\gamma^{\text{eq}}_{\Phi\phi_1 \to \phi_1\phi_1\phi_2} \sim - \tag{11}$$



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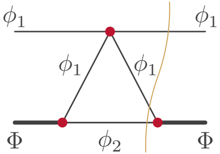

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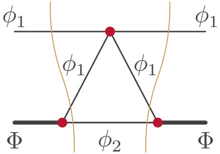

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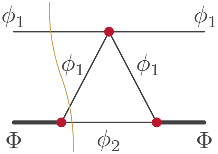

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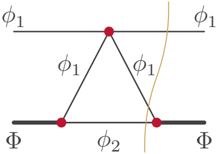

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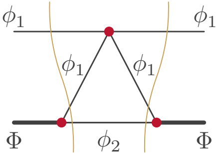

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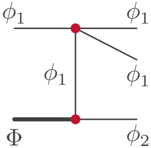
$$\frac{1}{k^2 + \mathrm{i}\epsilon} = \text{P.V.}\frac{1}{k^2} - \mathrm{i}\pi\delta\left(k^2\right)$$



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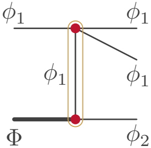
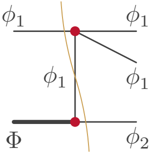


$+ \frac{1}{2}$



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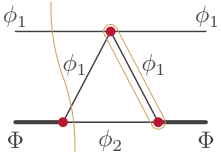
Anomalous thresholds and thermal corrections

$$\mathcal{L}_{\text{int.}} = -\mu \Phi \phi_1 \phi_2 - \frac{1}{4!} \lambda \phi_1^4 \tag{10}$$

$$\gamma^{\text{eq}}_{\Phi\phi_1 \to \phi_1 \phi_1 \phi_2} \sim -2$$



$$= \mathring{m}^2_{\phi_1}(T) \times \frac{\partial \gamma^{\text{eq}}_{\Phi \to \phi_1 \phi_2}}{\partial m^2_{\phi_1}} \tag{13}$$

[Blažek, Maták '22]

$$2\theta(k^0)\delta\left(k^2\right)\text{P.V.}\frac{1}{k^2} = -\frac{1}{\left(k^0 + |\boldsymbol{k}|\right)^2} \frac{\partial \delta\left(k^0 - |\boldsymbol{k}|\right)}{\partial k^0} \tag{14}$$

[Frye, *et al.* '19, Racker '19]

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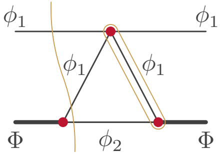

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[Blažek, Maták '22]

$$\mathring{m}^2_{\phi_1}(T) = \lambda \int [\mathrm{d}\boldsymbol{k}_1] \mathrm{e}^{-E_1/T} = \frac{\lambda}{4\pi^2} T^2 \tag{15}$$

Anomalous thresholds and thermal corrections

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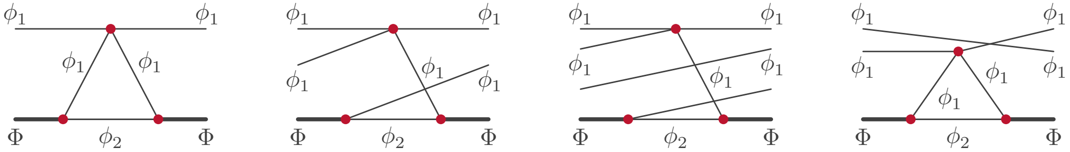

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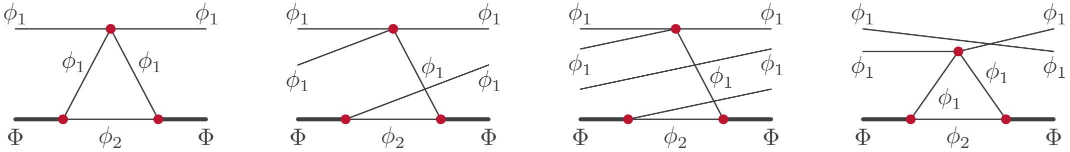


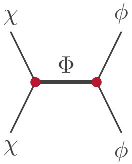
$$\downarrow$$

$$m^2_{\phi_1}(T) \times \frac{\partial \gamma^{\text{eq}}_{\Phi \to \phi_1 \phi_2}}{\partial m^2_{\phi_1}} \text{ with } m^2_{\phi_1}(T) = \frac{\lambda}{24} T^2 \text{ and quantum statistics in } \gamma^{\text{eq}}_{\Phi \to \phi_1 \phi_2}$$

[Blažek, Maták '22]

On-shell intermediate states

$$\mathcal{L}_{\mathrm{int.}} = -\frac{1}{2}\lambda_{\mathrm{DM}}\Phi\chi^2 - \frac{1}{2}\lambda_{\mathrm{SM}}\Phi\phi^2 \tag{16}$$



$$\left|\frac{1}{s - M^2 + \mathrm{i}\epsilon}\right|^2 \quad \rightarrow \quad \left|\frac{1}{s - M^2 + \mathrm{i}M\Gamma}\right|^2 \tag{17}$$

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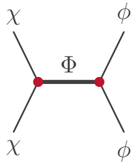

$$\left|\frac{1}{s - M^2 + \mathrm{i}\epsilon}\right|^2 \quad \to \quad \left|\frac{1}{s - M^2 + \mathrm{i}M\Gamma}\right|^2 \approx \frac{\pi}{M\Gamma}\delta\left(s - M^2\right) \tag{17}$$

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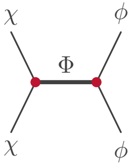

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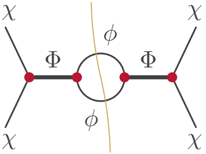
$$\gamma^{\text{LO}}_{\chi\chi\to\phi\phi} \stackrel{\text{def.}}{=} \gamma_{\chi\chi\to\Phi} \times \text{Br}(\Phi \to \phi\phi) \qquad (18)$$

Resonance in $\chi\chi \to \phi\phi$ already included in $\chi\chi \to \Phi$ and $\Phi \to \phi\phi$.

The double-counting has to be removed!

[Kolb, Wolfram '80]

On-shell intermediate states

$$\gamma^{\mathrm{NLO}}_{\chi\chi\to\phi\phi} \sim -$$  $$? \tag{19}$$

On-shell intermediate states

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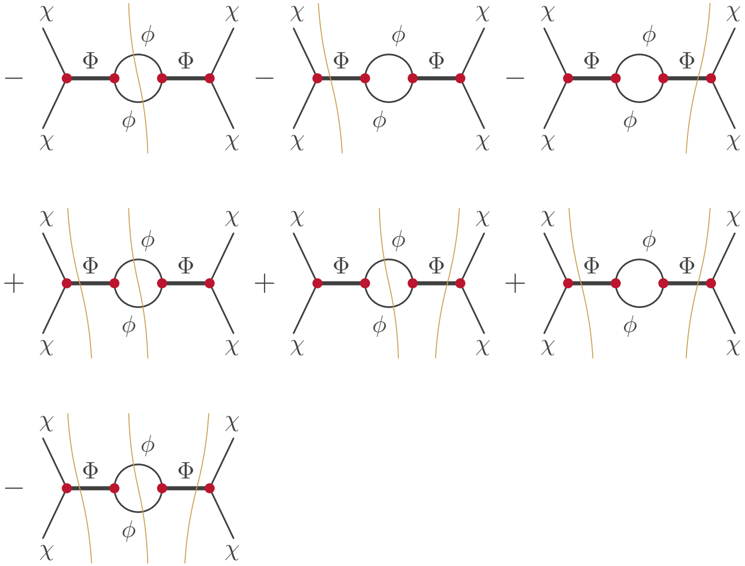
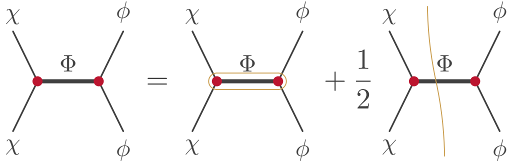


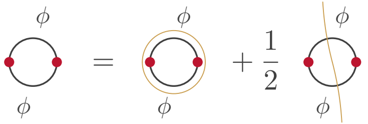
On-shell intermediate states

$$\frac{1}{s - M^2 + \mathrm{i}\epsilon} = \mathcal{P}\frac{1}{s - M^2} - \mathrm{i}\pi\delta(s - M^2)$$



(21)

$$\Sigma(s) = \Sigma_R(s) + \mathrm{i}\Sigma_I(s)$$



(22)

On-shell intermediate states

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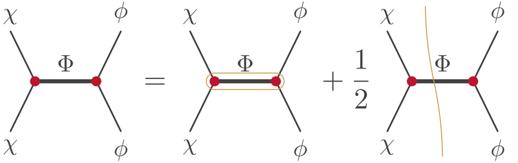

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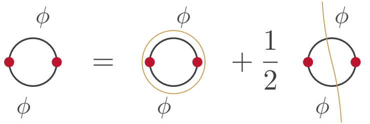

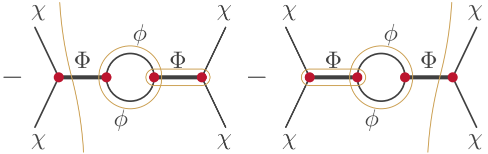

$$\propto -\frac{\partial}{\partial s}\pi\delta\left(s - M^2\right) \tag{20}$$

[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

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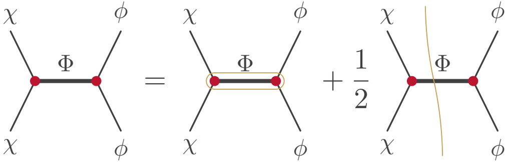
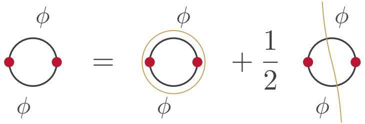
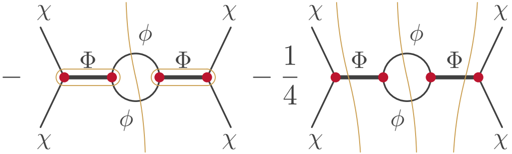


$$\Sigma(s) = \Sigma_R(s) + \mathrm{i}\Sigma_I(s) \tag{22}$$





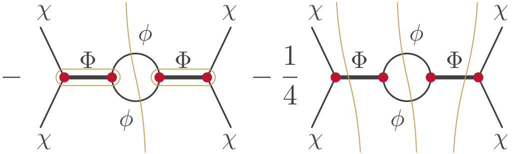
$$\propto -\frac{\partial}{\partial s}\mathcal{P}\frac{1}{s - M^2} \tag{23}$$

[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

On-shell intermediate states

$$\frac{\epsilon}{M\Gamma}\left|\frac{1}{s-M^2+\mathrm{i}\epsilon}\right|^2 \quad\to\quad \frac{\pi}{M\Gamma}\delta(s-M^2) \tag{24}$$

$$\frac{\epsilon}{M\Gamma}\left[\frac{1}{(s-M^2)^2-\epsilon^2}-\frac{2\epsilon^2}{[(s-M^2)^2-\epsilon^2]^2}\right] \quad\to\quad 0 \tag{25}$$

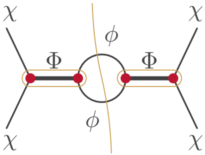


$$\propto\ -\frac{\partial}{\partial s}\mathcal{P}\frac{1}{s-M^2} \tag{23}$$

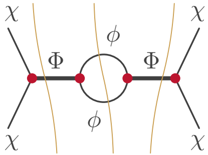
[Maták ’24, see also Tkachov ’98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen ’24]

On-shell intermediate states

$$\gamma^{\mathrm{NLO}}_{\chi\chi\to\phi\phi} \sim -$$



$$-\frac{1}{4}$$



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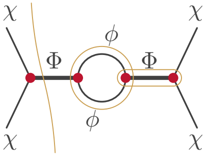

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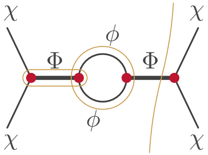


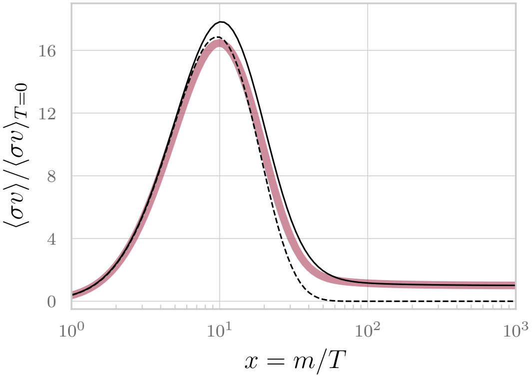
$$\sim 2\lambda^2_{\mathrm{DM}}\left[\Sigma_I(s)\frac{\partial}{\partial s}\mathcal{P}\frac{1}{s-M^2} - \Sigma_R(s)\frac{\partial}{\partial s}\pi\delta\left(s-M^2\right)\right] \times \mathrm{Br}(\Phi\to\phi\phi) \tag{27}$$

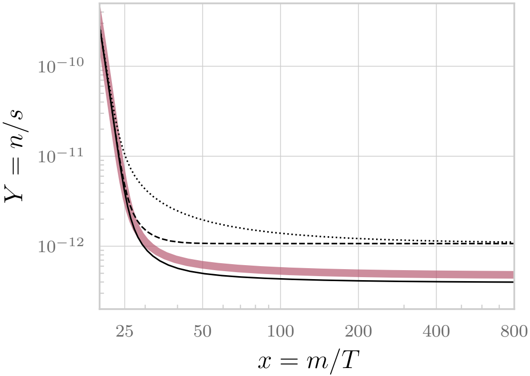
[Maták '24, see also Tkachov '98, Ala-Mattinen, Heikinheimo, Tuominen, Kainulainen '24]

Resonant dark matter annihilation

$$\frac{dY}{dx} = -\left(\frac{45}{\pi}G\right)^{-1/2}\frac{g_*^{1/2}m}{x^2}\langle\sigma v\rangle\left(Y^2 - Y_{\rm eq}^2\right) \tag{28}$$

[Bernstein '88; Gondolo, Gelmini '91]





$$M = 100\ \text{TeV}, m = 45\ \text{TeV}, \Gamma/M = 10^{-2}$$

[Maták '24]

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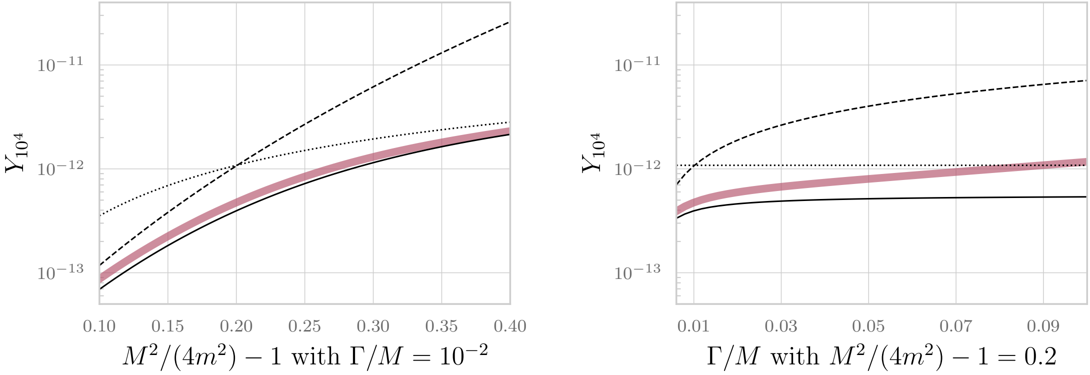

[Maták '24]

Where it all comes from?

$$\rho = \prod_p \rho_p = \frac{1}{Z} \exp\left\{ -\sum_p F_p a_p^\dagger a_p \right\} \quad \leftarrow \quad Z = \prod_p Z_p = \prod_p \frac{\exp F_p}{\exp F_p - 1} \tag{29}$$

$$\exp\{-E_p/T\} \quad \rightarrow \quad \exp\{-F_p\} = \frac{f_p}{1 \pm f_p} \qquad f_p = \mathrm{Tr}\left[a_p^\dagger a_p \rho\right] \tag{30}$$

[Wagner '91]

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[Wagner '91]

$$\rho' = S\rho S^\dagger \quad \to \quad (1 + \mathrm{i}T)\rho(1 - \mathrm{i}T + \mathrm{i}T\mathrm{i}T - \ldots) \tag{31}$$

The collision term for the Boltzmann equation is obtained as $\mathrm{Tr}\left[a_p^\dagger a_p(\rho - \rho')\right]/V_4$.

[McKellar, Thomson '94, Blažek, Maták '21b]

Summary I.

- Unitarity may help in calculating reaction rates for the Boltzmann equation.

$$\gamma_{fi}^{\text{eq}} = \frac{1}{V_4} \int \prod_{k=1}^{p} [\mathrm{d}\boldsymbol{p}_k]\, f_{i_k}^{\text{eq}}(\boldsymbol{p}_k) \int \prod_{l=1}^{q} [\mathrm{d}\boldsymbol{p}_l] \Big(-\mathrm{i}\,T_{if}\,\mathrm{i}\,T_{fi} + \sum_n \mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nf}\,\mathrm{i}\,T_{fi} + \ldots \Big)$$

- Completing diagrams by all possible winding numbers accounts for quantum statistics.

- Anomalous thresholds approximate thermal-mass effects in lower-order process kinematics.

- There is no double-counting of on-shell intermediate states in fixed-order results.

Part II.

Imaginary kinematics in Feynman diagrams

$$\mathrm{i}\,T_{if}^{*} - \mathrm{i}\,T_{fi} = \sum_{n} T_{nf}^{*}\,T_{ni} \qquad T_{fi} = \sum_{\text{diagrams}} C_{fi} K_{fi} \qquad C_{fi} = C_{if}^{*},\, K_{fi} = K_{if} \tag{32}$$

Imaginary kinematics in Feynman diagrams

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$$2\,\mathrm{Im}\,K_{fi} = \sum_n K^*_{fn} K_{ni} \tag{33}$$

[Cutkosky '60; Veltman '63]

CP asymetries and unitarity constraints

$$S^\dagger S = S S^\dagger \quad \rightarrow \quad \sum_f |T_{fi}|^2 = \sum_f |T_{if}|^2 \quad \text{for} \quad \mathrm{i}T = S - \mathbb{1} \tag{34}$$

$$CPT \text{ symmetry} \quad \rightarrow \quad \Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{\overline{fi}}|^2 = |T_{fi}|^2 - |T_{if}|^2 \tag{35}$$

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[Dolgov '79; Kolb, Wolfram '80, See also Hook '11, Baldes, Bell, Petraki, Volkas '14]

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[Dolgov '79; Kolb, Wolfram '80, See also Hook '11, Baldes, Bell, Petraki, Volkas '14]

$$\left.\begin{aligned} T_{fi} &= C_{\text{tree}} K_{\text{tree}} + C_{\text{loop}} K_{\text{loop}} \\ T_{if} &= C^*_{\text{tree}} K_{\text{tree}} + C^*_{\text{loop}} K_{\text{loop}} \end{aligned}\right\} \quad \Delta|T_{fi}|^2 = -4\,\mathrm{Im}[C_{\text{tree}} C^*_{\text{loop}}]\,\mathrm{Im}[K_{\text{tree}} K^*_{\text{loop}}] \tag{37}$$

CP asymetries and unitarity constraints

$$S^\dagger S \quad \to \quad T = T^\dagger + \mathrm{i}\, T^\dagger\, T \tag{38}$$

$$\Delta \left| T_{fi} \right|^2 = \left| T_{if}^* + \mathrm{i} \sum_n T_{fn}^\dagger\, T_{ni} \right|^2 - \left| T_{if} \right|^2 = -2\,\mathrm{Im}\left[\sum_n T_{if}\, T_{fn}^\dagger\, T_{ni} \right] + \left| \sum_n T_{fn}^\dagger\, T_{ni} \right|^2 \tag{39}$$

[Kolb, Wolfram ’80]

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[Kolb, Wolfram '80]

No further on-shell cuts means $T_{if}^* = T_{fi} \quad \to \quad \Delta \left| T_{fi} \right|^2 = -2\, \mathrm{Im} \left[\sum_n T_{if}\, T_{fn}\, T_{ni} \right]$ (40)

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$$S^\dagger S \quad \to \quad T = T^\dagger + \mathrm{i}\, T^\dagger\, T \tag{38}$$

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$$\Delta\left|\, T_{fi}\right|^2 = \sum_n \mathrm{i}\, T_{in} \mathrm{i}\, T_{nf} \mathrm{i}\, T_{fi} - \sum_n \mathrm{i}\, T_{if} \mathrm{i}\, T_{fn} \mathrm{i}\, T_{ni} \tag{41}$$

[Covi, Roulet, Vissani '98]

CP asymetries and unitarity constraints

2 April 1998

PHYSICS LETTERS B

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Physics Letters B 424 (1998) 101–105

On the CP asymmetries in Majorana neutrino decays

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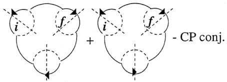

Fig. 3. Pictorial representation of the self-energy contribution to the cross section asymmetry ϵ_σ.

Therefore, one can pictorially represent the self-energy contributions to $\sigma(\ell^c H^* \to \ell H)$ as in Fig. 2, where the cut blobs are the initial (i) and final (f) states, while the remaining blob stands for the one-loop self-energy. This last is actually the sum of two contributions, one with a lepton and one with an antilepton.

Now, in the computation of ϵ_σ, only the absorptive part of the loop will contribute, and the Cutkoski

Turning now to the vertex contributions, to see the cancellations we need to add the three contributions shown in Fig. 4 (including the tree level u-channel interfering with the one-loop self-energy diagram). In terms of cut diagrams, this can be expressed as in Fig. 5, where CP-conjugate stands for the same four diagrams with all the arrows reversed. Hence, the CP-conjugate contribution will exactly cancel the four diagrams, since changing the directions of the arrows just exchanges among themselves the first and fourth diagrams, as well as the second and third ones. We then see explicitly that the absorptive part of the self-energies are also playing here a crucial role, enforcing the cancellation of the CP violation produced by the vertex diagrams.

The only remaining diagrams to be considered are the interference of the one-loop vertex diagrams with the tree level u-channel. Pictorially, they are represented in Fig. 6, and they again cancel since the two

$$\Delta\left|T_{fi}\right|^2 = \sum_n \mathrm{i}\,T_{in}\,\mathrm{i}\,T_{nf}\,\mathrm{i}\,T_{fi} - \sum_n \mathrm{i}\,T_{if}\,\mathrm{i}\,T_{fn}\,\mathrm{i}\,T_{ni} \qquad (41)$$

[Covi, Roulet, Vissani ’98]

Asymmetries with holomorphic cuts

$$S = 1 + \mathrm{i}T \qquad\qquad T_{fi} = (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi} \tag{6}$$

$$S^\dagger S = 1 \quad \to \quad \mathrm{i}T^\dagger = \mathrm{i}T - \mathrm{i}T\mathrm{i}T^\dagger \tag{7}$$

$$|T_{fi}|^2 = -\mathrm{i}T^\dagger_{if}\mathrm{i}T_{fi} = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \sum_{n,k} \mathrm{i}T_{in}\mathrm{i}T_{nk}\mathrm{i}T_{kf}\mathrm{i}T_{fi} + \dots \tag{8}$$

[Coster, Stapp '70, Bourjaily, Hannesdottir, *et al.* '21, Hannesdottir, Mizera '22, Blažek, Maták '21a]

Asymmetries with holomorphic cuts

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$$\begin{aligned}\Delta |T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2 = &\sum_n \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fn}\mathrm{i}\,T_{ni}\right) \\ &- \sum_{n,k} \left(\mathrm{i}\,T_{in}\mathrm{i}\,T_{nk}\mathrm{i}\,T_{kf}\mathrm{i}\,T_{fi} - \mathrm{i}\,T_{if}\mathrm{i}\,T_{fk}\mathrm{i}\,T_{kn}\mathrm{i}\,T_{ni}\right) \\ &+ \ldots\end{aligned} \tag{42}$$

[Blažek, Maták '21a, see also Roulet, Covi, Vissani '98]

Consequences for the asymmetry generation

$$\Delta\dot{n}_{f_1} + 3H\Delta n_{f_1} = \sum_i \sum_{f \ni f_1} \left(\frac{n_{i_1}}{n_{i_1}^{\rm eq}} \frac{n_{i_2}}{n_{i_2}^{\rm eq}} \cdots \frac{n_{i_p}}{n_{i_p}^{\rm eq}} - 1 \right) \times \Delta\gamma_{fi}^{\rm eq} + \text{wash-out terms} \tag{43}$$

$$\left.\begin{array}{l} f_1 \text{ in the final state of the contributing processes} \\ \\ \text{out-of-equilibrium initial state} \end{array}\right\} \quad \Delta n_{f_1} \text{ source term}$$

[detailed derivation in Racker '19]

Example: Leptogenesis with Dirac neutrinos

- Introduced in Phys. Rev. Lett. **84** (2000) 4039 [Dick, Lindner, Ratz, and Wright 2000]

- Lepton-number conserving decays of heavy particles

- Right-handed neutrinos decoupled from the bath develop asymmetry opposite to that of standard-model leptons

$$Y_B = \frac{28}{79} Y_{B-L_{\mathrm{SM}}} = \frac{28}{79} \Delta_{\nu_R} \tag{44}$$

[Kuzmin, Rubakov, Shaposhnikov '85; Harvey, Turner '90]

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[Heeck, Heisig, Thapa '23a]

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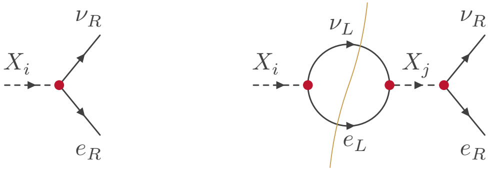

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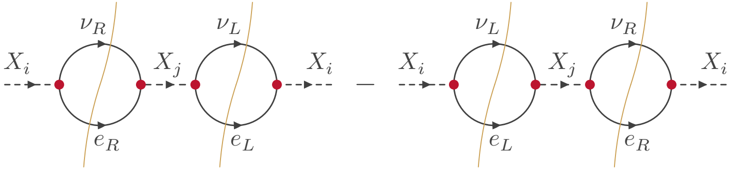
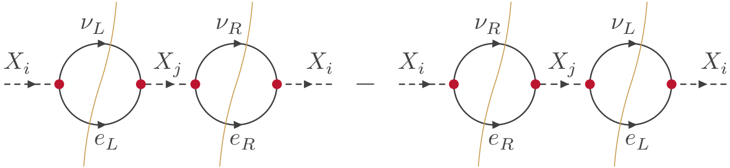
$$\Delta |T_{X_i \to \nu_R e_R}|^2 + \Delta |T_{X_i \to \nu_L e_L}|^2 = 0 \tag{46}$$

Example: Leptogenesis with Dirac neutrinos

$$\Delta|T_{X_i\to\nu_R e_R}|^2 = \quad (47)$$



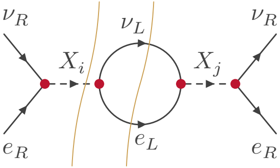
$$\Delta|T_{X_i\to\nu_L e_L}|^2 = \quad (48)$$



$$\Delta|T_{X_i\to\nu_R e_R}|^2 + \Delta|T_{X_i\to\nu_L e_L}|^2 = 0 \quad (46)$$

Example: Leptogenesis with Dirac neutrinos

$$\Delta|T_{\nu_R e_R\to X_i}|^2 = \tag{49}$$



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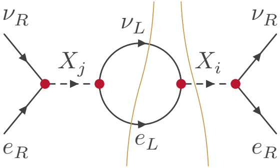
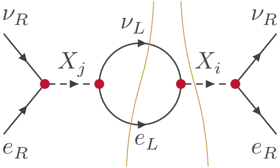


$$\Delta|T_{\nu_R e_R\to \nu_L e_L}|^2 = \tag{50}$$



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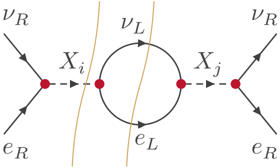

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Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \frac{1}{2}\bar{L}^c F_i L X_i^\dagger + \bar{e}_R^c G_i \nu_R X_i^\dagger + \text{H.c.} \qquad M_X \gg T_\text{reh} \tag{52}$$

[Heeck, Heisig, Thapa '23b]

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[Heeck, Heisig, Thapa '23b]

$$\cancel{\Delta|T_{\nu_R e_R \to X_i}|^2} + \Delta|T_{\nu_R e_R \to \nu_L e_L}|^2 = 0 \tag{53}$$

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[Heeck, Heisig, Thapa '23b]

$SU(3) \times SU(2) \times U(1)$	spin	$(B-L)(X)$	asymmetry-generating operators
$(\mathbf{1}, \mathbf{1}, -1)$	0	-2	$\nu_R e_R X^\dagger$, $LLX^\dagger$
$(\mathbf{1}, \mathbf{2}, 1/2)$	0	0	$\bar{H}X, \bar{\nu}_R L X, \bar{L} e_R X, \bar{Q} d_R X, \bar{u}_R Q X, X^\dagger H^\dagger H H$
$(\mathbf{3}, \mathbf{1}, -1/3)$	0	$-2/3$	$d_R \nu_R X^\dagger$, $u_R e_R X^\dagger$, $QLX^\dagger$, $u_R d_R X$, QQX
$(\mathbf{3}, \mathbf{1}, 2/3)$	0	$-2/3$	$u_R \nu_R X^\dagger$, $d_R d_R X$
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$4/3$	$\bar{Q} \nu_R X$, $\bar{d}_R L X$
$(\mathbf{1}, \mathbf{2}, -1/2)$	$1/2$	-1	$\bar{X} L, \bar{\nu}_R X H, \bar{X} e_R H$

[Heeck, Heisig, Thapa '23a]

Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \qquad M_X \gg T_\text{reh} \tag{54}$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

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$(\mathbf{1}, \mathbf{2}, 1/2)$	0	0	$\bar{H}X, \bar{\nu}_R LX, \bar{L}e_R X, \bar{Q}d_R X, \bar{u}_R QX, X^\dagger H^\dagger HH$
$(\mathbf{3}, \mathbf{1}, -1/3)$	0	$-2/3$	$d_R\nu_R X^\dagger, u_R e_R X^\dagger, QLX^\dagger, u_R d_R X, QQX$
$(\mathbf{3}, \mathbf{1}, 2/3)$	0	$-2/3$	$u_R\nu_R X^\dagger, d_R d_R X$
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	$4/3$	$\bar{Q}\nu_R X, \bar{d}_R LX$
$(\mathbf{1}, \mathbf{2}, -1/2)$	$1/2$	-1	$\bar{X}L, \bar{\nu}_R XH, \bar{X}e_R H$

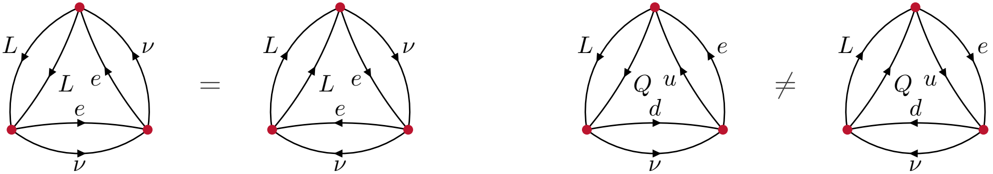
[Heeck, Heisig, Thapa '23a]

Dirac leptogenesis without heavy particles?

$$\mathcal{L} = \bar{Q}^c F_i L X_i^\dagger + \bar{d}_R^c G_i \nu_R X_i^\dagger + \bar{u}_R^c K_i e_R X_i^\dagger + \text{H.c.} \qquad M_X \gg T_\text{reh} \tag{54}$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

$$\Delta|T_{fi}|^2 = \sum_n \left(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}\right) - \dots$$



[see also Roulet, Covi, Vissani '98, Botella, Nebot, Vives '06]

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[Blažek, Heeck, Heisig, Maták, Zaujec '24]

- B and L individually conserved
- first generation only, ignoring SM interactions at $T_\text{reh} > 3 \times 10^{13}$ GeV

[Bento '03, Garbrecht, Schwaller '14]

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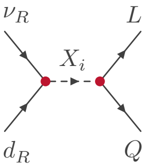
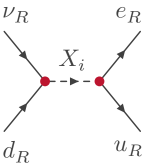
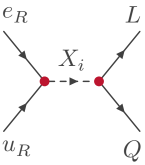
[Bento '03, Garbrecht, Schwaller '14]

$$\left.\begin{array}{lll} \Delta_{d_R} + \Delta_{u_R} + \Delta_Q = 0 & & \Delta_{\nu_R} + \Delta_{e_R} + \Delta_L = 0 \\ \Delta_{\nu_R} = \Delta_{d_R} & \Delta_L = \Delta_Q & \Delta_{e_R} = \Delta_{u_R} \end{array}\right\} \quad \Delta_a \equiv \frac{n_a - n_{\bar{a}}}{s} \tag{55}$$

Dirac leptogenesis without heavy particles?







$$\langle \sigma_1 v \rangle = \frac{16}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{F_i^* F_j G_j^* G_i}{M_i^2 M_j^2} \equiv \frac{16}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_1}{\zeta(3)^2} \approx \frac{T^2}{T_{\text{reh}}^4} \alpha_1 \tag{56}$$

$$\langle \sigma_2 v \rangle = \frac{8}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{K_i^* K_j G_j^* G_i}{M_i^2 M_j^2} \equiv \frac{8}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_2}{\zeta(3)^2} \approx \frac{1}{2} \frac{T^2}{T_{\text{reh}}^4} \alpha_2 \tag{57}$$

$$\langle \sigma_3 v \rangle = \frac{16}{3\pi} \frac{T^2}{\zeta(3)^2} \sum_{i,j} \frac{F_i^* F_j K_j^* K_i}{M_i^2 M_j^2} \equiv \frac{16}{3\pi} \frac{T^2}{T_{\text{reh}}^4} \frac{\alpha_3}{\zeta(3)^2} \approx \frac{T^2}{T_{\text{reh}}^4} \alpha_3 \tag{58}$$

Dirac leptogenesis without heavy particles?

$$\frac{\mathrm{d}\,Y_{\nu_R}}{\mathrm{d}x} = -\,\frac{1}{x^4}\,\frac{\Gamma}{H}\bigg|_{T_{\mathrm{reh}}}\left(Y_{\nu_R} - \,Y_{\nu_R}^{\mathrm{eq}}\right) \qquad \Gamma = \frac{5}{9}\,s\,Y_{\nu_R}^{\mathrm{eq}}\Big(\langle\sigma_1\,v\rangle + \langle\sigma_2\,v\rangle\Big) \tag{59}$$

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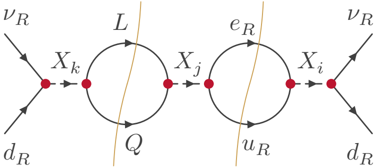
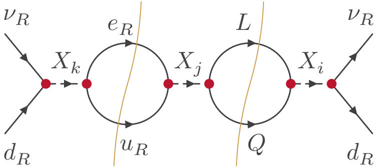
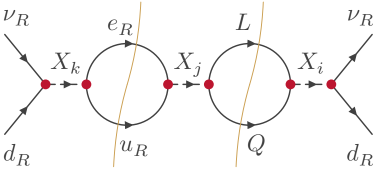
$$Y_{\nu_R}(x) = \frac{135\zeta(3)}{8\pi^4 h_*}\left(1 - \exp\left[-\,\frac{\Gamma}{\mathcal{H}}\bigg|_{T_{\rm reh}}\frac{x^3-1}{3x^3}\right]\right) \tag{60}$$

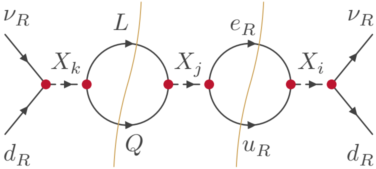
Dirac leptogenesis without heavy particles?

$\Delta|T_{\nu_R d_R \to LQ}|^2 =$ 

$-$ 

(61)

$\Delta|T_{\nu_R d_R \to e_R u_R}|^2 =$ 

$-$ 

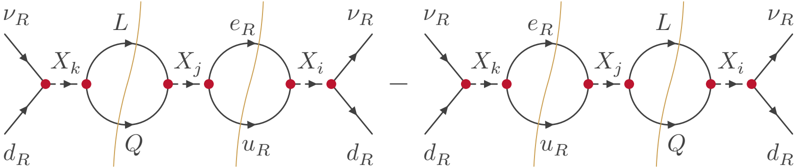
(62)

$$\Delta|T_{\nu_R d_R \to LQ}|^2 + \Delta|T_{\nu_R d_R \to e_R u_R}|^2 = 0 \tag{63}$$

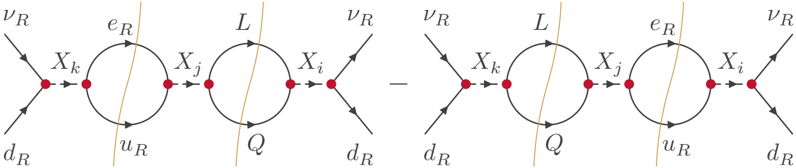
[Blažek, Heeck, Heisig, Maták, Zaujec '24]

Dirac leptogenesis without heavy particles?

$$\Delta|T_{\nu_R d_R \to LQ}|^2 = \quad (61)$$



$$\Delta|T_{\nu_R d_R \to e_R u_R}|^2 = \quad (64)$$



$$\Delta\langle\sigma_1 v\rangle = -\Delta\langle\sigma_2 v\rangle \equiv \frac{64}{\pi^2}\frac{T^4}{T_{\rm reh}^6}\frac{\epsilon}{\zeta(3)^2} \approx \frac{T^4}{T_{\rm reh}^6}\epsilon \quad (65)$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

Freeze-in and wash-in

$$\left(\frac{\mathrm{d}\Delta_L}{\mathrm{d}x}\right)_{\text{source}} = -\left(\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x}\right)_{\text{source}} \quad \rightarrow \quad \left(\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x}\right)_{\text{source}} = 0 \tag{66}$$

$$\left(\frac{\mathrm{d}\Delta_L}{\mathrm{d}x}\right)_{\text{wash-out}} \neq -\left(\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x}\right)_{\text{wash-out}} \quad \rightarrow \quad \left(\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x}\right)_{\text{wash-in}} \neq 0 \tag{67}$$

[see also Domcke, Kamada, Mukaida, Schmitz, Yamada '21, Aristizabal, Nardi, Muñoz '09]

Freeze-in and wash-in

$$\frac{\mathrm{d}\Delta_L}{\mathrm{d}x} = \frac{Y_{\nu_R}^{\mathrm{eq}}}{H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg\{\Delta\langle\sigma_1 v\rangle\left(Y_{\nu_R}^{\mathrm{eq}} - Y_{\nu_R}\right) + \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) + \frac{8}{9}\langle\sigma_1 v\rangle\left[\Delta_L - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\left(\Delta_L - \frac{3}{2}\Delta_{\nu_R}\right)\right]\Bigg\} \tag{68}$$

$$\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x} = \frac{Y_{\nu_R}^{\mathrm{eq}}}{H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg\{\Delta\langle\sigma_2 v\rangle\left(Y_{\nu_R}^{\mathrm{eq}} - Y_{\nu_R}\right) - \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) + \frac{8}{9}\langle\sigma_2 v\rangle\left[2\Delta_{e_R} - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\left(2\Delta_{e_R} - \frac{3}{2}\Delta_{\nu_R}\right)\right]\Bigg\} \tag{69}$$

[Blažek, Heeck, Heisig, Maták, Zaujec ’24]

Freeze-in and wash-in

$$\frac{\mathrm{d}\Delta_L}{\mathrm{d}x} = \frac{Y_{\nu_R}^{\mathrm{eq}}}{H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg\{\Delta\langle\sigma_1 v\rangle\left(Y_{\nu_R}^{\mathrm{eq}} - Y_{\nu_R}\right) + \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) \\ \boxed{+\frac{8}{9}\langle\sigma_1 v\rangle\left[\Delta_L - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\left(\Delta_L - \frac{3}{2}\Delta_{\nu_R}\right)\right]}\Bigg\} \tag{68}$$

$$\frac{\mathrm{d}\Delta_{e_R}}{\mathrm{d}x} = \frac{Y_{\nu_R}^{\mathrm{eq}}}{H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg\{\Delta\langle\sigma_2 v\rangle\left(Y_{\nu_R}^{\mathrm{eq}} - Y_{\nu_R}\right) - \frac{10}{9}\langle\sigma_3 v\rangle\left(\Delta_L - 2\Delta_{e_R}\right) \\ \boxed{+\frac{8}{9}\langle\sigma_2 v\rangle\left[2\Delta_{e_R} - \frac{17}{8}\Delta_{\nu_R} + \frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\left(2\Delta_{e_R} - \frac{3}{2}\Delta_{\nu_R}\right)\right]}\Bigg\} \tag{69}$$

[Blažek, Heeck, Heisig, Maták, Zaujec '24]

Freeze-in and wash-in

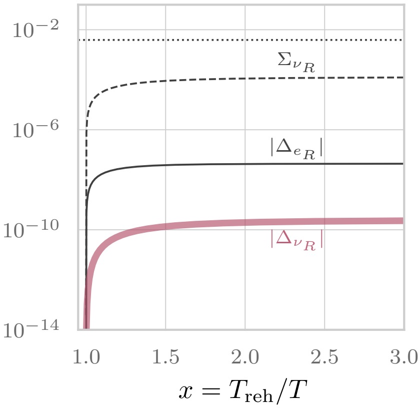
$$\frac{\mathrm{d}\Delta_{\nu_R}}{\mathrm{d}x} = \frac{Y_{\nu_R}^{\mathrm{eq}}}{H}\frac{\mathrm{d}s}{\mathrm{d}x}\left\{\frac{5}{9}\langle\sigma_1 v\rangle\left(5+\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\right)\Delta_{\nu_R} + \frac{1}{9}\langle\sigma_2 v\rangle\left(17+3\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\right)\Delta_{\nu_R} + \frac{8}{9}(\langle\sigma_1 v\rangle - 2\langle\sigma_2 v\rangle)\left(1+\frac{1}{4}\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}}\right)\Delta_{e_R}\right\} \tag{70}$$

$$\frac{Y_{\nu_R}}{Y_{\nu_R}^{\mathrm{eq}}} = 1 - \exp\left[-\left.\frac{\Gamma}{\mathcal{H}}\right|_{T_{\mathrm{reh}}}\frac{x^3-1}{3x^3}\right] \tag{71}$$

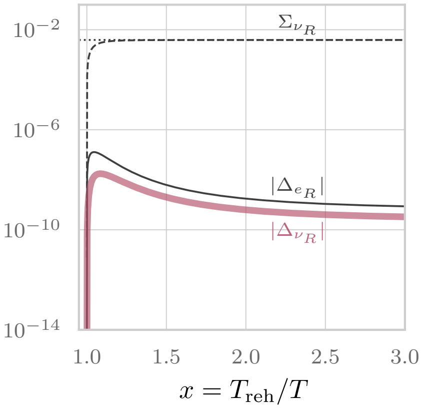
[Blažek, Heeck, Heisig, Maták, Zaujec ’24]

Numerical solution for $T_{\rm reh} = 10^{14}$ GeV

$\alpha_1 = 1.3 \times 10^{-5}, \epsilon = 1.3 \times 10^{-8}$



$\alpha_1 = 2.6 \times 10^{-3}, \epsilon = 4.9 \times 10^{-7}$



$$\langle \sigma_1 v \rangle = 1.5 \times 10^{-33}\ \text{GeV}^{-2}/x^2$$
$$|\Delta \langle \sigma_1 v \rangle| = 6.0 \times 10^{-36}\ \text{GeV}^{-2}/x^4$$

$$\langle \sigma_1 v \rangle = 3.1 \times 10^{-31}\ \text{GeV}^{-2}/x^2$$
$$|\Delta \langle \sigma_1 v \rangle| = 2.2 \times 10^{-34}\ \text{GeV}^{-2}/x^4$$

Summary II.

- Holomorphic cutting rules allow for easy tracking of asymmetry cancellations due to the CPT and unitarity constraints.

- Leptogenesis with ν_R as the only out-of-equilibrium particles is possible. Their asymmetry is washed in, although the source term vanishes.

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Thank you for your attention!