Clockwork inspired extra dimension models at future lepton colliders, beam dumps, and SN1987

Seminar “HECA”

High-Energy, Cosmology and Astro-particle physics

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Outline

- Theory
  - Clockwork mechanism → axion/relaxion
  - Generalized Continuous Clockwork → hierarchy problem

- Phenomenology
  - Future lepton colliders, beam dumps, cosmology
    - Randall-Sundrum
    - Linear Dilaton
    - Generalized Linear Dilaton (with heterotic M-theory UV completion)

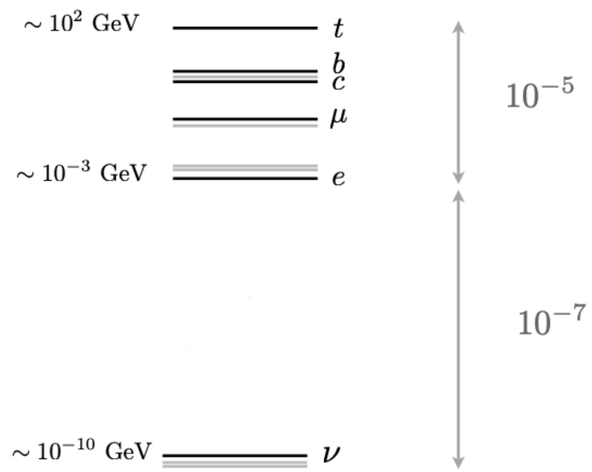
- RS with 3 branes and the NANOGrav signal

Physics Beyond the Standard Model

SM contains several *hierarchies* between energy scales

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + gA_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2 + \ldots$$ d=4

- cosmological constant — $+c_0\,\Lambda_{UV}^4\,\sqrt{g}$ — $c_0 \sim -10^{-60}\left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^4$ — d=0
- hierarchy problem — $+c_2\,\Lambda_{UV}^2\,H^\dagger H$ — $c_2 \simeq 0.008\left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^2$ — d=2
- strong CP problem — $+\theta\,G_{\mu\nu}\tilde{G}^{\mu\nu}$ — $\theta \lesssim 10^{-10}$ — d=4
- neutrino masses



Explaining *fine tuning*

- GUT, SUSY, XD, string theory (landscape), anthropic principle, relaxion, ...

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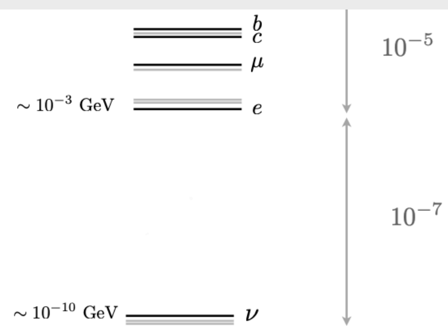

Explaining *fine tuning*

- GUT, SUSY, XD, string theory landscape, anthropic principle,

Clockwork: generating hierarchy from O(1) numbers by asymmetric NN interactions

Clockwork mechanism

- First studied in context of pseudoscalar model building (axion/relaxion)

  Kim, Nilles, Peloso 0409138, Dvali 0706.2050, K. Choi, H. Kim, S. Yun 1404.6209

  also related to dimensional deconstruction

  Arkani-Hamed, Cohen, Georgi 0104005, Hill, Pokorski, Wang 0104035

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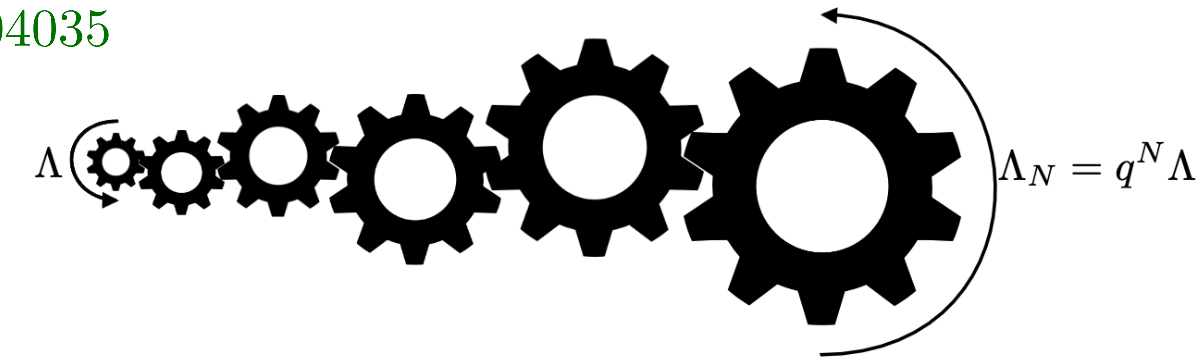
  - exp. small couplings without large scales
    (e.g. large, super-Planckian, axion decay constant)



K. Choi, S. H. Im 1511.00132; Kaplan, Rattazzi 1511.01827

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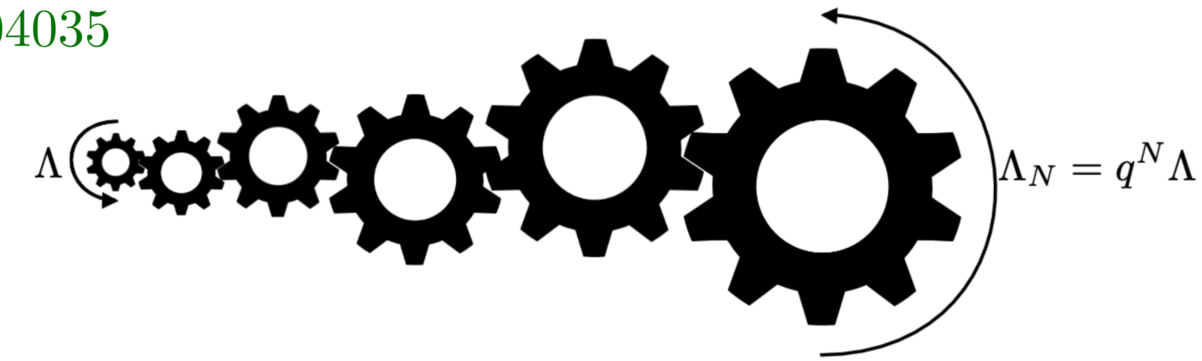

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- General Clockwork → other spins and continuous limit

  Giudice, McCullough 1610.07962

  Only abelian symmetry can be Clockworked? Craig, Garcia, Sutherland, 1704.07831

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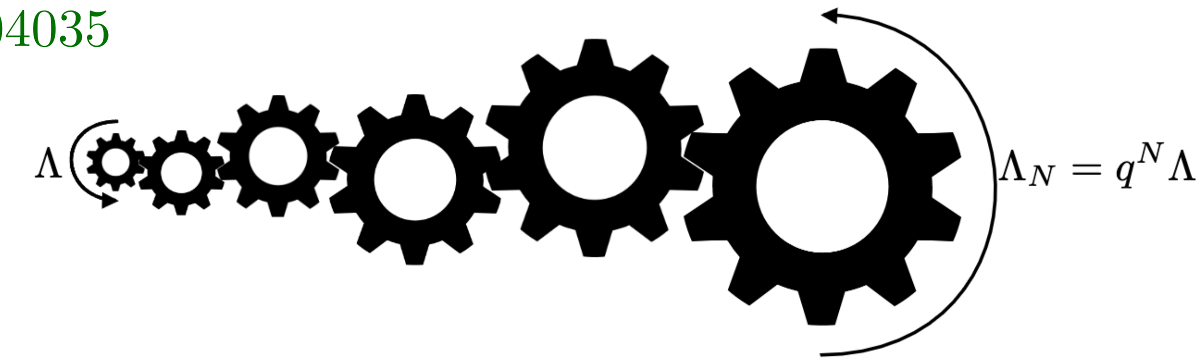

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- Generalized Continuous Clockwork → XD model with bulk & boundary mass terms or nontrivial 5D geometry

  K. Choi, S.H. Im, C. Shin 1711.06228, Giudice, Katz, McCullough, Torre, and Urbano 1711.08437

- UV completions

  SUGRA: Kehagias, Riotto 1710.04175, Antoniadis, Delgado, Markou, Pokorski 1710.05568

  heterotic M-theory: Im, Nilles, Olechowski 1811.11838

Clockwork pseudoscalar (axion)

K. Choi and S. H. Im 1511.00132; Kaplan and Rattazzi 1511.01827

- $N+1$ complex scalars whose dynamics generate an exponentially suppressed interaction scale
    - $U(1)^{N+1}$ global symmetry broken at high scale f
- Add explicit breaking to $U(1)^0$ by asymmetric nearest neighbors couplings; SM coupled *only* to the last site

$$\mathscr{L} = -\frac{f^2}{2}\sum_{j=0}^{N}\partial_\mu U_j^\dagger \partial^\mu U_j + \frac{m^2 f^2}{2}\sum_{j=0}^{N-1}\Big(U_j^\dagger U_{j+1}^q + \text{ h.c. }\Big), \qquad q > 1$$

where $U_j(x) = e^{i\pi_j/j}$

$\pi_j \to \pi_j + a/q^j$ remains → N pseudo-Goldstone bosons, 1 massless Goldstone boson

Promote $a \to a(x)$. If π_N coupled to SM by $1/f$, a coupled by $1/(q^N f)$.

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$\pi_j \to \pi_j + a/q^j$ remains

$$\mathscr{L}_{int} \simeq \frac{m^2}{2}\sum_{j=0}^{N}(\pi_j - q\pi_{j+1})^2 \simeq \frac{1}{2}\sum_{i,j=0}^{N}\pi_i\left(M_\pi^2\right)_{ij}\pi_j$$

$$M_\pi^2 = m^2\begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}$$

$$m_0^2 = 0, \quad m_k^2 = m^2\left[q^2 + 1 - 2q\cos\frac{k\pi}{N+1}\right]$$

massless Goldstone and massive gears

Spectrum - gapped or continuous

- At n-th site: $\pi_n = \sum_j O_{nj}\, a_j$

$$O_{j0} = \frac{\mathcal{N}_0}{q^j}, \quad O_{jk} = \mathcal{N}_k \left[q \sin\frac{jk\pi}{N+1} - \sin\frac{(j+1)k\pi}{N+1} \right], \quad j = 0,..,N; \quad k = 1,..,N$$

localization of massless mode component along the gear → exponentially small coupling to SM (Nth site)

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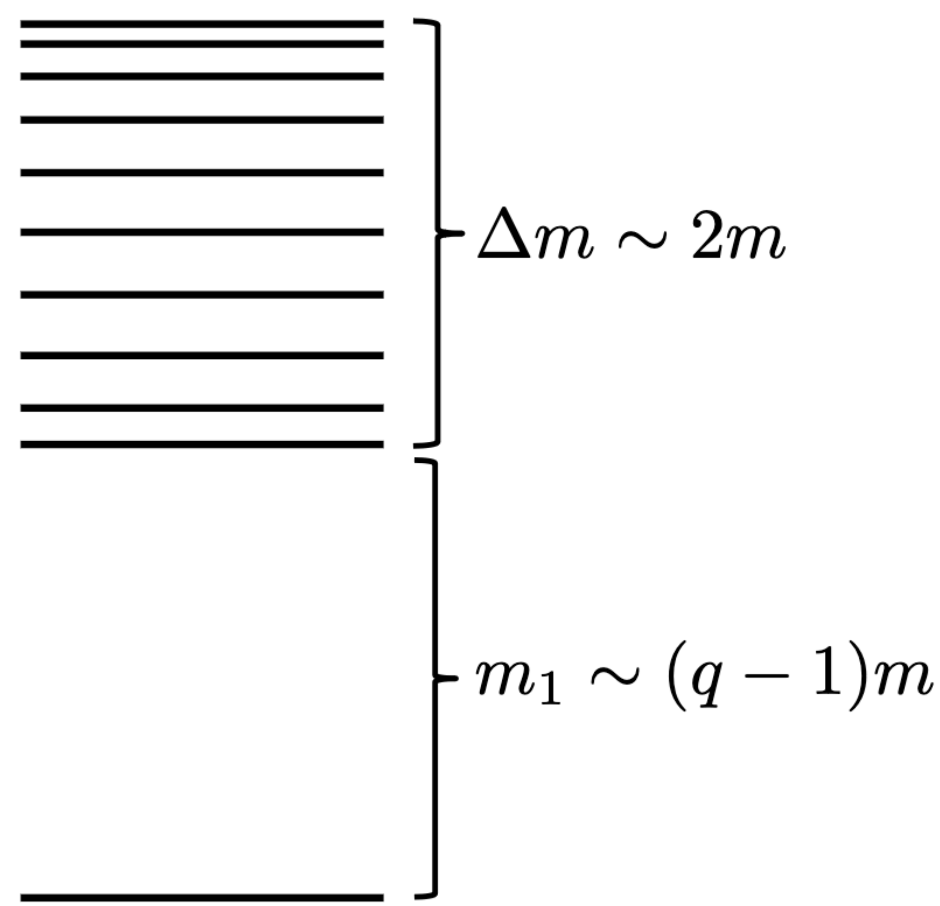
$$\mathcal{N}_0 \equiv \sqrt{\frac{q^2-1}{q^2-q^{-2N}}}, \quad \mathcal{N}_k \equiv \sqrt{\frac{2}{(N+1)\lambda_k}}$$

$$\lambda_k = 1 + q^2 - 2q\cos\theta_k, \qquad \theta_k = \frac{k\pi}{N+1}$$



- Mass gap Δm and the state density δm

$$\frac{\Delta m}{m_{a_1}} = 2(q-1),$$

$$\frac{\delta m_k}{m_{a_k}} \sim \frac{q\pi}{N\lambda} \sin \frac{k\pi}{N+1} \sim \mathcal{O}(1/N).$$

Towards Continuous Clockwork

- How to take $n \to \infty$ limit (the chain as extra dim.)?

$$\mathcal{L}_{int} \simeq -\frac{m^2 f^2}{2} \sum_{j=0}^{N} \exp\left(\frac{i}{f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{f} F\tilde{F}, \quad \pi_i \to \pi_i + \alpha/q^j$$

Craig, Garcia, Sutherland, 1704.07831

- $U(1)$ with charges: 1,...,$1/q^N$. Continuum limit with q^N fixed $\to$ symmetry is non-compact (no charge quantization).
- Moreover, if the CW sym. group was not abelian, the generators would mix in the zero mode $\to$ all couplings have to be equal to 1.

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- Notice: by field redefinition ($\pi_i \to \pi_i/q^i$)

$$\mathcal{L}_{int} \to -\frac{m^2 f^2}{2} \sum_{j=0}^{N} q^{-2j} (\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2} \exp\left(\frac{i}{q^j f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{q^j f} F\tilde{F}$$

warping — position-dependent coupling

Giudice, McCullough 1610.07962, K. Choi, S.H. Im, C. Shin 1711.06228, Giudice, Katz, McCullough, Torre, and Urbano 1711.08437

Clockwork from 5th dim with LD

- Sites at $i = 0,\ldots,N \leftrightarrow$ points in 5-th dimension

$$y \leftrightarrow ja, \quad Na = \pi R, \quad \int \mathrm{dy} \leftrightarrow \sum, \quad \partial_y \phi \leftrightarrow \frac{1}{a}\left(\phi_{j+1} - \phi_j\right)$$

$$\mathscr{L}_{int} \to -\frac{m^2 f^2}{2} \sum_{j=0}^{N} q^{-2j} (\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2} \exp\left(\frac{i}{q^j f}(\pi_j - q\pi_{j+1})^2\right) + \frac{\pi_j}{q^j f} F\tilde{F}$$

$$\mathcal{L} = -\frac{1}{2} \int dy\, e^{-2ky} \left[\partial_\mu \pi\, \partial^\mu \pi + (\partial_y \pi)^2\right] + e^{-ky} \frac{\pi}{f} F\tilde{F}$$

- *Interpret* as Linear Dilaton (S) background

$$\mathcal{S} = \int d^4x\, dy \sqrt{-g}\, \frac{M_5^3}{2} e^S \left(\mathcal{R} + g^{MN} \partial_M S\, \partial_N S + 4k^2\right)$$

From Einstein eqs: $\langle S \rangle = \pm\, 2ky$

- Works for any massless field (scalar, fermion, vector, graviton)
    - position-dependent hierarchy, zero-mode localisation, gear mass spectrum

Clockwork for gravitons

$$\mathcal{L} = -\frac{m^2}{2}\sum_{j=0}^{N-1}\left(\left[h_j^{\mu\nu} - q h_{j+1}^{\mu\nu}\right]^2 - \left[\eta_{\mu\nu}(h_j^{\mu\nu} - q h_{j+1}^{\mu\nu})\right]^2\right)$$

- N+1 gravitons with diffeomorphism invariance

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \nabla_{(\mu} A^i_{\nu)}$$

NN interactions break it to (Linearized gravity $g^i_{\mu\nu} = \eta^i_{\mu\nu} + 2h^i_{\mu\nu}/M_{Pl}$)

$$g^i_{\mu\nu} \to g^i_{\mu\nu} + \frac{1}{q^i}\nabla_{(\mu}\tilde{A}_{\nu)}$$

SM fields only couple to the last graviton → hierarchy problem solved

$$-\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \to -\frac{1}{M_P}\tilde{h}_0^{\mu\nu} T_{\mu\nu} \qquad M_P = q^N\, M_N$$

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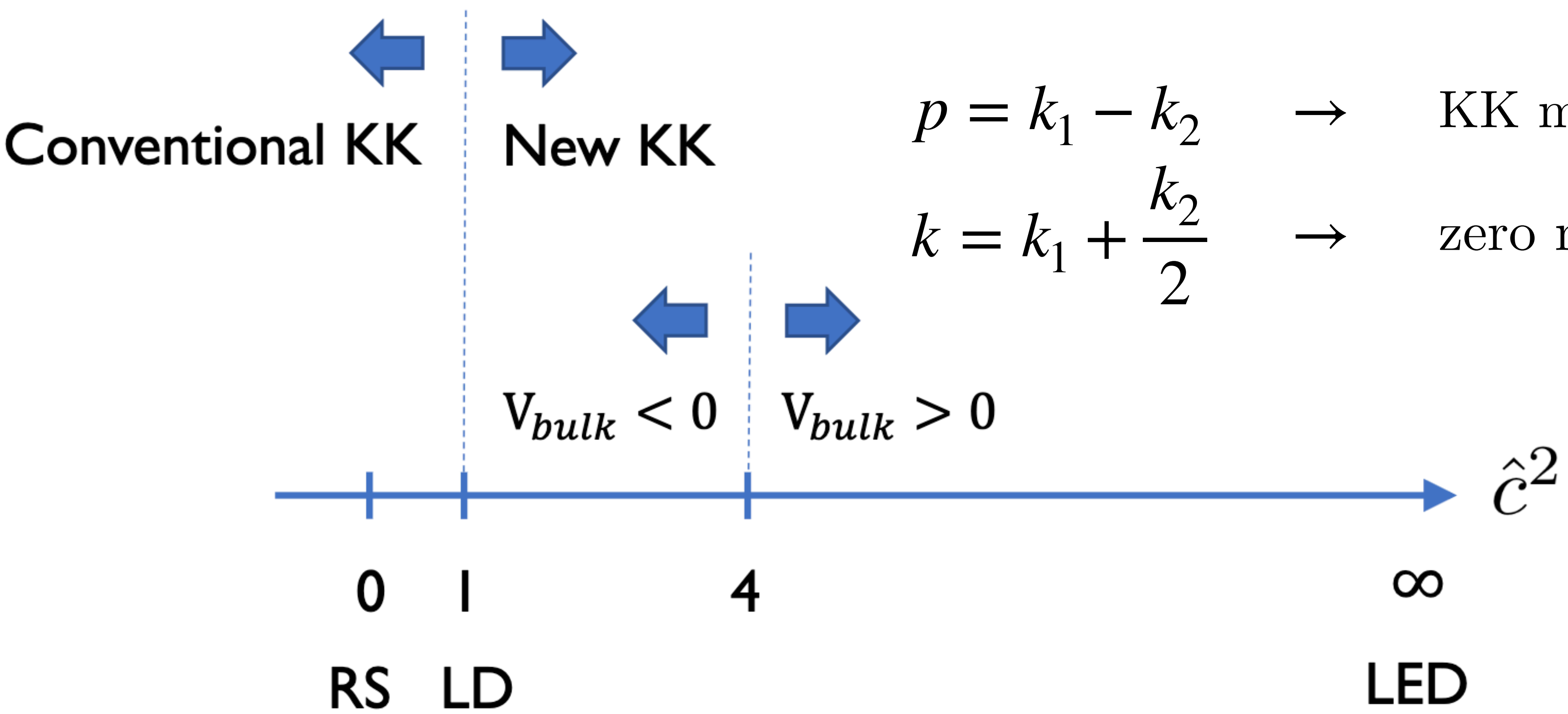
Clockwork/5th dim. with GLD

$$ds^2 = e^{2k_1 y} dx^2 + e^{2k_2 y} dy^2, \qquad c^2 = \frac{k_2}{k_1}$$

gravity + dilaton + cosmological constants on 5D orbifold $M_4 \times S^1/Z_2$

$p = k_1 - k_2 \quad \rightarrow \quad$ KK masses

$k = k_1 + \frac{k_2}{2} \quad \rightarrow \quad$ zero mode coupling hierarchy



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Clockwork from Heterotic M-theory

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- Linear dilaton profile

$$\mathcal{S} = \int d^5x\sqrt{-g}\; e^S\left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}\left(\partial_M S\right)^2 - \Lambda\right)$$

EOM for S: $\frac{Z_S}{2}\left(\partial_M S\right)^2 = -\Lambda + \frac{\mathcal{R}}{2} + \dots$

Solution if $\Lambda < 0$: $S = k\,y$

F_{MN}: $E_8 \times E_8$ gauge field strength

G_{IJKL}: 4-form field strength

11D SUGRA with two 10 D boundaries

$\mu \sim \langle F^2 \rangle$: magnetic flux over internal space

$$\mathcal{S}_5 = M_5^3 \int_{\mathcal{M}^5} \sqrt{-g}\,\hat{V}\left[\frac{1}{2}\mathcal{R}_5 + \frac{5}{12}\hat{V}^{-2}\partial_\alpha \hat{V}\,\partial^\alpha \hat{V} - \frac{1}{384}\hat{V}^{-4/3}\mu^2 - \frac{\sqrt{2}}{8}\hat{V}^{-2/3}\mu\frac{1}{\sqrt{g_{11}}}\left[\delta\left(x_{11}\right) - \delta\left(x_{11} - \pi r_{11}\right)\right]\right]$$

Scale invariance under $x \to x\,t$:

$g_{\alpha\beta} \to g_{\alpha\beta}\,t^2$, $\hat{V} \to \hat{V}\,t^{3/2}$, $\mathcal{S}_5 \to \mathcal{S}_5\,t^{9/2}$

$$\mathcal{S}_5 = M_5^3 \int_{\mathcal{M}^5} \sqrt{-g}\; e^S\left[\frac{1}{2}\mathcal{R}_5 + \frac{23}{36}\left(\partial_M S\right)^2 - \frac{\mu^2}{384} - \frac{\sqrt{2}}{8}\mu\frac{1}{\sqrt{g_{11}}}\left[\delta\left(x_{11}\right) - \delta\left(x_{11} - \pi r_{11}\right)\right]\right]$$

Clockwork from Heterotic M-theory

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μ: magnetic flux over the internal space

Standard embedding: $\mu < 0$

Non-standard embedding: $\mu > 0$:

- V (6D internal volume) grows with $|x^{11}|$
- 4D Planck Mass dilution as in LED with $n \simeq 2$: $M_{4Pl.}^2 \simeq M_{11Pl.}^4 (\pi R_{11})^2$

Heterotic M-theory with vector multiplets (in 5D)

supersymmetry → non-zero flux $\qquad G_{ABCD} = -\frac{\mu}{48}\epsilon_{ABCD}{}^{EF}\omega_{EF}$

$$\mu \equiv \frac{\sqrt{2}}{\pi V_0}\left(\frac{\kappa}{4\pi}\right)^{2/3}\int_{X^6}\omega \wedge \left(\mathrm{tr}\, F_{(1)} \wedge F_{(1)} - \frac{1}{2}\mathrm{tr}\,\mathscr{R} \wedge \mathscr{R}\right)$$

There are $h_{(1,1)} \geq 1$ (Hodge number) Kähler moduli t^i def. by: $\omega = \omega_i t^i$

Intersection numbers $\qquad d_{ijk} \equiv \frac{1}{V_0}\int_{X^6}\omega_i \wedge \omega_j \wedge \omega_k$

There are also $h_{(1,1)}$ intersection flux parameters $\mu_i = \mu(\omega_i)$

Clockwork from Heterotic M-theory

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Stringy origin of LD (Clockwork): Gravity dual of the type I Little String Theory

$$M_{4Pl.}^2 \simeq M_S^8 V_6/g_S^2 \text{ and } g_S^2 = e^{-S} \sim 10^{-30} \to M_S \sim V_6^{-1/6} \sim O(1)\,\text{TeV}$$

Aharony, Berkooz, Kutasov, Seiberg 98, Antoniadis, Dimopoulos, Giveon 2001

UV origin of GLD: Heterotic M-theory with non-standard embedding

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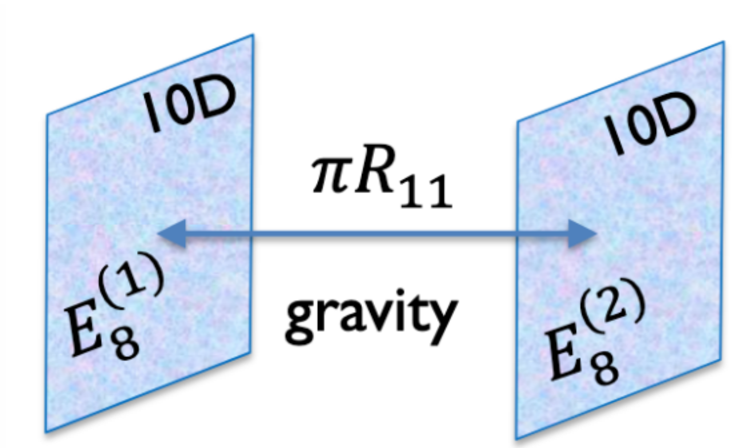

$g_S^2 \simeq 1/(R_{11} M_S)$, $M_{4Pl.}^2 \simeq M_S^8 V_6/g_S^2$,
and V_6/g_S^2 can be large $\to M_S \sim O(1)\,\text{TeV} \ll M_{4Pl.}$

Clockwork from Heterotic M-theory

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Non-standard embedding: $\mu > 0$:

- Calabi Yau 3-fold with Hodge number $h_{(1,1)} = 1$: $c^2 = 6$
- Calabi Yau 3-fold with Hodge number $h_{(1,1)} > 1$
  - $h_{(1,1)} = 2$ with $\mu_1 \neq 0$, and $\mu_2 \neq 0$: $c^2 = 6$
  - $h_{(1,1)} = 2$ with $\mu_1 \neq 0$, and $\mu_2 = 0$: $c^2 = 7$
  - $h_{(1,1)} = 2$ with $\mu_1 = 0$, and $\mu_2 \neq 0$: $c^2 = 10$

Clockwork bound from this stringy construction: $c^2 \geq 6.$

$c\,S = \sqrt{6}\,S_V + S_1$, where $c^2 = c_V^2 + c_1^2 = 6 + c_1^2 \geq 6$

$$V_{\text{bulk}} = M_5^3 \frac{1}{256} e^{-\frac{2}{\sqrt{3}}\sqrt{6}S_V} \left[\frac{\mu_1^2}{\beta^2} e^{-\frac{2}{\sqrt{3}}S_1} + 2\frac{\mu_2^2}{\beta^2} e^{\frac{4}{\sqrt{3}}S_1} \right]$$

$$V_{\text{boundary}} = M_5^3 \frac{\sqrt{2}}{8} e^{-\frac{1}{\sqrt{3}}\sqrt{6}S_V} \left[\frac{\mu_1}{\beta} e^{-\frac{1}{\sqrt{3}}S_1} + \frac{\mu_2}{\beta} e^{\frac{2}{\sqrt{3}}S_1} \right] \left[\delta\left(x_{11}\right) - \delta\left(x_{11} - \pi r_{11}\right) \right]$$

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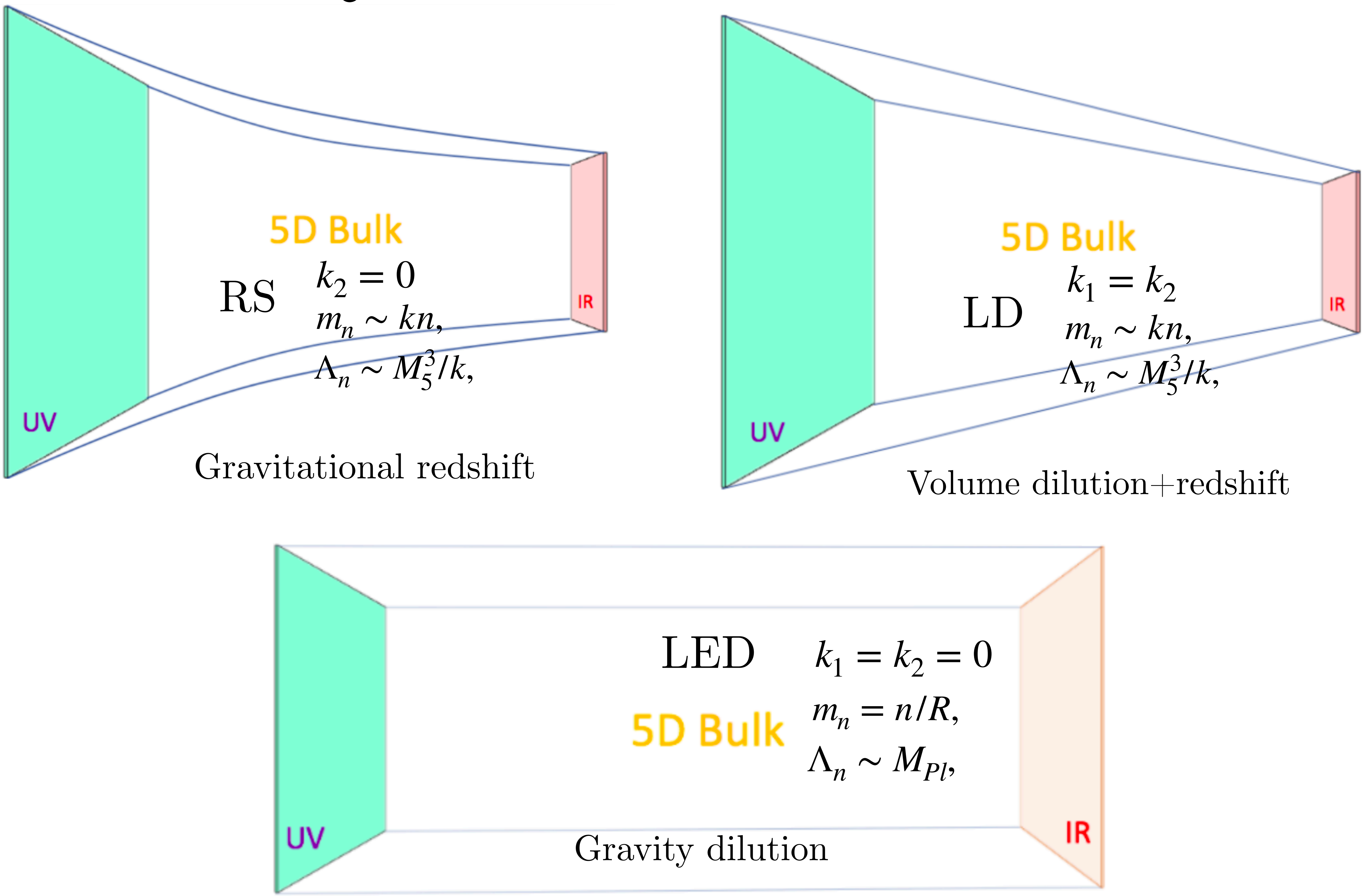
$c\,S = \sqrt{6}\,S_V + S_1$, where $c^2 = c_V^2 + c_1^2 = 6 + c_1^2 \geq 6$

We recover the case of $h_{(1,1)} = 1$, if $\mu_i \neq 0$ for all i, because all the Kähler moduli are stabilized.

μ_i: $h_{(1,1)}$ magnetic fluxes over independent cycles

If $\mu_1, \mu_2 \neq 0$: $S_1 = \dfrac{1}{\sqrt{3}} \ln\left(\dfrac{\mu_1}{2\mu_2}\right)$ stabilized; $c^2 = 6$.

Geometry of the extra dimension



Spectrum as a function of $k_{1,2}$

- There are three backgrounds as limiting cases

  Curved: Randall-Sundrum and Linear Dilaton

$$\text{RS}: \; m_n = \left(n + \frac{1}{4}\right)\pi k, \;\; \Lambda_n = \sqrt{\frac{M_5^3}{k}}, \qquad M_{\text{Pl.}} = \sqrt{\frac{M_5^3}{k}\left(e^{2k\pi R} - 1\right)}, \;\; k_2 = 0,$$

$$\text{LD}: \; m_n = \sqrt{k^2 + \frac{n^2}{R^2}}, \;\; \Lambda_n = \sqrt{M_5^3 \pi R\left(1 + \frac{k^2R^2}{n^2}\right)}, \;\; M_{\text{Pl.}} = \sqrt{\frac{M_5^3}{k}\left(e^{2k\pi R} - 1\right)}, \;\; k_1 = k_2.$$

$\sim$ Flat: GLD

For hierarchy problem $kR \simeq 10$.

GLD: (for $1 < c < 2$)

$$m_n = \frac{\pi}{2}\left(k + k(-1+4n)\frac{|c^2-1|}{c^2+2}\right)\left(1 + \frac{kM_{Pl}^2}{M_5^3}\right)^{\frac{-|c^2-1|}{c^2+2}} \sim \frac{n}{R},$$

$$\Lambda_n = \frac{M_5}{C_n}, \qquad C_n = \frac{M_5^{3/2}\, 2^{2-\frac{c^2+2}{2|c^2-1|}}\, \pi^{\frac{c^2+2}{2|c^2-1|}}\, |c^2-1| \left(\frac{c^2+2}{4|c^2-1|} + n - \frac{1}{4}\right)^{\frac{c^2+2}{2|c^2-1|}}}{\Gamma\left(\frac{c^2+2}{2|c^2-1|}\right)\sqrt{(c^2+2)\left((4n-1)|c^2-1| + c^2+2\right)\left(kM_{Pl}^2 + M_5^3\right)}} \sim \frac{M_5}{M_{Pl}},$$

Solving the hierarchy problem

- LED (ADD scenario)
    - gravity gets diluted in the large volume of the extra dimensions which explains its weakness
    - need R $\simeq$ 1mm, while we expect R $\simeq 1/M_{Pl} \simeq 10^{-33}$ cm (?)

- Add warping: *dynamical* mechanisms that stabilize the distance between IR and UV branes $\rightarrow kR \simeq 10$.
- Randall-Sundrum
    - Goldberger-Wise: additional massive scalar φ in the bulk with a potential V(φ); add potentials $V_1(\varphi)$ and $V_2(\varphi)$ on the two branes at the boundaries
- Linear Dilaton
    - Dilaton scalar - it *both* sources the background geometry and stabilizes the extra dimension

Signatures

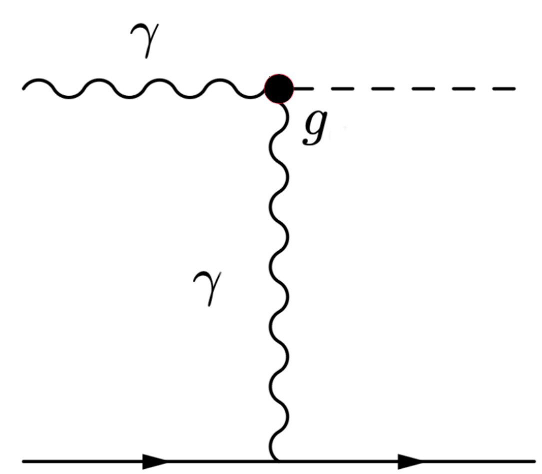
- We focus on two regimes for radion and KK gravitons: $\phi, G_k \to f\bar{f}, \gamma\gamma$
  - $c\tau\gamma \lesssim 1m$: FCC-ee, CLIC

$$\sigma\left(e^+e^- \to XG\right) = \int d\Omega \frac{d\sigma\left(e^+e^- \to GX\right)}{d\Omega}\left(1 - e^{-L_{\rm det}/L_G^\perp(\theta)}\right), \qquad (2.2)$$

where $L_G^\perp = c\tau\gamma\beta\sin\theta$, $\gamma = (s - m_X + m_G^2)/(2m_G\sqrt{s})$ for $X = \gamma$, Z, and θ is the angle measured from the collider axis.

  - $c\tau\gamma \gtrsim 1m$: SHiP, FASER, MATHUSLA and BBN/SN1987

$$N = \sum_{E,\theta} N_{\rm LLP}(E,\theta) \times \left(e^{-L_{\rm min}/d(E)} - e^{-L_{\rm max}/d(E)}\right)$$



Prod. from $\sigma_{\gamma N \to GN} \simeq \frac{\alpha_{em} g_{G\gamma\gamma}^2 Z^2}{2}\left(\log\left(\frac{d}{1/a^2 + t_{max}}\right) - 2\right)$ and $Z \to Gf\bar{f}$.

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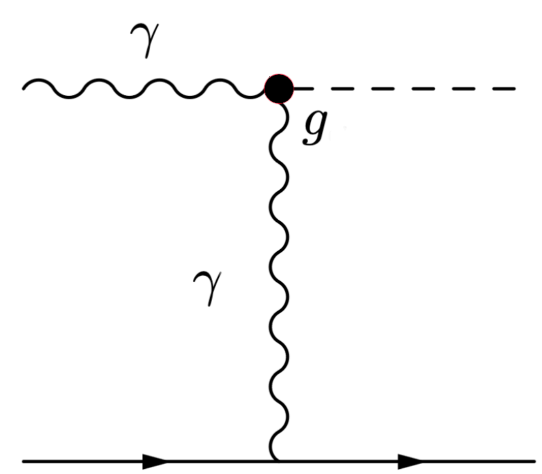

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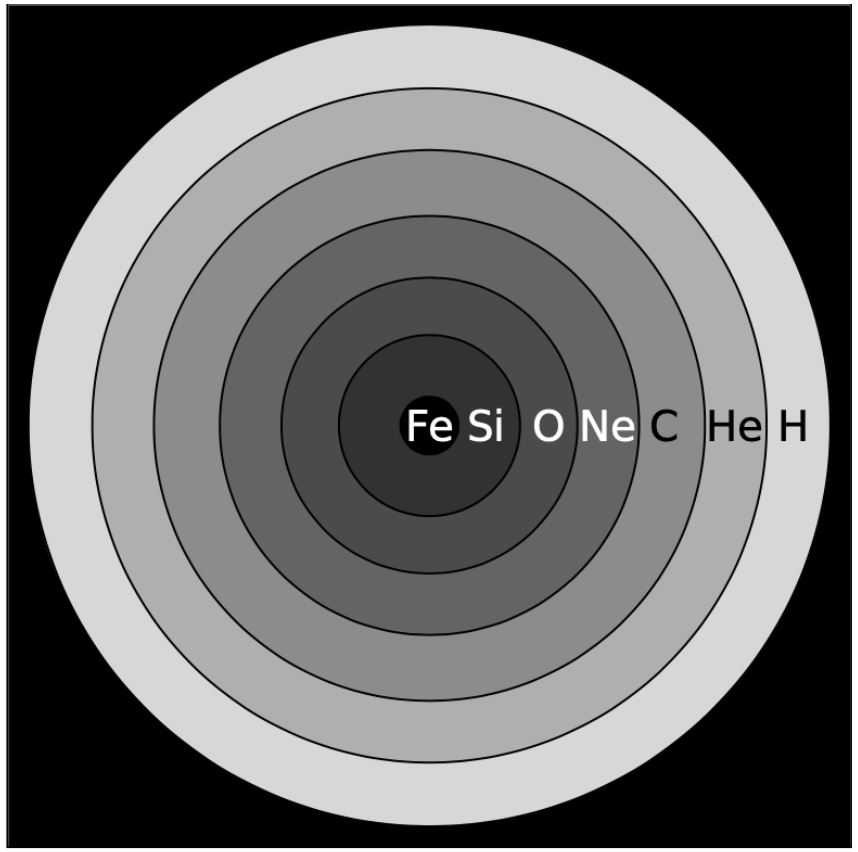
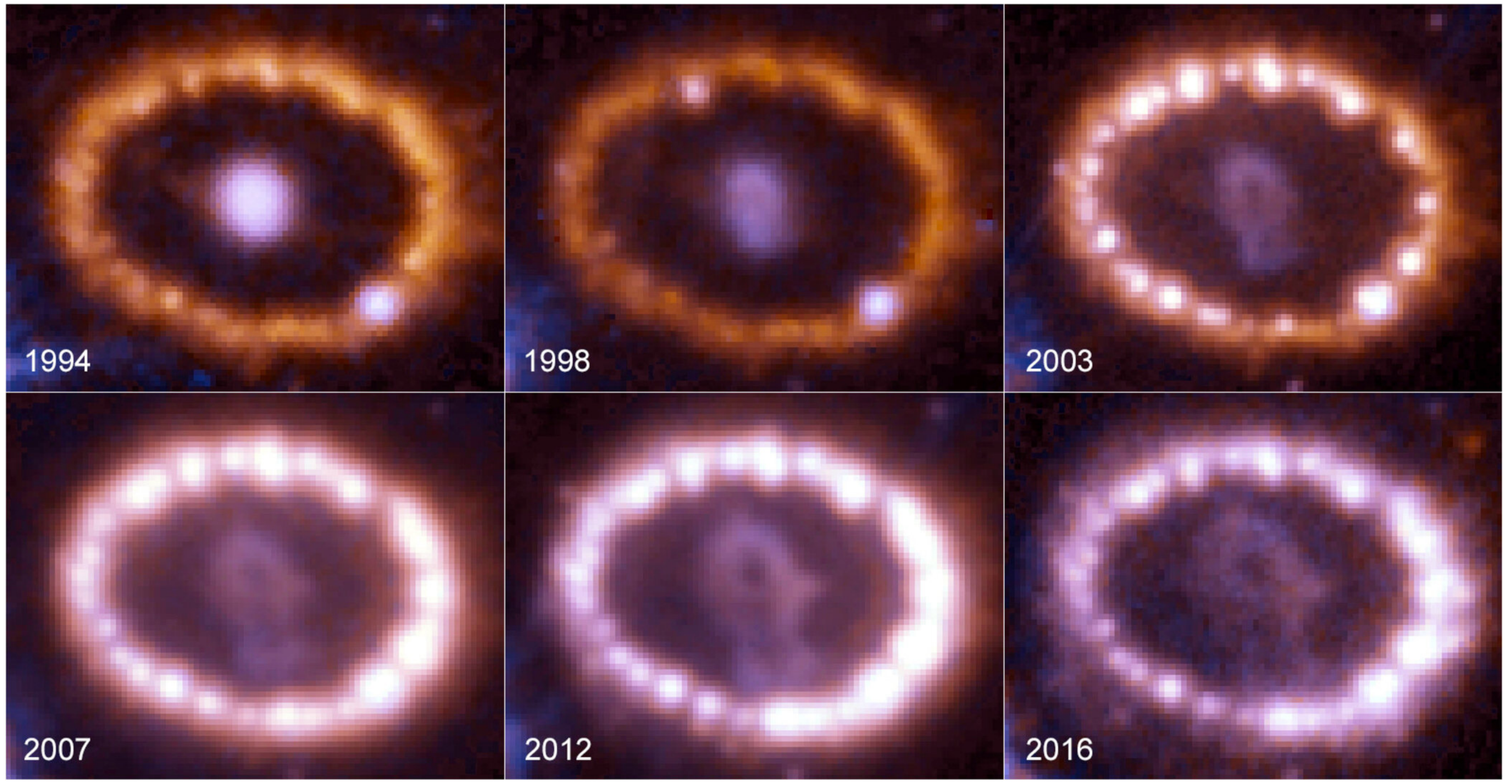
SN1987



Chandrasekhar limit

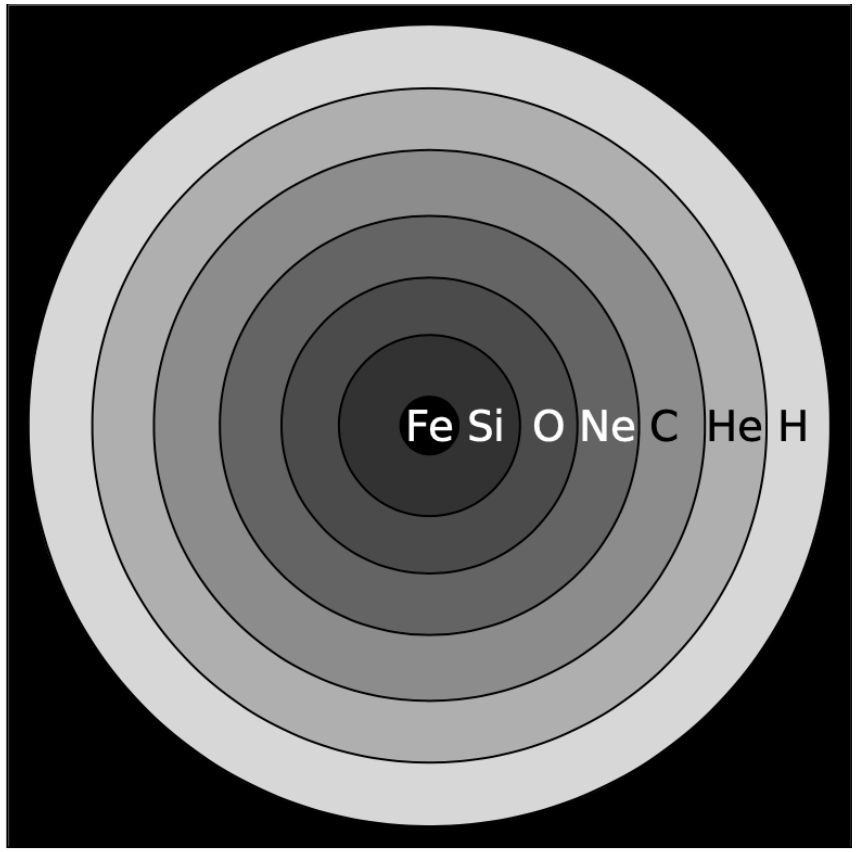
$M_{\text{core}} > 1.44\, M_{\text{Sun}}$

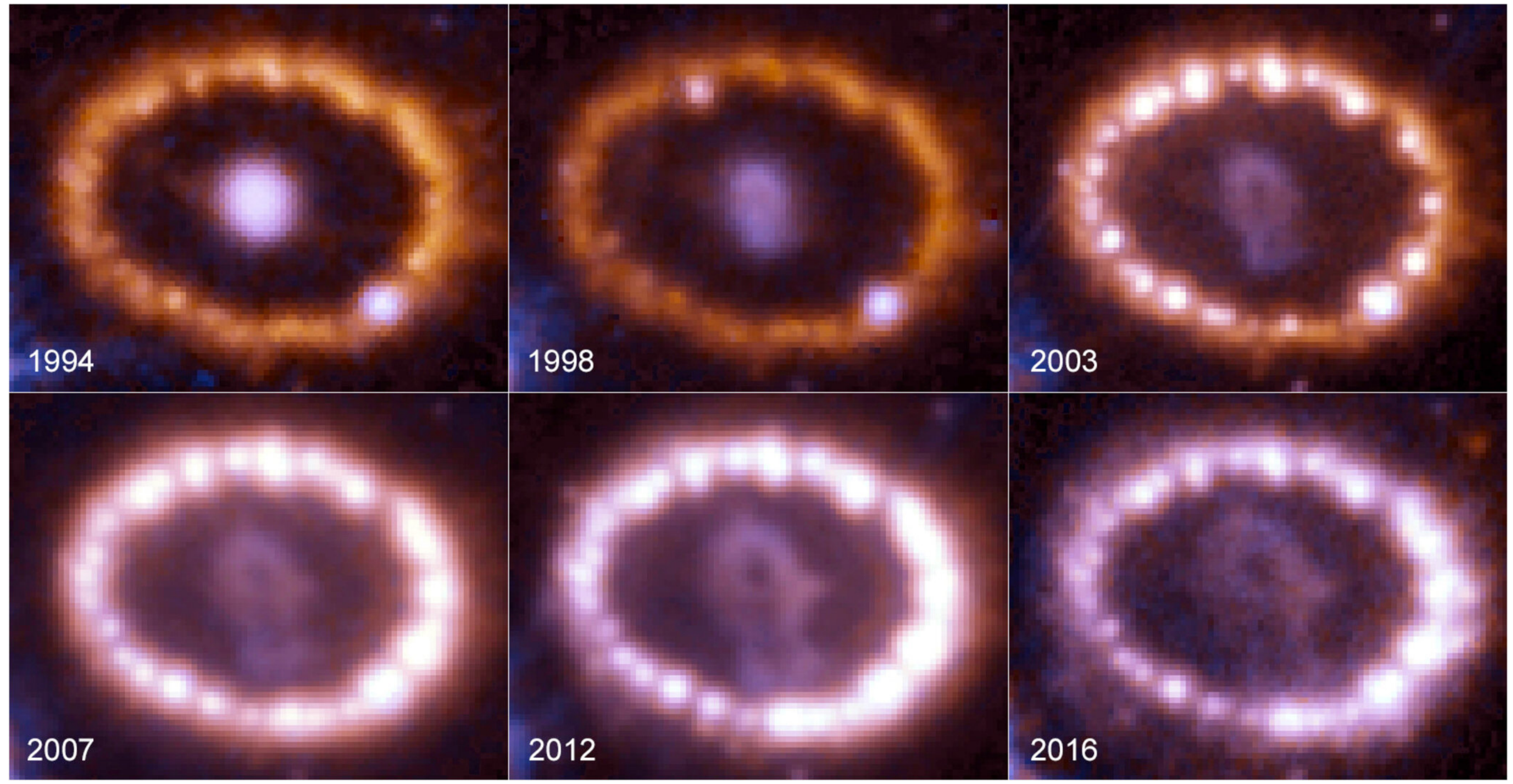


Core-collapse supernova

Binding energy $E \simeq \dfrac{G_N M^2}{R}$ gets released as neutrinos, which are emitted first, and photons.

SN1987





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Core-collapse supernova

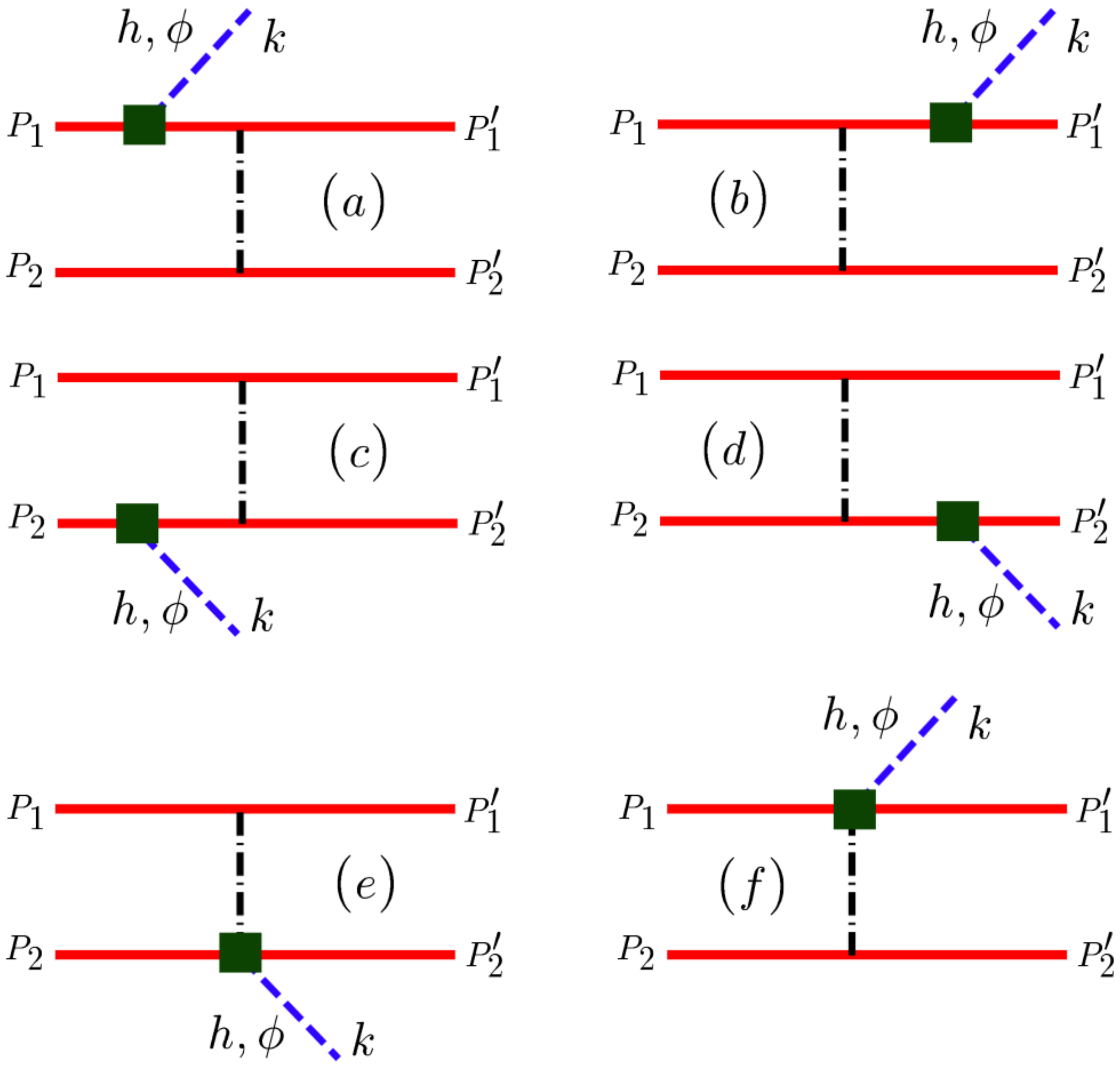
Binding energy $E \simeq \dfrac{G_N M^2}{R}$ gets released as neutrinos, which are emitted first, and photons.

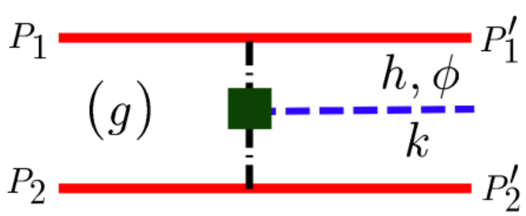
Neutrino luminosity $\mathscr{L}_\nu \simeq 3 \times 10^{53}\,\text{erg/s}$. Raffelt criterion: $\mathscr{L}_{\text{BSM}} < \mathscr{L}_\nu$.

(assuming KK gravitons are not trapped inside the SN core)

$\lambda_{\text{MFP}} \simeq 1/(n\sigma) < R$.

SN1987 for LED





$$N + N \to N + N + G_k$$

Energy loss rate for a single graviton

$$Q_m \propto \frac{1}{\Lambda(m)^2} \sigma_N n_B^2 T^{7/2} m_N^{-1/2}$$

$$T = 30\,\mathrm{MeV},\ \rho = 3\times 10^{14}\ \mathrm{g\ cm^{-3}},\ \sigma_N = 25\ \mathrm{mb}$$

Energy loss rate for all gravitons

$$S(-\omega) = \frac{1}{\omega^2} \frac{2}{1+e^{\omega/T}} \frac{1024\sqrt{\pi}}{5} \frac{\sigma_N n_B^2\, T^{5/2}}{m_N^{1/2}} \frac{1}{\Lambda_n^2}$$

$$Q = \frac{2R}{(2\pi)^2} \int_0^\infty d\omega\, \omega\, S(-\omega) \int_0^\omega dm\, \rho(m) \left(\frac{19}{18} + \frac{11}{9}\frac{m^2}{\omega^2} + \frac{2}{9}\frac{m^4}{\omega^4} \right)$$

λ_{MFP} due to $N + N + G_k \to N + N$ and decays

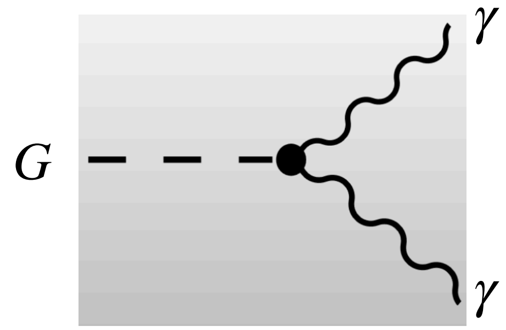
Hanhart, Phillips, Reddy, Savage 0007016

Hannestad, Raffelt 0103201, 0304029

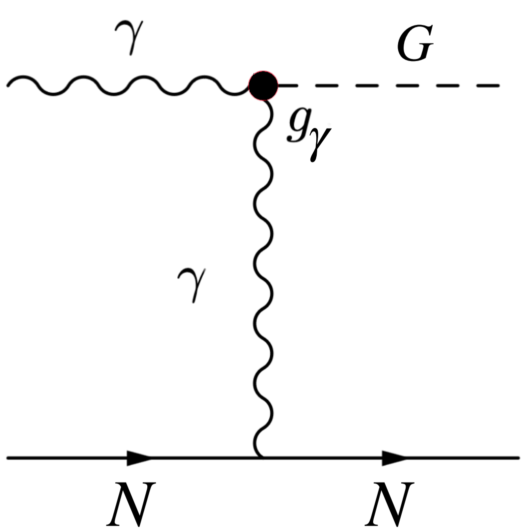
Beam dumps: massive spin-2 mediator $c\tau\gamma \sim 100\,m$

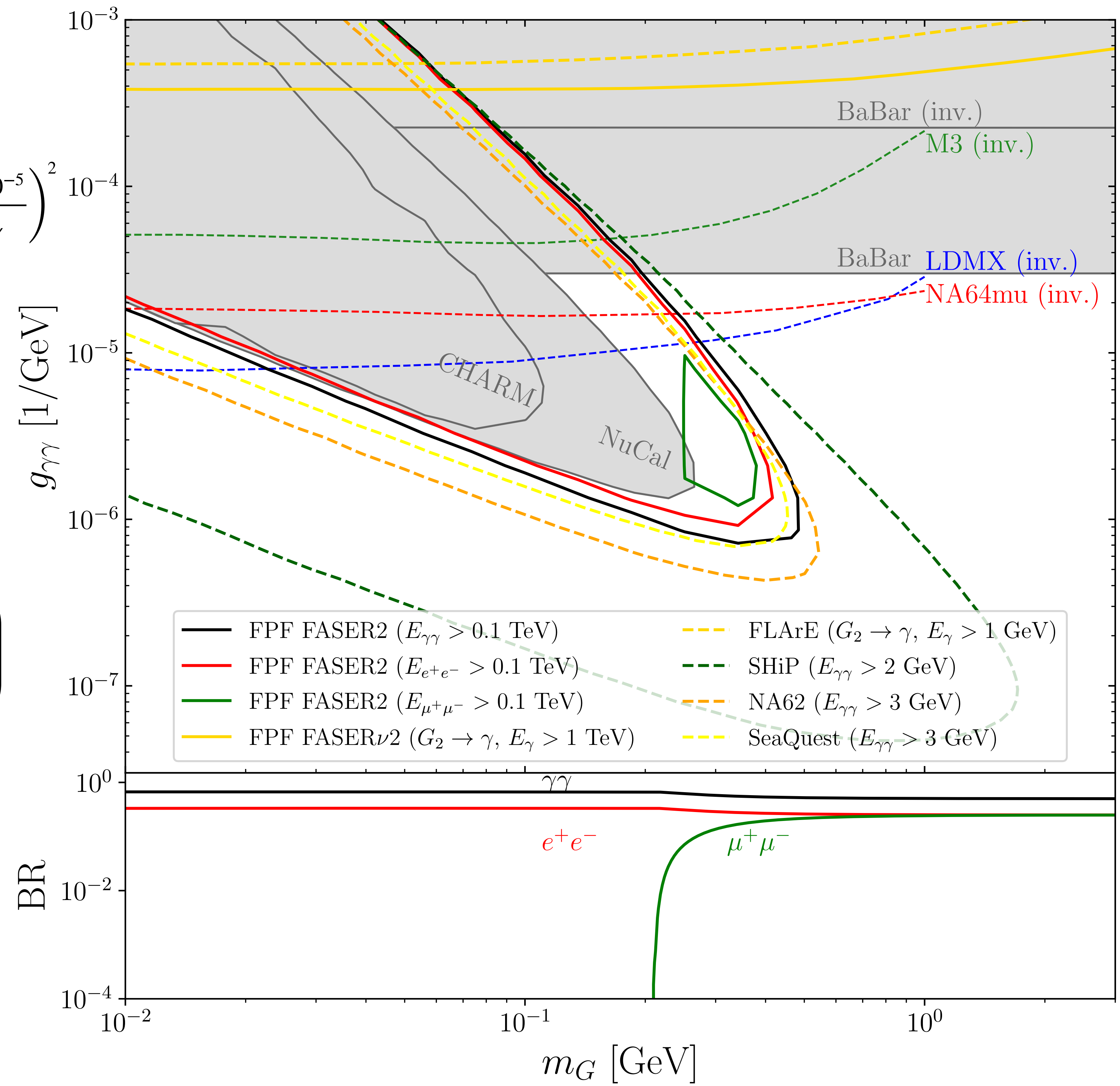
$$\mathscr{L} \supset g_\gamma G^{\mu\nu} T^{\rm EM}_{\mu\nu} + g_\ell G^{\mu\nu} T^{\rm matter}_{\mu\nu}$$

$$d_G = c\tau\beta\gamma \simeq 100\,m \times \left(\frac{E}{1000\,GeV}\right)\left(\frac{0.1\,GeV}{m_G}\right)^4\left(\frac{5\times10^{-5}}{g_{G\gamma\gamma}}\right)^2$$



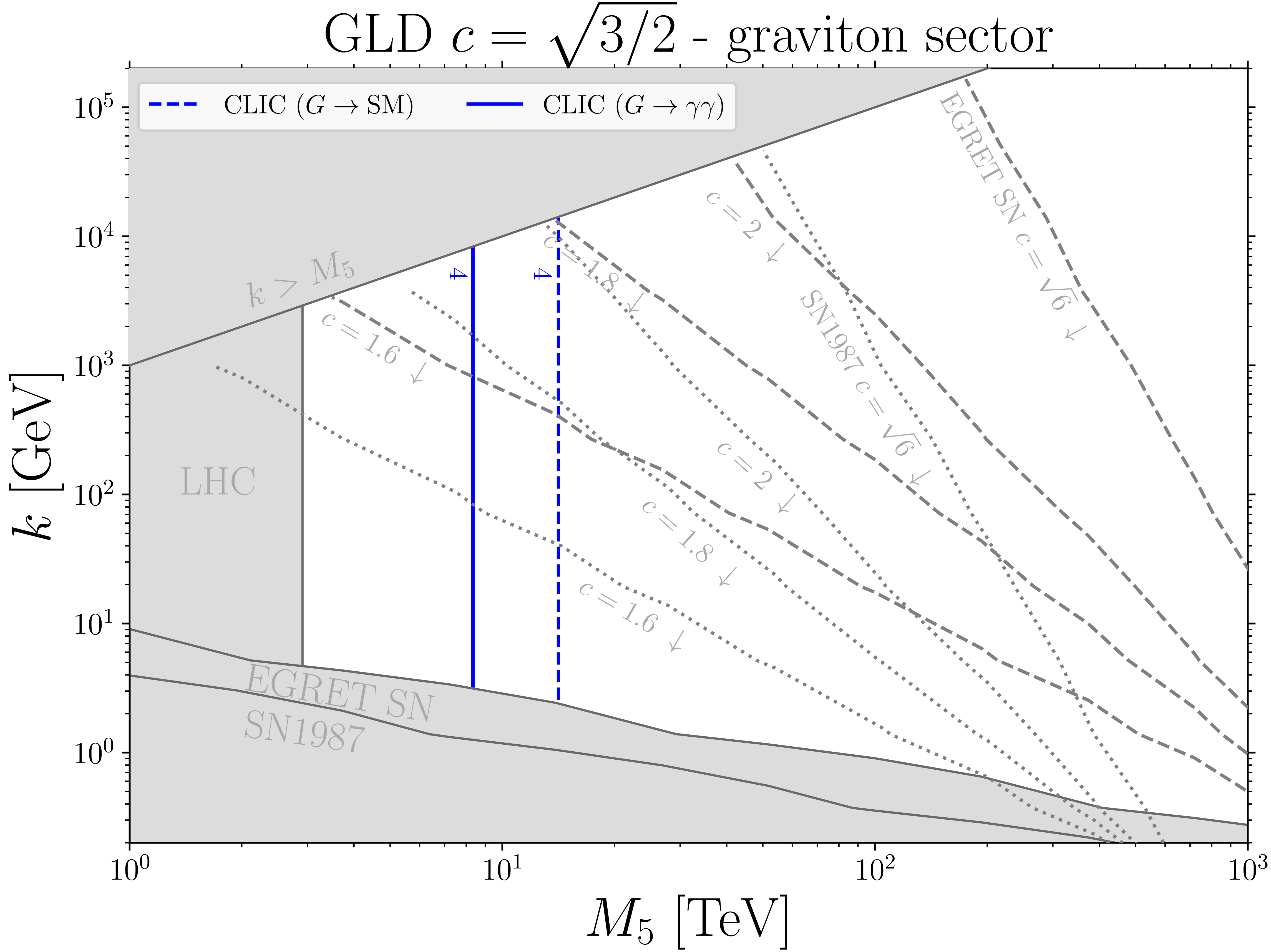
$$\sigma_{\gamma N\to GN} \simeq \frac{\alpha_{em} g^2_{G\gamma\gamma} Z^2}{2}\left[\log\left(\frac{d}{1/a^2 + t_{max}}\right) - 2\right]$$



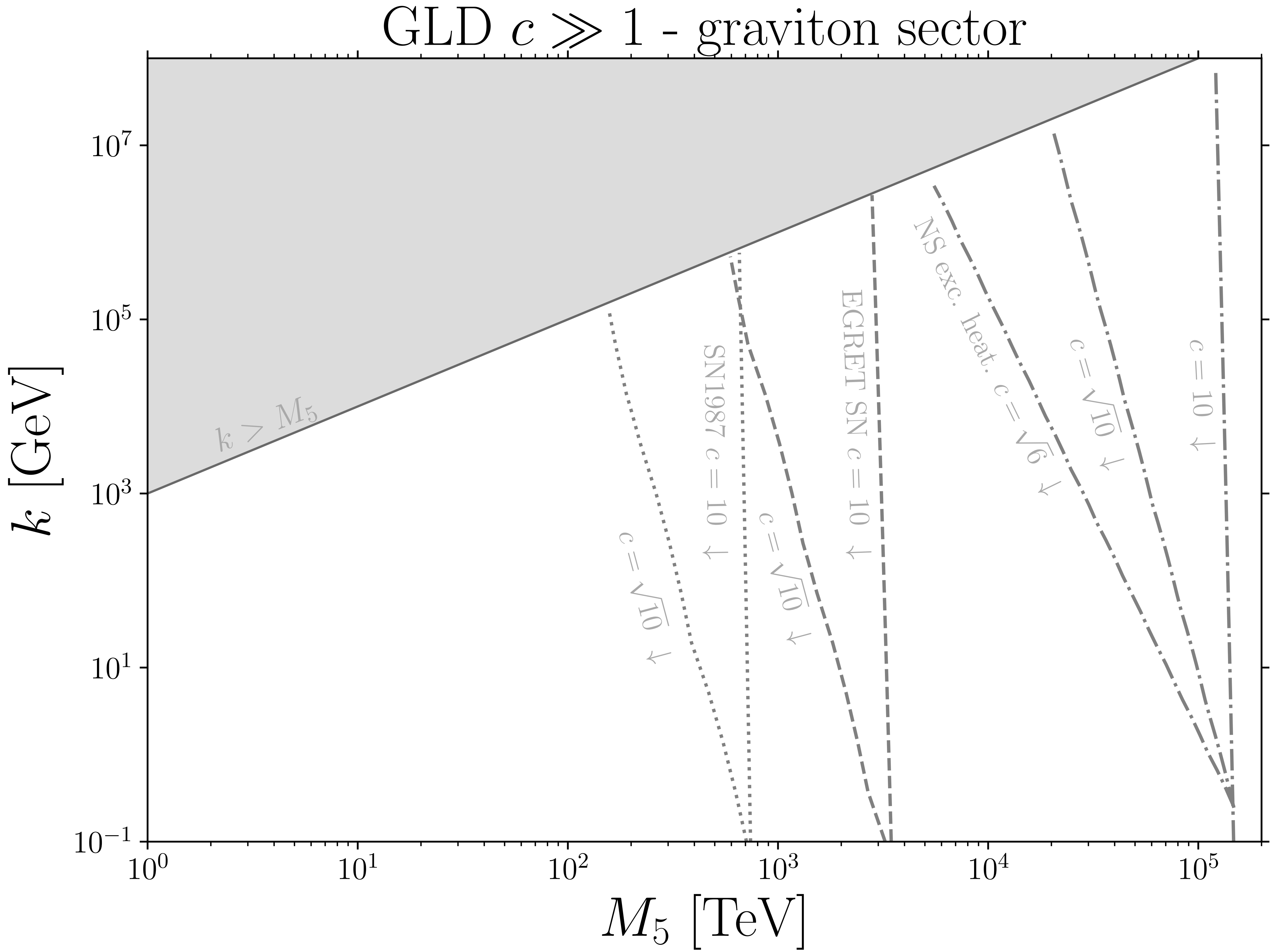


KJ, 2305.05710 (Phys. Rev. D 108, 115017)

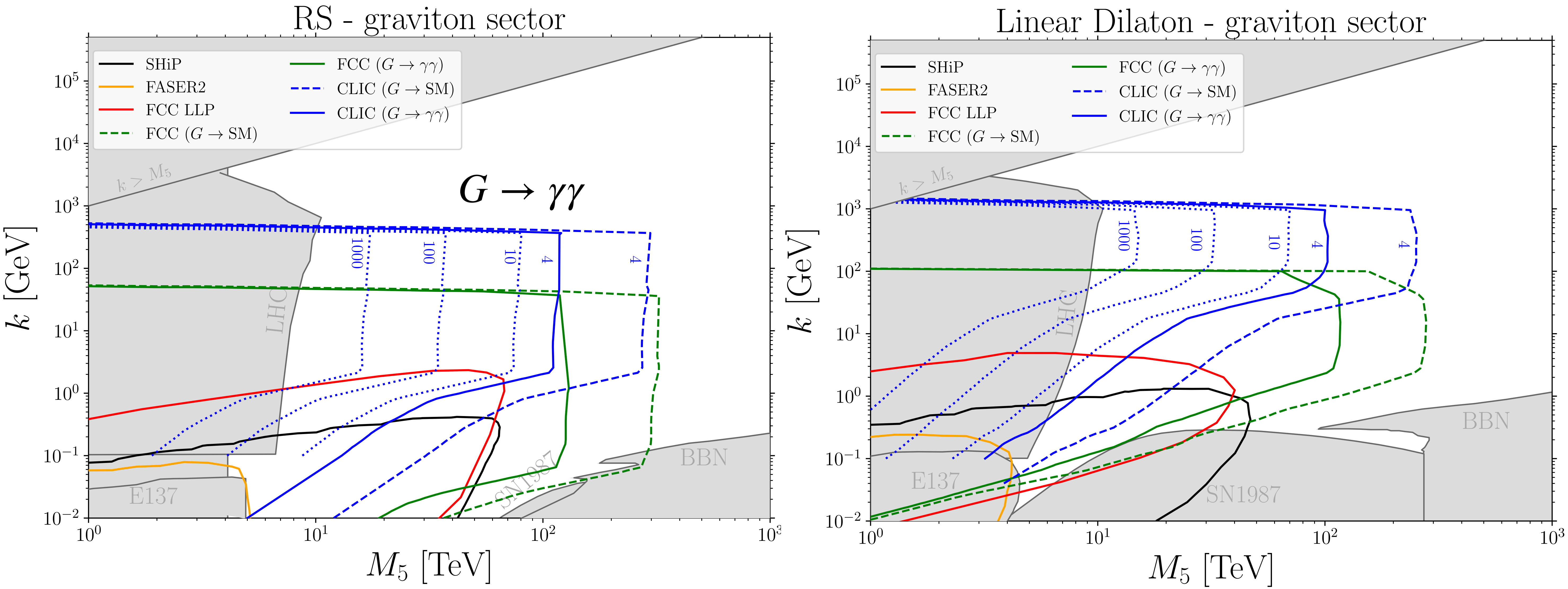
SN1987 - Clockwork



SN1987 - Clockwork



Results

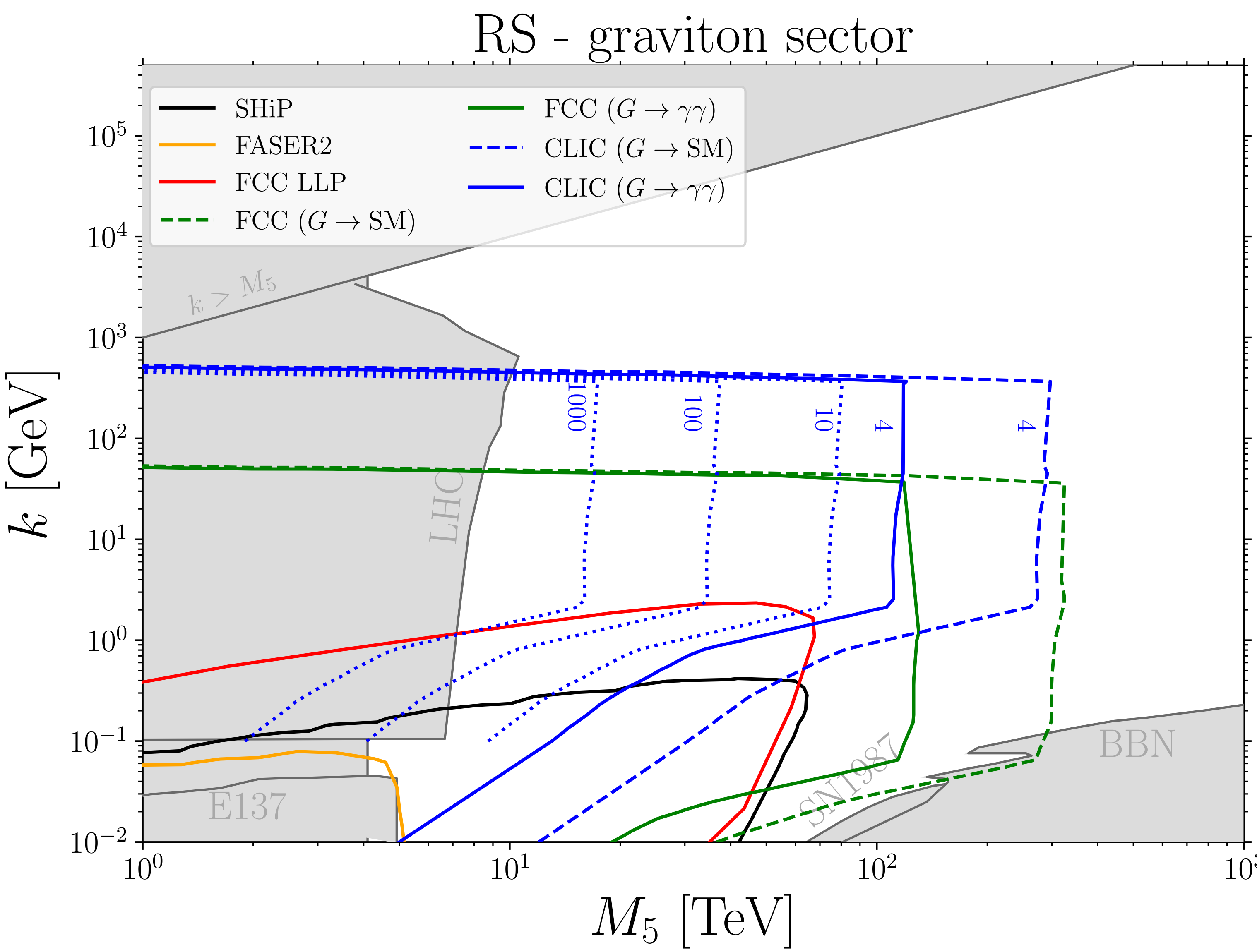


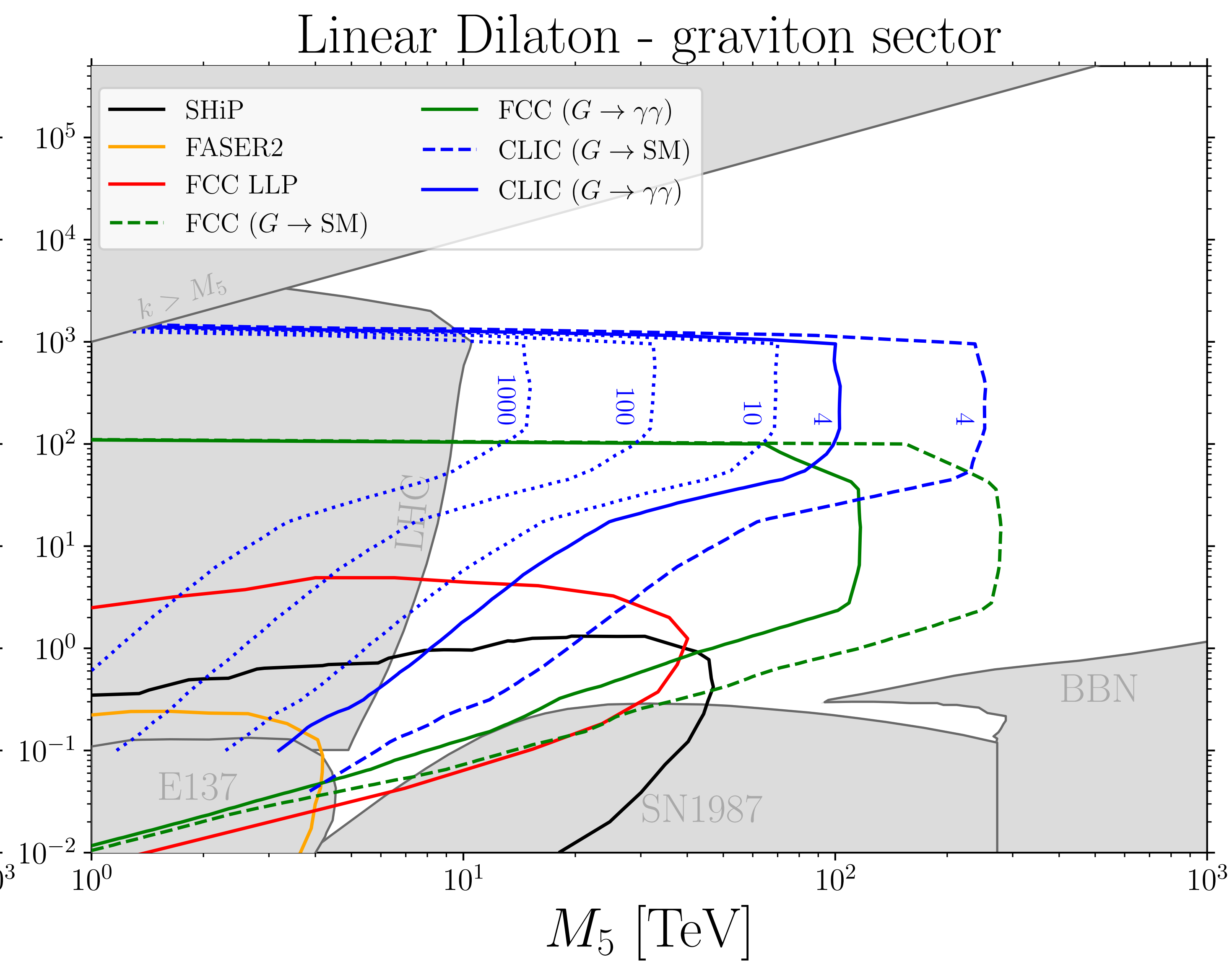
Short-lived regime → LHC, FCC-ee, CLIC $e^+e^- \to G\gamma, GZ, G$

$g + g \to G$

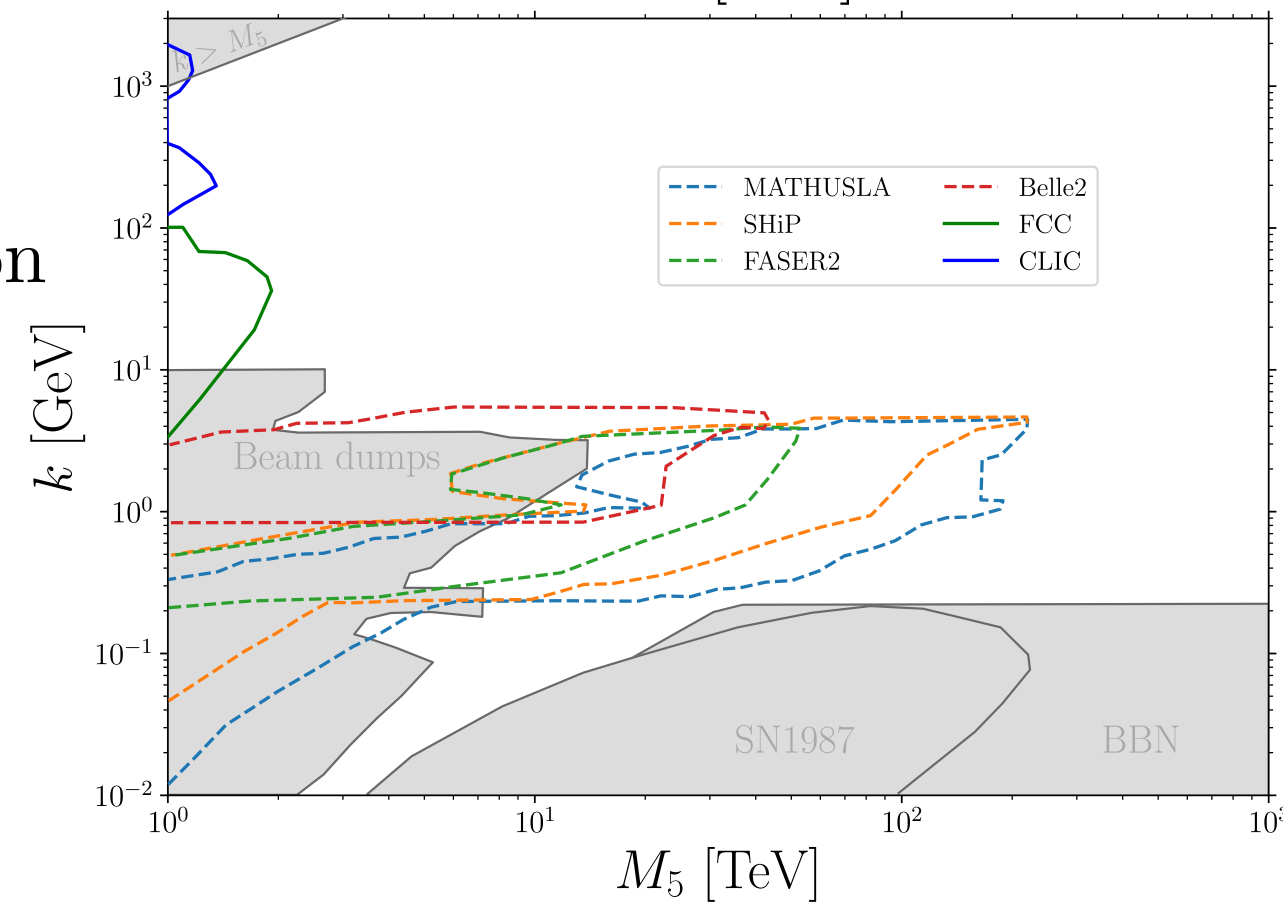
Long-lived regime → KK-gravitons from Primakoff scattering and Z decays

$e^+e^- \to Z, \;\; Z \to b\bar{b}G$





Radion beam dump → efficient production in $B \to K\phi$ due to top coupling $\propto m_t$

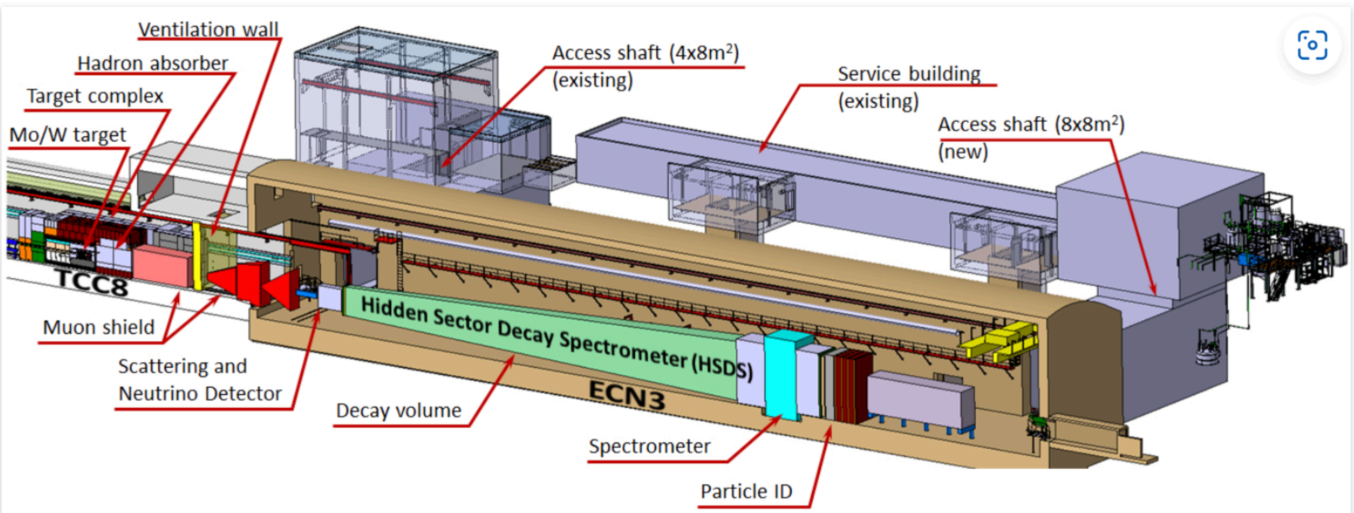


Timeline

SHiP sets sail to explore the hidden sector

The experiment is designed to detect very feebly interacting particles, including candidate dark-matter particles

19 APRIL, 2024 | By Corinne Pralavorio



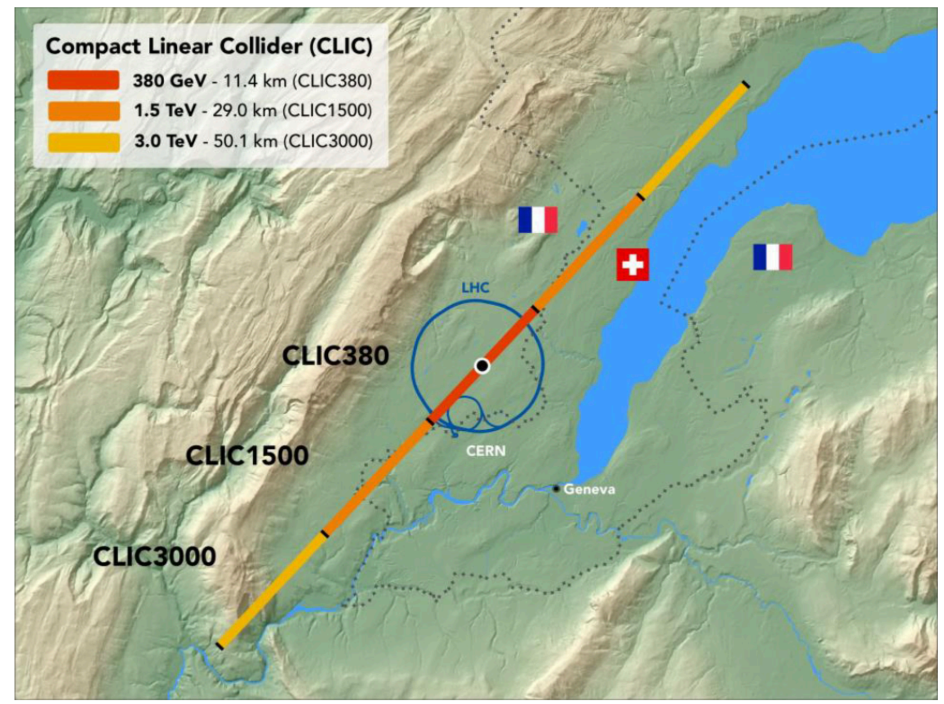
Layout of the SHiP experiment, with the target on the left. (Image : SHiP/CERN)

The SHiP (Search for Hidden Particles) collaboration was in high spirits at its annual meeting this week. Its project to develop a large detector and target to be installed in one of the underground caverns of the accelerator complex has been accepted by the CERN Research Board. Thus, SHiP plans to sail to explore the hidden sector in 2031. Scientists hope to capture particles that interact very feebly with ordinary matter – so feebly, in fact, that they have not yet been detected.

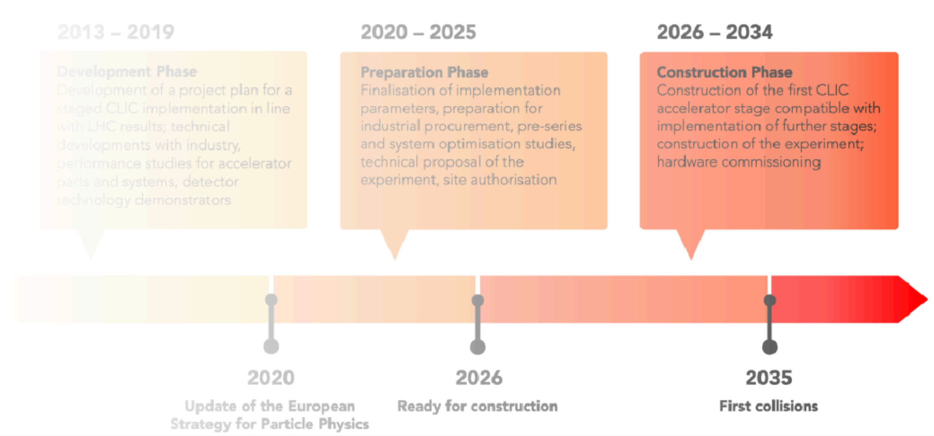
The tentative timeline is:

- **2025**: Completion of the FCC Feasibility Study
- **2027–2028**: Decision by the CERN Member States and international partners
- **2030s**: Start of construction
- **Mid-2040s**: **FCC-ee** begins operation and runs for approximately 15 years
- **2070s**: **FCC-hh** begins operation and runs for approximately 25 years

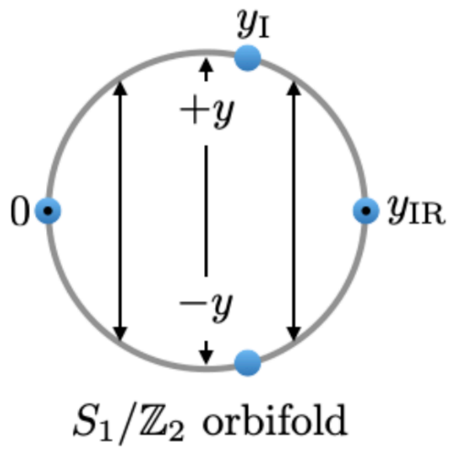
FCC-ee

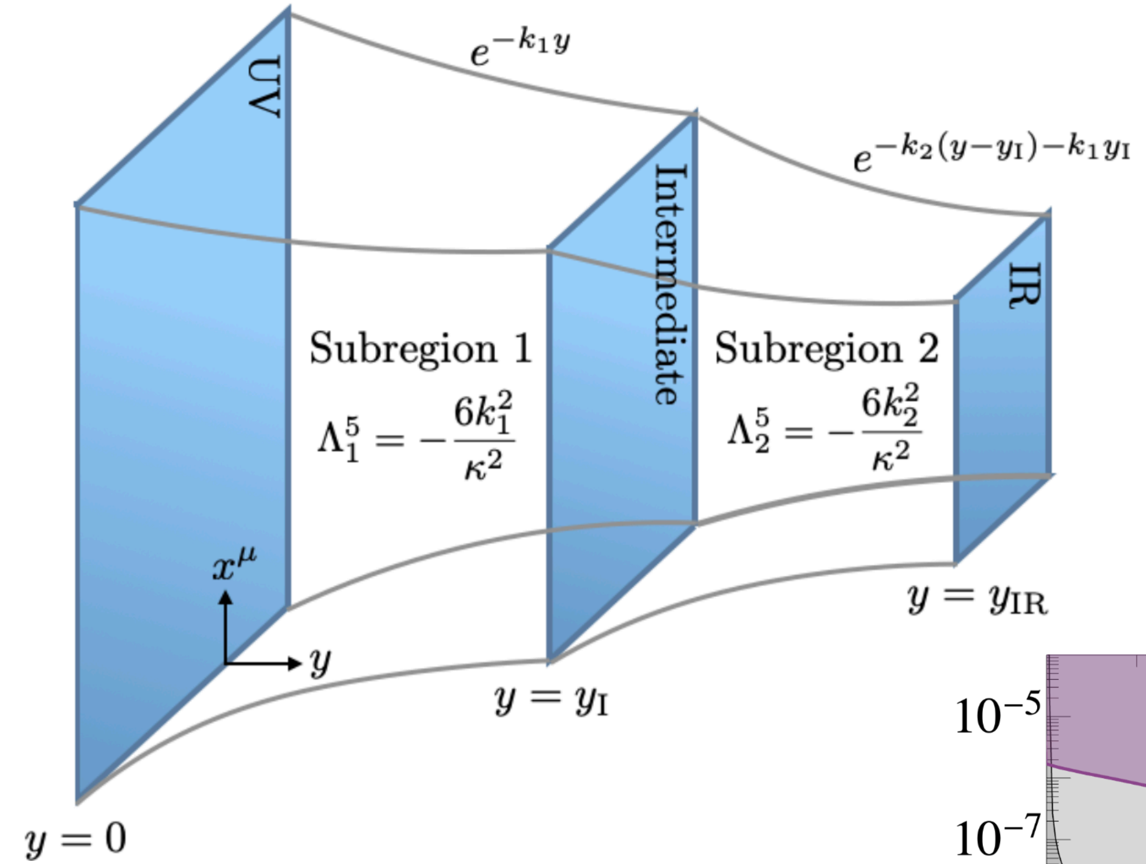


CLIC

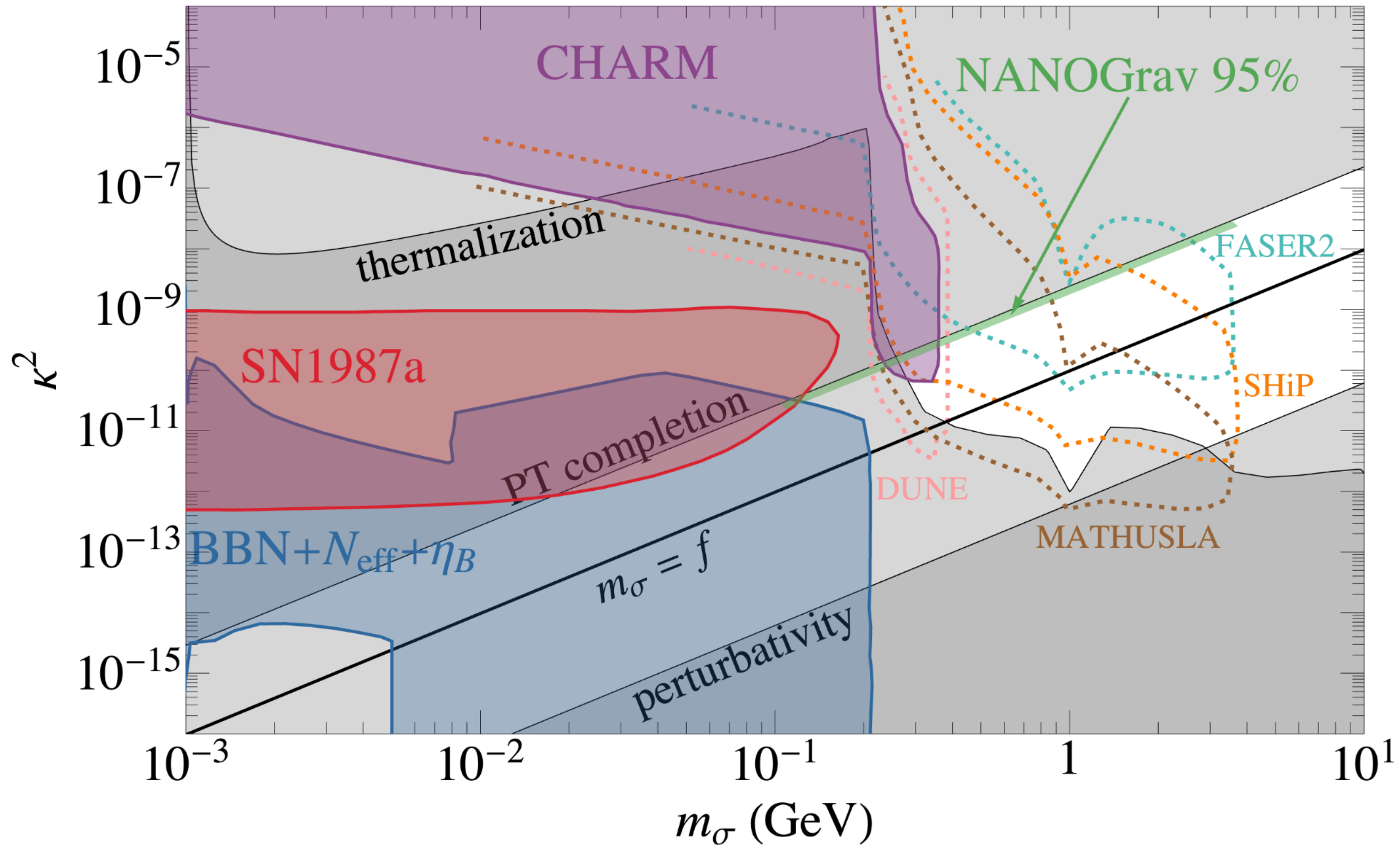


RS with 3 branes and the NANOGrav signal





Multiple Hierarchies from a Warped Extra Dimension,
S. Lee, Y. Nakai, M. Suzuki 2109.10938



Ferrante, Ismail, Lee, and Lee 2308.16219

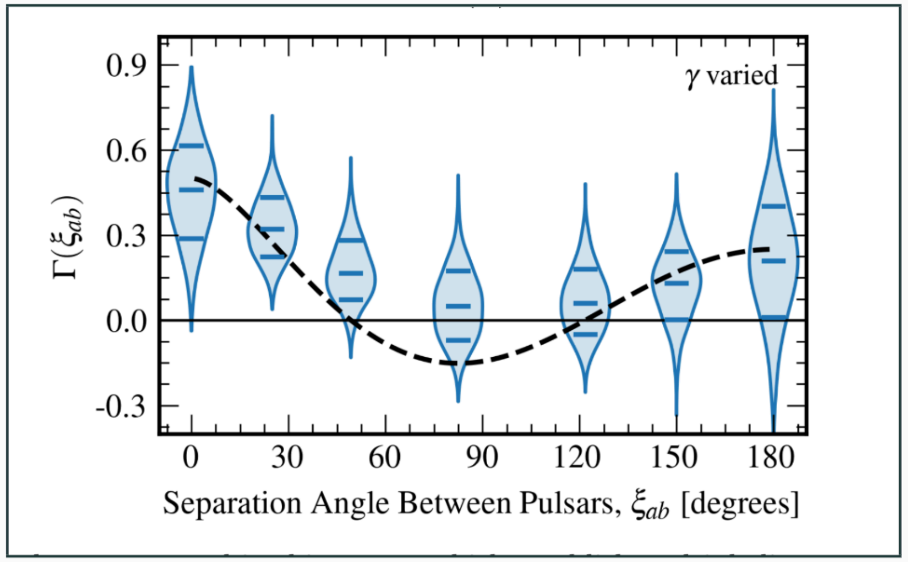
Couplings

graviton, heavy radion → suppressed

light radion → not suppressed

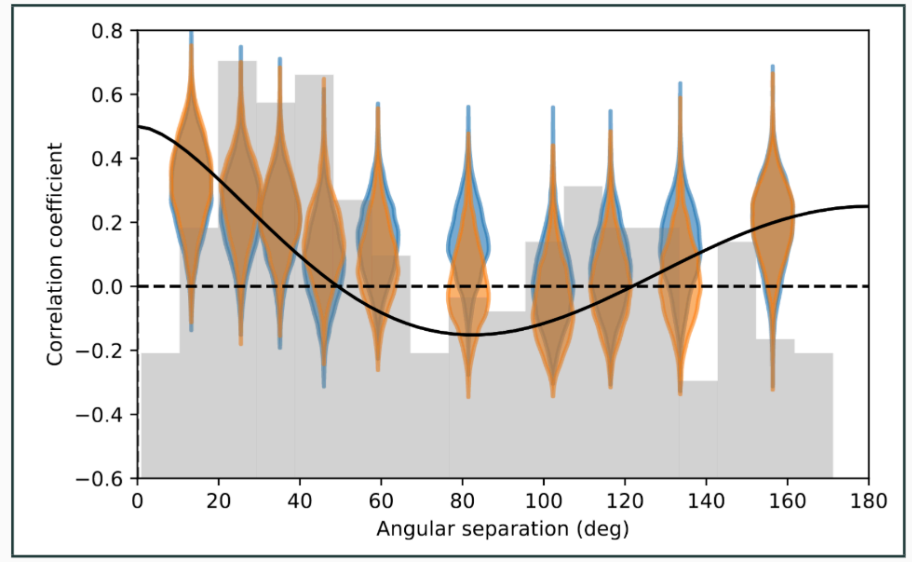
NANOGrav et al. stochastic GWB

2306.16213: NANOGrav



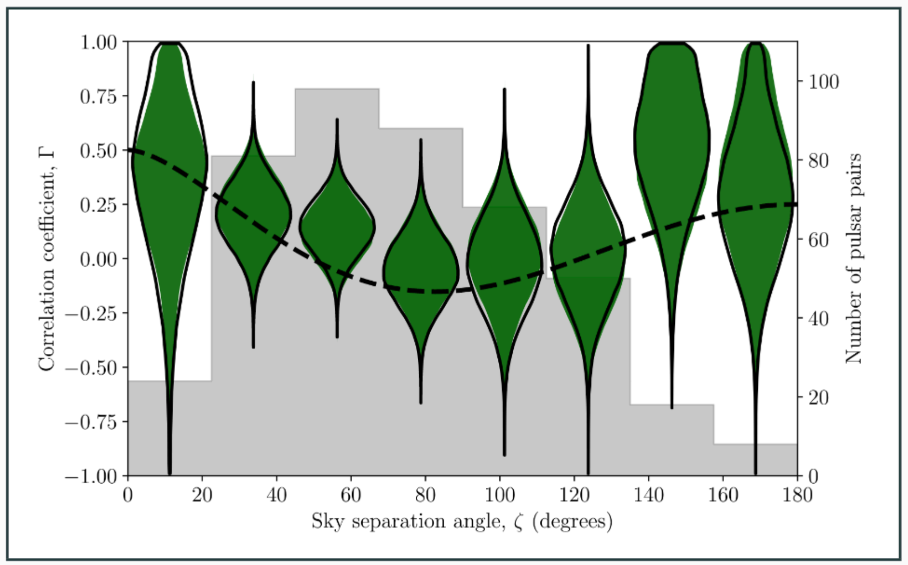
68 pulsars, 16 yr of data, HD at $\sim 3 \cdots 4\,\sigma$

2306.16214: EPTA+InPTA



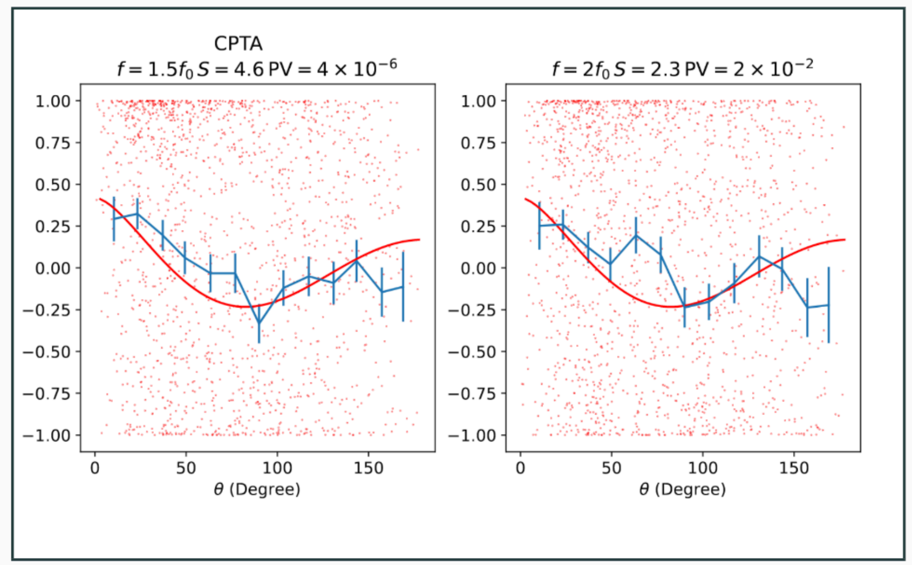
25 pulsars, 25 yr of data, HD at $\sim 3\,\sigma$

2306.16215: PPTA



32 pulsars, 18 yr of data, HD at $\sim 2\,\sigma$

2306.16216: CPTA



57 pulsars, 3.5 yr of data, HD at $\sim 4.6\,\sigma$

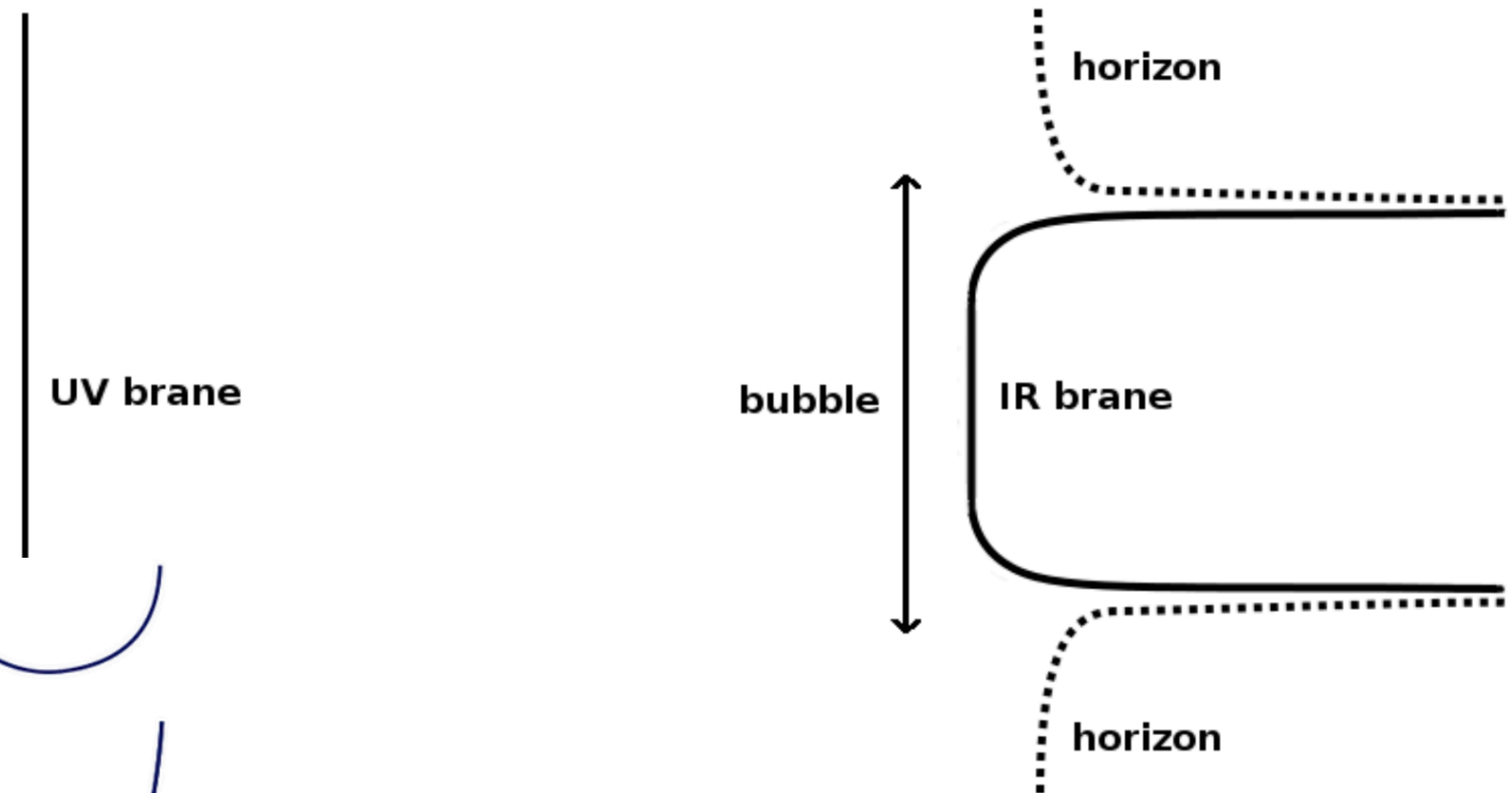
Quadrupolar correlations described by Hellings–Downs curve

Hellings, Downs Astrophys. J. 265 (1983) L39

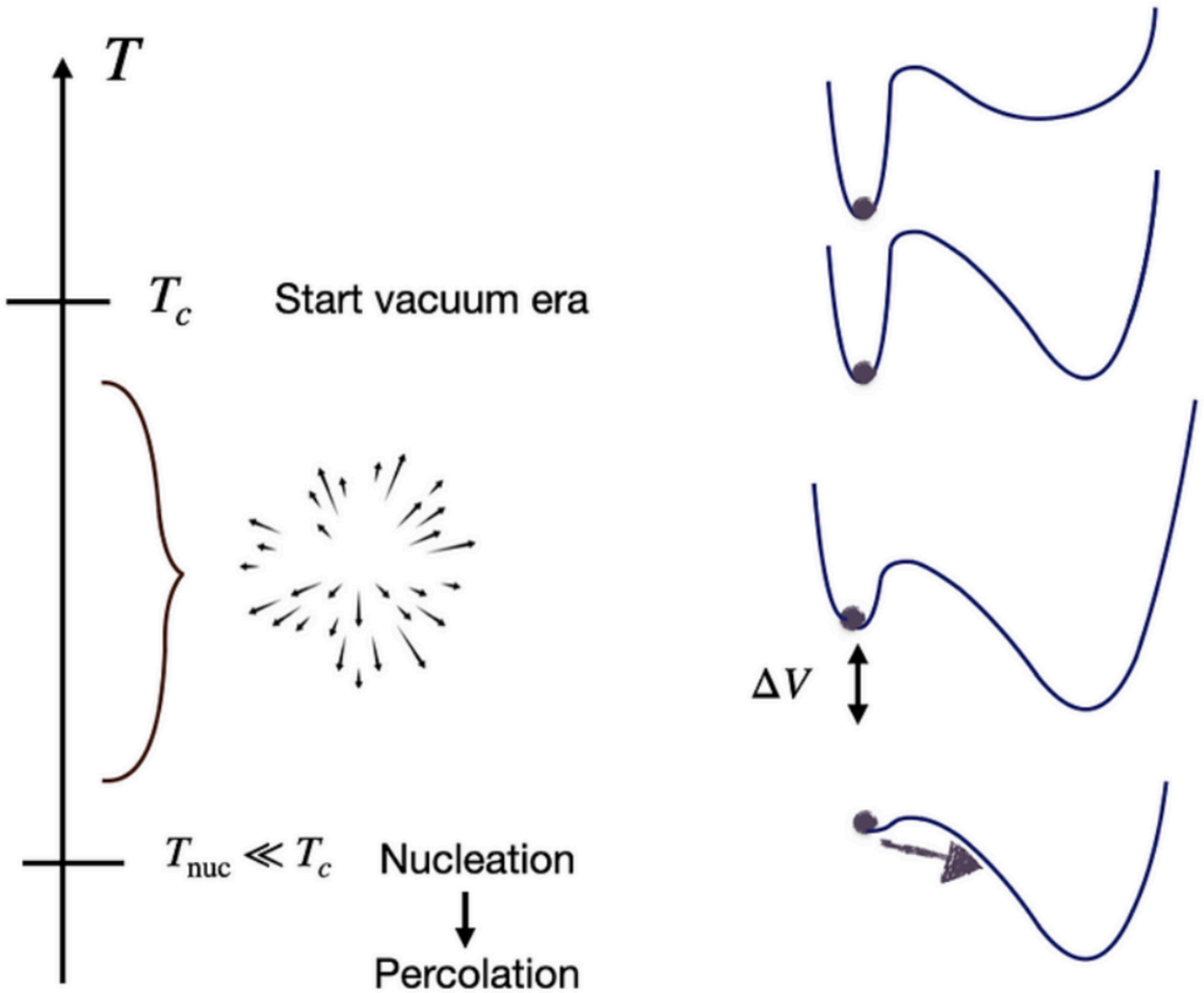
RS and the NANOGrav signal

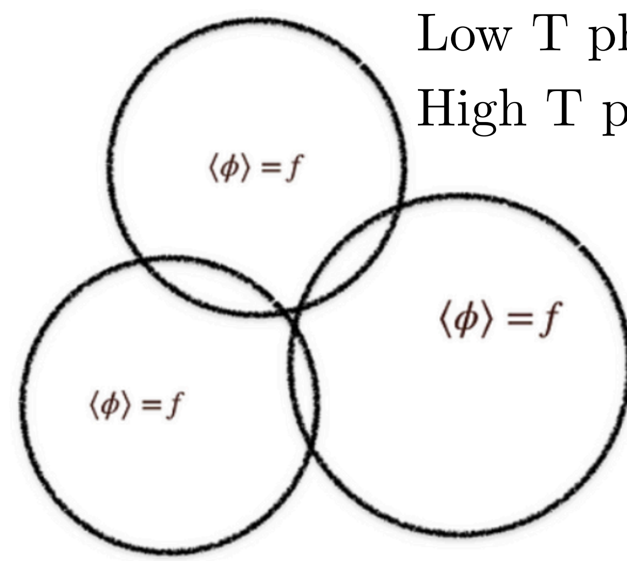
First-order phase transition (confinement-deconfinement) described by 4D EFT due to AdS/CFT.

Arkani-Hamed, Porrati, and Randall 0012148,
Rattazzi and Zaffaroni 0012248



von Harling, Servant 1612.02447





Low T phase: AdS_5 with UV and IR branes
High T phase: AdS_5-Schwarzschild

RS and the NANOGrav signal

Nearly conformal radion potential from GW bulk scalars   Goldberger, Wise 9907447

$$S \supset \int d^5x \sqrt{g} \left(\frac{1}{2}\partial_A\phi\, \partial^A\phi - \frac{m_\phi^2}{2}\phi^2 - \delta(y)\, V_{\rm UV} - \delta(y - y_{\rm IR})\, V_{\rm IR} \right)$$

$$V_{\rm UV} = \lambda_{\rm UV}(\phi^2 - v_{\rm UV}^2)^2, \qquad V_{\rm IR} = \lambda_{\rm IR}(\phi^2 - v_{\rm IR}^2)^2 .$$

$$V(\sigma_{\rm IR}) = (4+\epsilon)\, kA^2(\sigma_{\rm IR}^{-(4+2\epsilon)} - 1) + \epsilon\, kB^2(1 - \sigma_{\rm IR}^{4+2\epsilon}) + V_{\rm UV}\big(\phi(0)\big) + \sigma_{\rm IR}^4 V_{\rm IR}\big(\phi(y_{\rm IR})\big)$$

Radion (dilaton) obtains a vev stabilizing the 5th dim. → deconfined to confined PT.

$$F_{\rm confined}(\langle\chi\rangle) = F_{\rm deconfined}(T_c) \implies T_c = \sqrt{\frac{m_\sigma f}{\pi N}} \left(\frac{2}{4+\alpha}\right)^{1/4} \qquad \alpha = 2(\sqrt{4 + m_\phi^2} - 2)$$

PT completes if bubble nucleation rate $\Gamma \sim T_n^4 e^{-S_b} \gtrsim H \sim \sqrt{\rho}/M_{\rm Pl} \sim T_c^2/M_{\rm Pl}$

$$S_b \lesssim 170 + 4\log(T_n/T_c).$$

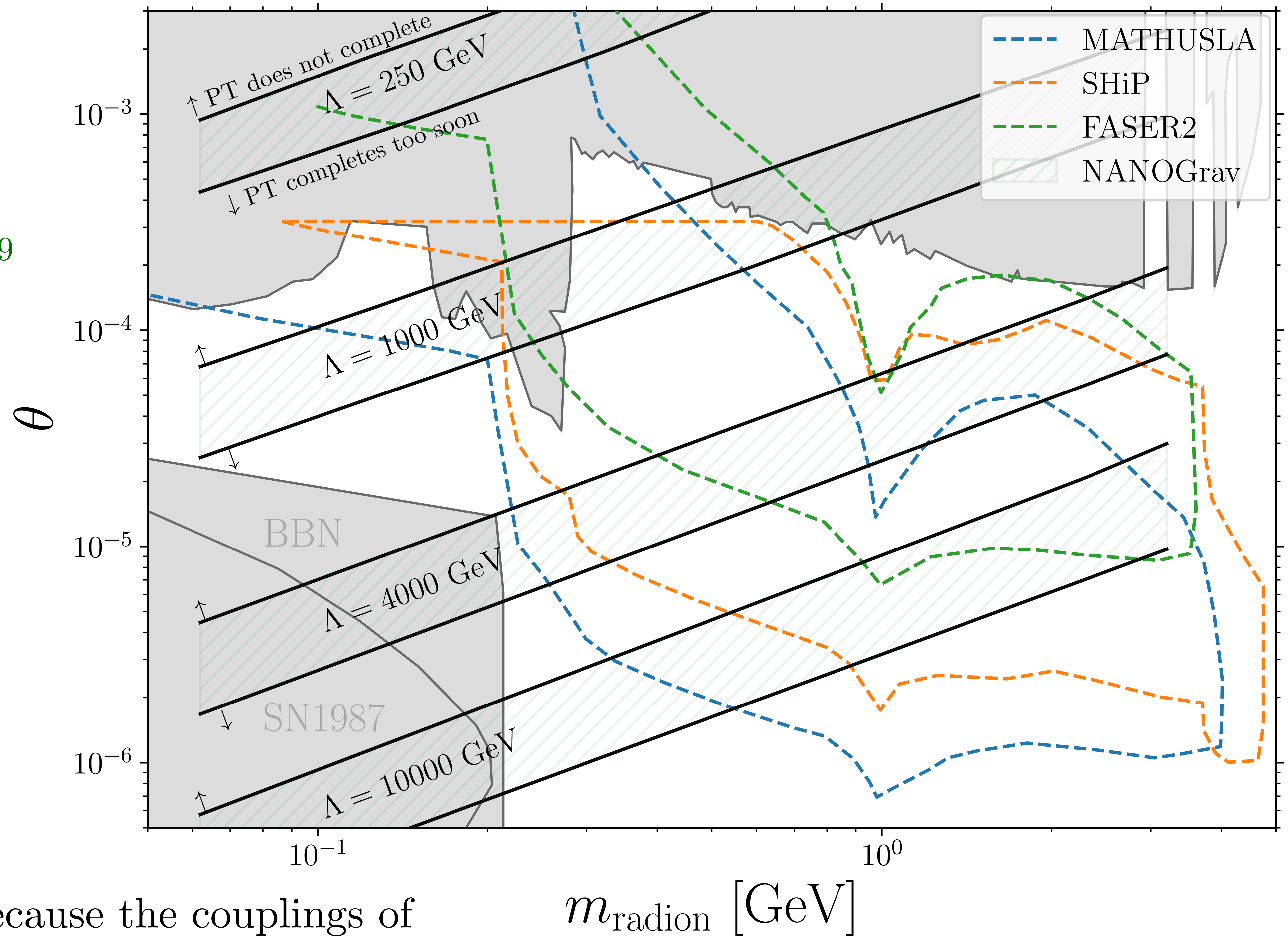
Ferrante, Ismail, Lee, and Lee 2308.16219

NANOGrav signal from RS with 3 branes

Because $T_R \sim \sqrt{mf} \sim 1\,\text{GeV}$ is fixed by fit to NANOGrav, this scenario is *impossible* to realize within standard RS.

Ferrante, Ismail, Lee, and Lee 2308.16219 introduced 3rd (dark) brane associated with scale $\sim 1\,\text{GeV}$

There is an extra light radion with Higgs-like couplings, but rescaled by $\theta = \dfrac{v_{SM} f}{\Lambda}$.
We extend previous results to arbitrary Λ and update the bounds on the radion.



Previous results do not apply because the couplings of KK gravitons are suppressed by $\sim (1\,\text{GeV}/10^3\,\text{GeV})^3 = 10^9$.

Conclusions

- Clockwork is an interesting mechanism that can solve hierarchy problem. We studied its three benchmarks: RS, LD, and LED-like scenario of GLD.
- We found the sensitivities of the future lepton colliders: FCC and CLIC, which will cover the short-live regime up to $M_5 \sim 200$ TeV complementary to the long-lived regime, which will be also probed by FCC-LLP.
- Low curvature of LD is technically natural (approximate shift symmetry) and leads to light LLPs which will be probed by SHiP and FCC-LLP.
- For the RS with third brane, we updated the prospects of a sub-GeV radion to explain the NANOGrav gravitational wave signal by FOPT.

Dziękuję!

Thank you!

감사합니다