HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

The dark in the white

White dwarfs as a portal to dark sectors

Jaime Hoefken Zink

Alma Mater Studiorum - Universitá di Bologna

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Contents

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White dwarfs

The long mean path of electrons due to their degeneration makes the temperature of the core the same everywhere.

WD cooling

$$C_V \frac{dT_{\mathrm{WD}}}{dt} = -L_\nu - L_\gamma + L_H \qquad (1)$$

Hot WDs: neutrino emission through plasmon decay

Cold WDs: photon surface emission

Equation of State: Salpeter

Salpeter, E.E., *Energy and pressure of a zero-temperature plasma*, DOI: 10.1086/147194.

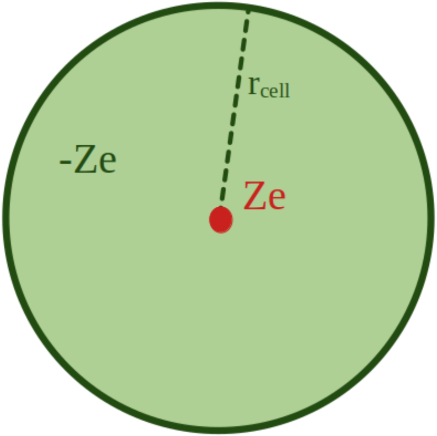
$$E_{\mathrm{tot}} = E_0 + E_C + E_{TF} + E_{Ex} + E_{Cor} \tag{2}$$

$$P = -\frac{1}{4\pi r_{\mathrm{e}}^2 a_0^3}\frac{dE}{dr_{\mathrm{e}}} \tag{3}$$

$E \rightarrow$ per electron, such that there is one electron per sphere of radius $r_e a_0$ ($a_0 \equiv 1/(\alpha m_e)$: Bohr radius)

Equation of State: (A) Salpeter

Wigner-Seitz cell



where $r_{\text{cell}} = Z^{1/3} r_e a_0$

Salpeter: (1) degenerated ideal gas

This is the simplest contribution, which at zero temperature is:

$$E_0 = \frac{g}{(2\pi)^3}\int d^3p \left(\sqrt{p^2+m_e^2}-m_e\right)\left(f_e(E_p)+f_{\bar{e}}(E_p)\right)$$
$$= \frac{1}{\pi^2}\int_0^{p_F} dp\ p^2\left(\sqrt{p^2+m_e^2}-m_e\right) \tag{4}$$

Salpeter: (2) classical Coulomb effect

Forces: e-e + e-N

$$E_C = \frac{1}{4\pi Z}\int_0^{V_{\text{cell}}} \left(\frac{\rho_e V \times \rho_e dV}{r} + \frac{Ze \times \rho_e dV}{r} \right) = -\frac{9}{5}\frac{Z^{2/3}}{r_e} r_y \tag{5}$$

where $r_y \equiv \frac{1}{2}\alpha^2 m_e$ is the Rydberg energy unit

Salpeter: (3) Thomas-Fermi correction

Non-uniform e-cloud: $n_e(r) \to n_{e,0}(1+\varepsilon(r))$
We fix the Fermi energy: $E_F = \frac{p_F}{2m_e} - eV$
We solve: $\nabla^2 V = 4\pi e n_e$

$$E_{TF} = -\frac{324}{175}\left(\frac{4}{9\pi}\right)^{2/3}\sqrt{1+x^2}Z^{4/3}r_y \qquad (6)$$

where $x \equiv p_F/m_e$

Salpeter: (4) Exchange energy

Fermi-Dirac statistics also affect [antisymmetrized] wave functions (not just kinetic energy)

Full contribution: $E_C + E_{Ex}$

$$E_{Ex} = \sum_{i,j} J_{ij} \langle \vec{s}_i \cdot \vec{s}_j \rangle \sim E_{\rm singlet} - E_{\rm triplet} = -\left(\frac{3}{4\pi}\right) \alpha m x \phi(x) \tag{7}$$

$\phi(x) = \frac{1}{4x^4}\left[\frac{9}{4} + 3\left(\beta^2 - \frac{1}{\beta^2}\right)\log\beta - 6(\log\beta)^2 - \left(\beta^2 + \frac{1}{\beta^2}\right) - \frac{1}{8}\left(\beta^4 + \frac{1}{\beta^2}\right)\right]$

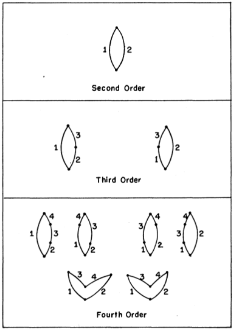
$\beta \equiv x + \sqrt{1+x^2}$

Salpeter: (5) Correlation energy

Next term in perturbation of interaction between electrons: computed by Gell-Mann and Brueckner, 10.1103/PhysRev.106.364 (DOI)

$$E_{Cor} = (0.0622 \log r_e - 0.096) r_y \qquad (8)$$

This energy takes into account an exchange of momenta among n electrons than then return to their original states:



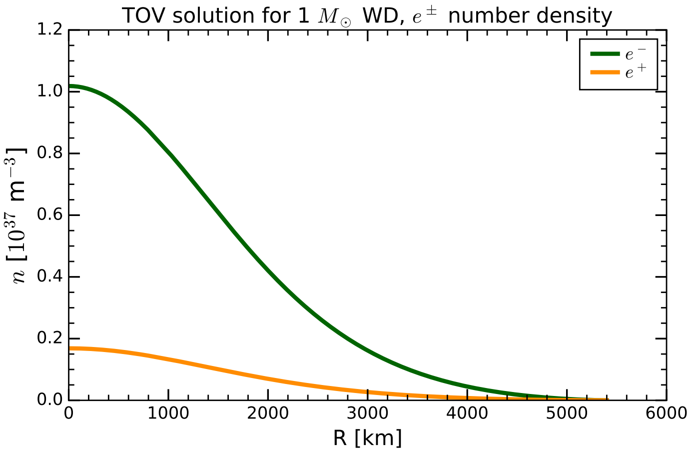
Equation of State: (B) TOV equations

Tolman-Oppenheimer-Volkoff (TOV) equations: Einstein field equations for a perfect fluid in the metric of the interior of a star

Tolman, R. C. 1939, Phys. Rev., 55, 364 / Oppenheimer, J. R., & Volkoff, G. M. 1939, Phys. Rev., 55, 374

$$\begin{aligned} \frac{dp(r)}{dr} &= -G\frac{\epsilon(r)+p(r)}{r\left(r-2Gm(r)\right)}\left[m(r)+4\pi p(r)r^3\right] \\ \frac{dm(r)}{dr} &= 4\pi\epsilon(r)r^2 \\ \epsilon &= \epsilon(r) \end{aligned} \tag{9}$$

Equation of State: (B) TOV equations



Thermal field theory

Thermal partition function:

$$
\begin{aligned}
Z(\beta) &= \int dq \, \langle q| \, e^{-\beta \hat{H}} \, |q\rangle \equiv \int dq \, \langle q| \, e^{-i\hat{H}(-i\beta)} \, |q\rangle \\
&= \int \mathcal{D}q(\tau) \exp\left[\int_0^\beta d\tau \left(\frac{1}{2} m\dot{q}^2(\tau) + V(q(\tau)) \right) \right] \\
&= \int \mathcal{D}q(\tau) \exp\left[-S_E(\beta) \right]
\end{aligned}
\tag{10}
$$

$$
\langle T(\hat{q}(-i\tau)\hat{q}(0)) \rangle_\beta = \frac{1}{Z(\beta)} \mathrm{Tr}\left[e^{-\beta \hat{H}} T(\hat{q}(-i\tau)\hat{q}(0)) \right] \tag{11}
$$

Thermal field theory

We can define 2-point functions:

$$\begin{aligned} D^{>}(t,t') &= \langle \hat{q}(t)\hat{q}(t') \rangle_{\beta} \\ D^{<}(t,t') &= \langle \hat{q}(t')\hat{q}(t) \rangle_{\beta} = D^{>}(t',t) \end{aligned} \tag{12}$$

And the time-ordered propagator:

$$\begin{aligned} D(t,t') &= \langle T(\hat{q}(t)\hat{q}(t')) \rangle_{\beta} \\ &= \theta(t-t')D^{>}(t,t') + \theta(t'-t)D^{<}(t,t') \end{aligned} \tag{13}$$

For the harmonic oscillator, the Fourier transform of $D^{>}$ is:

$$\begin{aligned} \Delta_F(i\omega_n) &= \frac{i}{\omega_n^2 + \omega^2} \\ \omega_n &= \frac{2\pi n}{\beta},\ n \in \mathbb{Z}\ \text{(Matsubara frequencies)} \end{aligned} \tag{14}$$

Thermal field theory

If we do the same process with scalar fields:

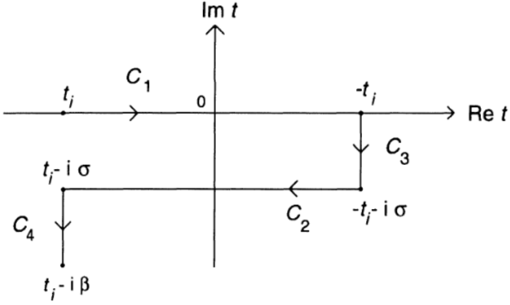
- Euclidean action:
  $S_E(\beta) = \int_0^\beta d^4x \left(\frac{1}{2}\left(\partial_\mu\phi\right)^2 + \frac{1}{2}m^2\phi^2 + \mathcal{V}(\phi)\right)$
- Generating functional:
  $Z(\beta, j) : \int \mathcal{D}\phi \exp\left[-S_E(\beta) + \int_0^\beta d^4x\, j(x)\phi(x)\right]$
- Mastubara propagator:
  $\Delta_F(i\omega_n, k) = \left(\omega_n^2 + k^2 + m^2\right)^{-1} \equiv \left(\omega_n^2 + \omega_k^2\right)^{-1}$
- Integral measure: $T\sum_n \int \frac{d^3k}{(2\pi)^3}$

This is the **Imaginary-time formalism**. Similar with fermions and vectors.

Thermal field theory

There's also the **real-time formalism**:

- Field operator: $\hat{\phi}(x) = e^{it\hat{H}}\hat{\phi}(0)e^{-it\hat{H}}$, $t = x^0 \in \mathbb{C}$
- Thermal Green functions:
  $G_C(x_1, ..., x_N) = \langle T_C(\hat{\phi}(x_1)...\hat{\phi}(x_N))\rangle_\beta$
- Generating functional:
  $Z_C(\beta, j) = \mathrm{Tr}\left[e^{-\beta\hat{H}} T_C \exp\left(i \int_C d^4\ x j(x)\hat{\phi}(x)\right)\right]$



Thermal field theory

A.4 Feynman rules in Minkowski space (real-time)

Boson propagator: diagonal elements

$$D^F_{11}(Q) = (D^F_{22}(Q))^* = \frac{i}{Q^2 - m^2 + i\eta} + 2\pi n(q_0)\delta(Q^2 - m^2)$$

Cut propagators

$$\begin{aligned} D^>_F(Q) &= 2\pi(\theta(q_0) + n(q_0))\delta(Q^2 - m^2) \\ &= 2\pi\varepsilon(q_0)(1 + f(q_0))\delta(Q^2 - m^2) \\ D^<_F(Q) &= 2\pi(\theta(-q_0) + n(q_0))\delta(Q^2 - m^2) \\ &= 2\pi\varepsilon(q_0)f(q_0)\delta(Q^2 - m^2) \end{aligned}$$

$D^>_F = D_{21}$ and $D^<_F = D_{12}$ if $\sigma = 0$

$$f(q_0) = \frac{1}{e^{\beta q_0} - 1} \qquad n(q_0) = \frac{1}{e^{\beta |q_0|} - 1}$$

Fermion propagators: diagonal elements

$$\begin{aligned} S^F_{11}(P) = (\not P + m)\tilde{S}^F_{11}(P) = (\not P + m)\bigg[&\frac{i}{P^2 - m^2 + i\eta} \\ &- 2\pi(\theta(-p_0) + \varepsilon(p_0)\tilde{f}(p_0 - \mu))\delta(P^2 - m^2)\bigg] \end{aligned}$$

$$S^F_{22}(P) = (\not P + m)(\tilde{S}^F_{11}(P))^*$$

Cut propagators

$$\begin{aligned} S^>_F(P) &= 2\pi\varepsilon(p_0)(1 - \tilde{f}(p_0 - \mu))(\not P + m)\delta(P^2 - m^2) \\ S^<_F(P) &= -2\pi\varepsilon(p_0)\tilde{f}(p_0 - \mu)(\not P + m)\delta(P^2 - m^2) \end{aligned}$$

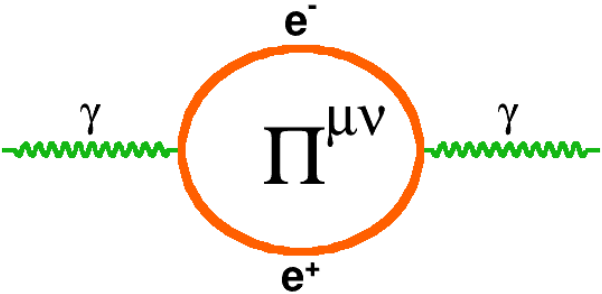
$$\tilde{f}(p_0) = \frac{1}{e^{\beta p_0} + 1}$$

Taken from Le Bellac

Photon self-energy

Importance of photon self-energy:

- Alters poles
- Modifies dispersion relations
- Changes field strength
- Appears in decay diagrams



Photon self-energy

$$\begin{aligned}
\Pi^{\mu\nu} =& - ie^2 \int \frac{d^4K}{(2\pi)^4} \mathrm{tr}[\gamma^\mu S^F_\beta(K)\gamma^\nu S^F_\beta(Q-K)] \\
=& - ie^2 \int \frac{d^4K}{(2\pi)^4} \mathrm{tr}[\gamma^\mu(\not{K}+m_e)\gamma^\nu(\not{Q}-\not{K}-m_e)] \\
& \times \left[\frac{i}{K^2-m_e^2} - 2\pi\big(\theta(-k^0)+\mathrm{sgn}(k^0)\tilde{f}(k^0-\mu_e)\big)\delta(K^2-m_e^2)\right] \\
& \times \Bigg[\frac{i}{(Q-K)^2-m_e^2} - 2\pi\Big(\theta(-q^0+k^0) \\
& + \mathrm{sgn}(Q^0-k^0)\tilde{f}(q^0-k^0+\mu_e)\Big)\delta((Q-K)^2-m_e^2)\Bigg]
\end{aligned} \tag{15}$$

where $\tilde{f}(x) = \left(e^{\beta x}+1\right)^{-1}$

Photon self-energy

Contribution just: thermal - non-thermal

$$\begin{aligned}\Pi^{\mu\nu} =& - e^2 \int \frac{d^4K}{(2\pi)^3} \mathrm{tr}[\gamma^\mu(\not{K} + m_e)\gamma^\nu(\not{Q} - \not{K} - m_e)] \\ &\times \left[\frac{\theta(-k^0) + \mathrm{sgn}(k^0)\tilde{f}(k^0 - \mu_e)}{(Q-K)^2 - m_e^2}\delta(K^2 - m_e^2)\right] \\ &\times \left[\frac{\theta(-q^0 + k^0) + \mathrm{sgn}(q^0 - k^0)\tilde{f}(q^0 - k^0 + \mu_e)}{K^2 - m_e^2}\right. \\ &\qquad \left.\times\, \delta((Q-K)^2 - m_e^2)\right]\end{aligned} \tag{16}$$

Photon self-energy

The first term:

$$\mathrm{tr}[\gamma^\mu(\not{K} + m_e)\gamma^\nu(\not{Q} - \not{K} - m_e)]$$
$$= 4(k^\mu q^\nu + k^\nu q^\mu - 2k^\mu k^\nu - K \cdot Q g^{\mu\nu}) \qquad (17)$$
$$\equiv 4A^{\mu\nu}$$

Ward identity: $Q_\mu \Pi^{\mu\nu} = 0 \rightarrow Q_\mu A^{\mu\nu} = 0 \rightarrow$
$Q^2 = 2Q \cdot K$

$$A^{\mu\nu} = \frac{(K \cdot Q)(k^\mu q^\nu + k^\nu q^\mu) - Q^2 k^\mu k^\nu - (K \cdot Q)^2 g^{\mu\nu}}{K \cdot Q} \qquad (18)$$

Due to delta: $(Q - K)^2 - m_e^2 = Q^2 - 2Q \cdot K$

Photon self-energy

With the other factors:

$$\theta(-k^0) + \mathrm{sgn}(k^0)\tilde{f}(k^0 - \mu_e)\delta(K^2 - m_e^2) = \frac{1}{2E_K}\left[\tilde{f}(k^0 - \mu_e)\delta^+ + \left(1 - \tilde{f}(k^0 - \mu_e)\right)\delta^-\right] \tag{19}$$

where $E_K \equiv \sqrt{\vec{K}^2 + m_e^2}$ and $\delta^\pm \equiv \delta(k^0 \mp E_K)$.
Therefore, the first term yields:

$$\frac{2A^{\mu\nu}}{E_K}\frac{\tilde{f}(k^0 - \mu_e)\delta^+ + \left(1 - \tilde{f}(k^0 - \mu_e)\right)\delta^-}{Q^2 - 2Q \cdot K} \tag{20}$$

Photon self-energy

Similar work with the second term, but we do:
$K \to K + Q$ and find:

$$\frac{-2A^{\mu\nu}}{E_K} \frac{\left(1 - \tilde{f}(-k^0 + \mu_e)\right)\delta^+ + \tilde{f}(-k^0 + \mu_e)\delta^-}{Q^2 + 2Q \cdot K} \quad (21)$$

To finish, we remember that under $k^\mu \to -k^\mu$, then:

1. $A^{\mu\nu} \to -A^{\mu\nu}$
2. $\int d^4K \to \int d^4K$
3. $\delta^- \to \delta^+$

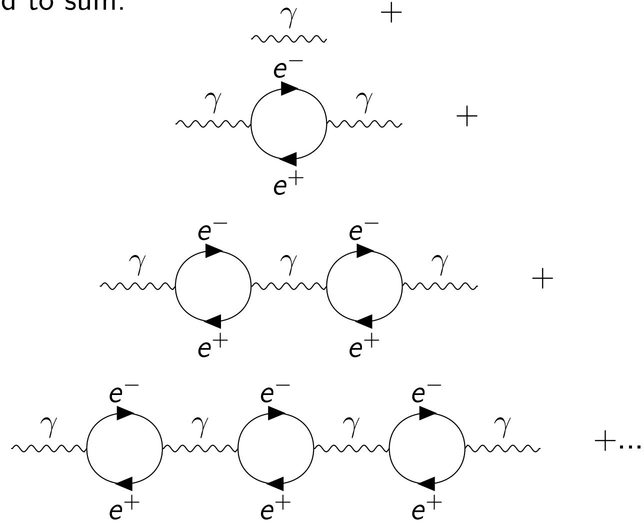
Photon self-energy

Final result:

$$\begin{aligned}\Pi^{\mu\nu} =& 4e^2 \int \frac{d^3k}{(2\pi)^3} \frac{f_e(E_K) + f_{\bar{e}}(E_K)}{2E_K} \\ &\times \frac{Q \cdot K(k^\mu q^\nu + k^\nu q^\mu) - Q^2 k^\mu k^\nu - (Q \cdot K)^2 g^{\mu\nu}}{(Q \cdot K)^2 - (Q^2)^2/4} \\ =& 4e^2 \int \frac{d^3k}{(2\pi)^3} \frac{f_e(E_K) + f_{\bar{e}}(E_K)}{2E_K} \\ &\times \frac{Q \cdot K(k^\mu q^\nu + k^\nu q^\mu) - Q^2 k^\mu k^\nu - (Q \cdot K)^2 g^{\mu\nu}}{(Q \cdot K)^2}\end{aligned} \tag{22}$$

Importance of $\Pi^{\mu\nu}$

We need to sum:



Importance of $\Pi^{\mu\nu}$

It is better to decompose $\Pi^{\mu\nu}$:

$$\Pi^{\mu\nu} = F P_L^{\mu\nu} + G P_T^{\mu\nu} \tag{23}$$

where the projectors are:

$$P_T^{\mu\nu} = \left(\delta^{ij} - \hat{q}^i \hat{q}^j\right)\delta_i^\mu \delta_j^\nu$$
$$P_L^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) - P_T^{\mu\nu} \tag{24}$$
$$\text{where:} \quad Q = (q_0, \vec{q})$$

Solution: $F = \frac{Q^2}{q^2}\Pi^{00}$ and $G = \Pi^{xx}$, such that $\vec{q} = q\hat{z}$

Importance of $\Pi^{\mu\nu}$

$$D^{\mu\nu}_{A'} = \frac{-i\,g^{\mu\nu}}{Q^2} + \frac{-i\,g^{\mu}_{\lambda}}{Q^2}\left(i\,\Pi^{\lambda\sigma}\right)\frac{-i\,g^{\nu}_{\sigma}}{Q^2} + \dots$$

$$= \frac{-i\,g^{\mu\lambda}}{Q^2}\left[\delta^{\nu}_{\lambda} + \sum_{n=1}^{\infty}\left(\frac{F}{Q^2}\right)^n P^{\nu}_{L\lambda} + \sum_{n=1}^{\infty}\left(\frac{G}{Q^2}\right)^n P^{\nu}_{T\lambda}\right] \quad (25)$$

$$= \frac{-i\,g^{\mu\lambda}}{Q^2 - F}\,P^{\nu}_{L\lambda} + \frac{-i\,g^{\mu\lambda}}{Q^2 - G}\,P^{\nu}_{T\lambda}\,,$$

Importance of $\Pi^{\mu\nu}$

The energy on-shell: $\omega_\lambda(q)$

Longitudinal: $D^{00} = \frac{1}{q^2 - \Pi_L(Q)}$

$$\lim_{q_0 \to \omega_l(q)} D^{00} = \frac{\omega_l^2(q)}{q^2} \frac{Z_l(q)}{q_0^2 - \omega_l(q)^2} \tag{26}$$

Transverse: $D^{xx} = \frac{1}{q_0^2 - q^2 - \Pi_T(Q)}$

$$\lim_{q_0 \to \omega_t(q)} D^{xx} = \frac{Z_t(q)}{q_0^2 - \omega_t(q)^2} \tag{27}$$

Solution

$$\begin{aligned} Z_l(q) &= \frac{q^2}{\omega_l(q)^2} \left[-\frac{\partial \Pi_L}{\partial q_0^2} (\omega_l(q), q) \right]^{-1} \\ Z_t(q) &= \left[1 - \frac{\partial \Pi_T}{\partial q_0^2} (\omega_t(q), q) \right]^{-1} \end{aligned} \tag{28}$$

Importance of $\Pi^{\mu\nu}$

The residue of a pole in q_0^2 of $D^{\mu\nu}(q_0, q)$ can be identified with $\varepsilon^\mu(q)\varepsilon^\nu(q)^*$. So we have:

$$\mathrm{Res}\, D^{00} = \mathrm{Res}\left(\frac{\omega_l(q)^2}{q^2}\frac{Z_l(q)}{q_0^2 - \omega_l(q)^2}\right) = \frac{\omega_l(q)^2}{q^2} Z_l(q)$$
$$\mathrm{Res}\, D^{xx} = \mathrm{Res}\left(\frac{Z_t(q)}{q_0^2 - \omega_t(q)^2}\right) = Z_t(q) \tag{29}$$

From these expressions, we can find the polarization 4-vectors:

$$\varepsilon^\mu(q, \lambda = 0) = \frac{\omega_l(q)}{q}\sqrt{Z_l(q)}(1, 0)^\mu$$
$$\varepsilon^\mu(q, \lambda = \pm 1) = \sqrt{Z_t(q)}(0, \varepsilon_\pm(q))^\mu \tag{30}$$

Importance of $\Pi^{\mu\nu}$

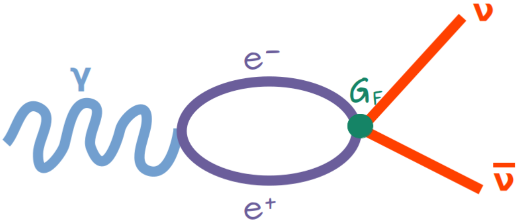
The dispersion relations are:

$$\begin{aligned} \omega_l(q)^2 &= \frac{\omega_l(q)^2}{q^2}\Pi_L(\omega_l(q), q) \\ \omega_t(q)^2 &= q^2 + \Pi_T(\omega_t(q), q) \end{aligned} \tag{31}$$

Plasmon decay

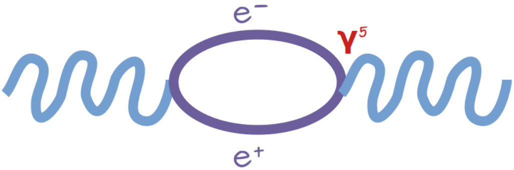
Decay of plasmon (SM):

But $Q^2 \ll m_W^2,\ m_Z^2$



Plasmon decay

We need to compute:



$$\Pi_A^{\mu\nu} = g^{\mu i} g^{\nu j} [\Pi_A(Q)(i\varepsilon^{ijm}\hat{q}^m)]$$

$$\Pi_A(Q) = 8\pi\alpha \frac{Q^2}{q} \int \frac{d^3k}{(2\pi)^3} \frac{f_e(E_k) - f_{\bar{e}}(E_k)}{2E_k} \frac{Q \cdot K q_0 - Q^2 E_k}{(Q \cdot K)^2} \tag{32}$$

Plasmon decay

Amplitudes:

$$\mathcal{M}_W = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \varepsilon_\mu(Q) \left[\Pi^{\mu\nu} - \Pi_A^{\mu\nu}\right] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2) \quad (33)$$

$$\mathcal{M}_Z = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \varepsilon_\mu(Q) \left[C_V^{(e)}\Pi^{\mu\nu} - C_A^{(e)}\Pi_A^{\mu\nu}\right] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2) \quad (34)$$

$$\mathcal{M}_{\text{tot}} = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \varepsilon_\mu(Q) \left[C_V\Pi^{\mu\nu} - C_A\Pi_A^{\mu\nu}\right] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2) \quad (35)$$

where $C_V = 2\sin^2\theta_W + 1/2$ (ν_e), and $2\sin^2\theta_W - 1/2$ for the rest, while $C_A = 1/2$ (ν_e) and $-1/2$ for the rest.

Plasmon decay

Total:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \Bigg[\varepsilon_\mu(\omega_l, q) C_V \left(\Pi_L(\omega_l, q) \left(1, \frac{\omega_l}{q}\hat{q}\right)^\mu \left(1, \frac{\omega_l}{q}\hat{q}\right)^\nu \right)$$
$$+ \varepsilon_\mu(\omega_t, q) g^{\mu i} \Bigg(C_V \Pi_T(\omega_t, q) \left(\delta^{ij} - \hat{q}^i \hat{q}^j\right)$$
$$+ C_A \Pi_A(\omega_t, q)(i\varepsilon^{ijm}\hat{q}^m) \Bigg) g^{\nu j} \Bigg] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2) \tag{36}$$

Short notation:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\Gamma^{\mu\nu}_\lambda \varepsilon_\nu(\vec{q}, \lambda) \right) \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2) \tag{37}$$

Plasmon decay

The decay width is:

$$\Gamma_\lambda(q) = \frac{1}{2\omega_\lambda(q)} \int \frac{d^3p_1}{(2\pi)^3} \frac{1}{2p_1} \int \frac{d^3p_2}{(2\pi)^3} \frac{1}{2p_2} (2\pi)^4 \delta^{(4)}(P_1 + P_2 - Q) |\mathcal{M}|^2 \tag{38}$$

We can perform:

$$\begin{aligned} I^{\mu\nu} &= \frac{1}{2\pi^2} \int \frac{d^3p_1}{p_1} \frac{d^3p_2}{p_2} \delta^{(4)}(Q - P_1 - P_2) \\ &\qquad \times \left[P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - P_1 \cdot P_2 g^{\mu\nu} - i\varepsilon^{\mu\nu\lambda\sigma} P_{1\lambda} P_{2\sigma} \right] \\ &= \frac{1}{3\pi} \left(q^\mu q^\nu - Q^2 g^{\mu\nu} \right) = -\frac{1}{3\pi} Q^2 g^{\mu\nu} \end{aligned} \tag{39}$$

$$\Gamma_\lambda(q) = -\frac{G_F^2}{12\pi} \frac{\omega_\lambda(q)^2 - q^2}{\omega_\lambda(q)} \left(\Gamma_\lambda^{\alpha\mu} \varepsilon_\mu(q, \lambda) \right) \left(\Gamma_{\alpha\rho}^\lambda \varepsilon^\rho(q, \lambda) \right)^* \tag{40}$$

Plasmon decay

Final results:

$$\Gamma_L(q) = C_V^2 \frac{G_F^2}{48\pi^2\alpha} Z_l(q) \left(\omega_l(q)^2 - q^2\right)^2 \omega_l(q) \tag{41}$$

$$\Gamma_T(q) = \frac{G_F^2}{48\pi^2\alpha} Z_t(q) \frac{\omega_t(q)^2 - q^2}{\omega_t(q)} C_V^2 \left(\omega_t(q)^2 - q^2\right)^2 \tag{42}$$

$$\Gamma_A(q) = \frac{G_F^2}{48\pi^2\alpha} Z_t(q) \frac{\omega_t(q)^2 - q^2}{\omega_t(q)} C_A^2 \Pi_A(\omega_t(q), q)^2 \tag{43}$$

Plasmon decay

Emissivity of plasma:

$\mathcal{Q}_\lambda \equiv \int d^3\vec{q}\ \Gamma_\lambda(q)\ \omega_\lambda(q) n_B(\omega_\lambda(q), T)$, where

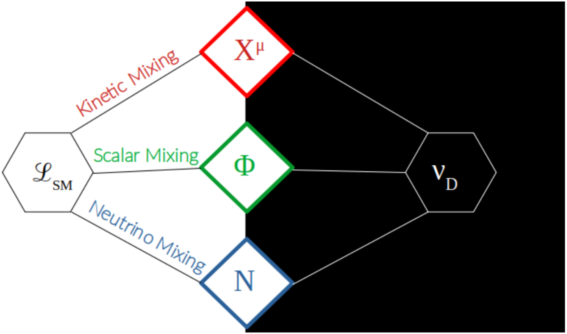
$n_B(\omega) = \frac{1}{e^{\omega/T}-1}$

$$\mathcal{Q}_T = 2\left(\sum_\nu C_V^2\right)\frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq\ q^2 Z_t(q)\left(\omega_t(q)^2 - q^2\right)^3 n_B(\omega_t(q))$$
$$\mathcal{Q}_A = 2\left(\sum_\nu C_A^2\right)\frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq\ q^2 Z_t(q)\left(\omega_t(q)^2 - q^2\right)\Pi_A(\omega_t(q), q)^2 n_B(\omega_t(q))$$
$$\mathcal{Q}_L = \left(\sum_\nu C_V^2\right)\frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq\ q^2 Z_l(q)\omega_l(q)^2\left(\omega_l(q)^2 - q^2\right)^2 n_B(\omega_l(q)) \tag{44}$$

Luminosity: $L_\nu = 4\pi\ \int_0^{R_{\mathrm{WD}}} \mathcal{Q}(r)\ r^2\ dr$

Dark sectors

Three Portal Model



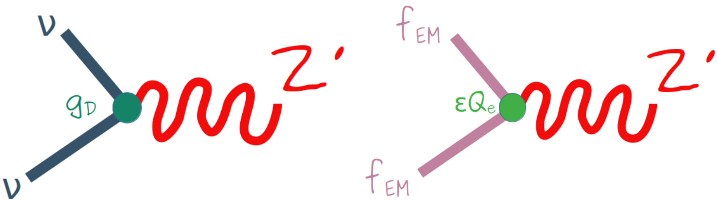
$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} + \overline{\nu}_D i \not{D}^x \nu_D$$
$$+ (D^x_\mu \Phi)^\dagger (D^{x\mu}\Phi) - V(\Phi, H)$$
$$- \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\epsilon}{2c_W} B_{\mu\nu} X^{\mu\nu}$$
$$+ \overline{N} i \not{\partial} N - [y^\alpha_\nu (\overline{L}_\alpha \cdot \widetilde{H}) N^C + \frac{\mu'}{2} \overline{N} N^C + y_N \overline{N} \nu^C_D \Phi + \mathrm{h.c.}]$$

Dark sectors

Three Portal Model

Effective interactions of the dark photon after SSB:

$$\mathcal{L}_I = -\epsilon e J_\mu^{\mathrm{EM}} Z'^\mu + g_D J_\mu^{\mathrm{D}} Z'^\mu \quad (45)$$



Dark sectors

$L_\mu - L_\tau$

$$\mathcal{L} \supset -\frac{1}{4}\,(B_{\alpha\beta}, W^3_{\alpha\beta}, X_{\alpha\beta}) \begin{pmatrix} 1 & 0 & \epsilon_B \\ 0 & 1 & \epsilon_W \\ \epsilon_B & \epsilon_W & 1 \end{pmatrix} \begin{pmatrix} B^{\alpha\beta} \\ W^{3\alpha\beta} \\ X^{\alpha\beta} \end{pmatrix}$$
$$+ \frac{1}{2}\,(B_\alpha, W^3_\alpha, X_\alpha)\,\frac{v^2}{4} \begin{pmatrix} g'^2 & g'\,g & 0 \\ g'\,g & g^2 & 0 \\ 0 & 0 & \frac{4\,M_X^2}{v^2} \end{pmatrix} \begin{pmatrix} B^\alpha \\ W^{3\alpha} \\ X^\alpha \end{pmatrix}$$
$$-\,(g'\,j^\alpha_Y,\; g\,j^\alpha_3,\; g_{\mu\tau}\,j^\alpha_{\mu\tau}) \begin{pmatrix} B_\alpha \\ W^3_\alpha \\ X_\alpha \end{pmatrix} \tag{46}$$

$$j^\alpha_{\mu\tau} = \bar{L}_2\gamma^\alpha L_2 + \bar{\mu}_R\gamma^\alpha\mu_R - \bar{L}_3\gamma^\alpha L_3 - \bar{\tau}_R\gamma^\alpha\tau_R \tag{47}$$

$$\mathcal{L}_{\rm int} = -g_{\mu\tau}\,j^\alpha_{\mu\tau}\,A'_\alpha + e\,\epsilon_A\left(j^\alpha_{\rm EM} - \frac{1}{2}\tan^2\theta_W\,j^\alpha_Z\right)A'_\alpha \tag{48}$$

$$\epsilon_A \simeq \frac{e\,g_{\mu\tau}}{6\pi^2}\,\log\left(\frac{m_\mu}{m_\tau}\right) \sim -\frac{g_{\mu\tau}}{70} \tag{49}$$

Dark sectors

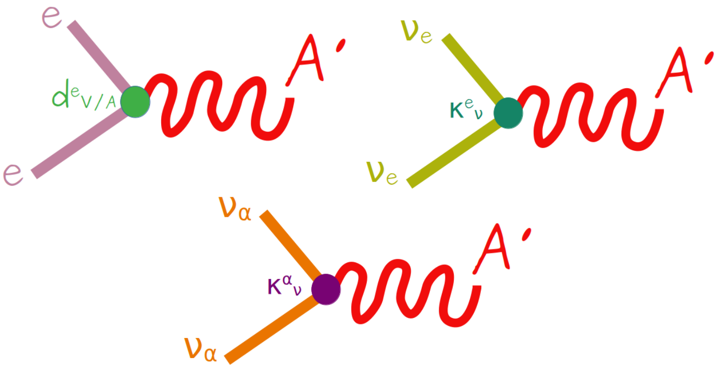
$L_\mu - L_\tau$

$$d_V^e = e\,\epsilon_A\Big(1 - \tan^2\theta_W(1 - 4\sin^2\theta_W)/8\Big)\,, \tag{50}$$

$$d_A^e = e\,\epsilon_A\ \tan^2\theta_W/8\,, \tag{51}$$

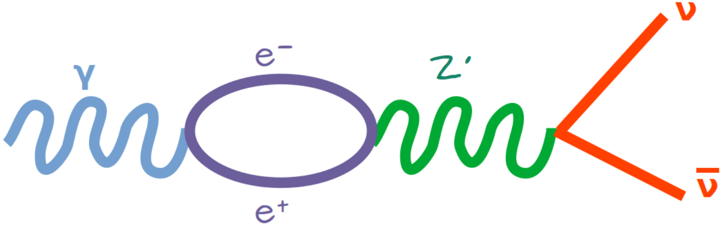
$$k_\nu^\alpha = s_\alpha\, g_{\mu\tau}/2 + d_A^e\,, \tag{52}$$

$$s_\alpha = 0, 1, -1,\ \text{for}\ \alpha = e, \mu, \tau \tag{53}$$



BSM WD cooling

Extra contribution:



BSM WD cooling

Three Portal Model

$$\mathcal{M}^{ij}_{Z'} = -\varepsilon_\mu(Q) \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\gamma^\mu S^F(K) \gamma^\nu S^F(K-Q)\right] \\ \times \epsilon e^2 \frac{1}{Q^2 - M^2_{Z'}} \frac{g_\mathrm{D}}{2} U^*_{i\mathrm{D}} U_{j\mathrm{D}} \\ \times \overline{u}_j(p_1)\gamma_\nu(1-\gamma_5)v_i(p_2) \\ = \frac{G_F}{2}\frac{1}{\sqrt{4\pi\alpha}}\left[C^\mathrm{D}_{\nu,ij}\Pi^{\mu\nu}\right]\overline{u}_j(p_1)\gamma_\nu(1-\gamma_5)v_i(p_2) \tag{54}$$

where U is the mixing matrix for neutrino states and:

$$C^\mathrm{D}_{\nu,ij} = \frac{\sqrt{2\pi\alpha}}{G_F}\frac{\epsilon g_\mathrm{D} U^*_{i\mathrm{D}} U_{j\mathrm{D}}}{M^2_{Z'} - Q^2} \tag{55}$$

BSM WD cooling

Three Portal Model

We need to sum it with the SM:

$$\sum_{ij} C_V^2 = \sum_\alpha \left(C_V^{\mathrm{SM}}\right)^2 + \frac{\sqrt{8\pi\alpha}}{G_F} \frac{\epsilon g_{\mathrm{D}} \left|U_{i\mathrm{D}}\right|^2}{M_{Z'}^2 - Q^2} \Re\Big(\sum_{\alpha,i,j} C_{V,\alpha}^{\mathrm{SM}} U_{\alpha i}^* U_{\alpha j} \Big) + \frac{18\pi\alpha}{G_F^2} \frac{\epsilon^2 g_{\mathrm{D}}^2 \left|U_{\mathrm{D}}\right|^4}{(M_{Z'}^2 - Q^2)^2} \tag{56}$$

where we have assumed that $U_{i\mathrm{D}} \equiv U_{\mathrm{D}}$ is equal for every i and $\sum_\alpha \left(C_V^{\mathrm{SM}}\right) = 6\sin^2\theta_W - 1/2$

BSM WD cooling

Three Portal Model

Three Portal Model contribution (it includes SM)

$$\begin{aligned}
\mathcal{Q}_T =& 2\frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq\ q^2 Z_t(q) \\
& \times \left(\sum_{\alpha\beta}\left(C_V(\omega_t(q),q)\right)^2\right)\left(\omega_t(q)^2 - q^2\right)^3 n_B(\omega_t(q)) \\
\mathcal{Q}_L =& \frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq\ q^2 Z_l(q)\left(\sum_{\alpha\beta}\left(C_V(\omega_l(q),q)\right)^2\right) \\
& \times \omega_l(q)^2\left(\omega_l(q)^2 - q^2\right)^2 n_B(\omega_l(q))
\end{aligned} \tag{57}$$

BSM WD cooling

$L_\mu - L_\tau$

Flavor basis, so just change of coefficients needed:

$$C_a^{\alpha,\text{SM+BSM}}(Q) \to C_a^\alpha + b_a \frac{\sqrt{2}}{G_F} \frac{k_\nu^\alpha\, d_a^e}{Q^2 - m_{A'}^2} \tag{58}$$

where $a = V, A$ are the vectorial and axial components with $b_V = 1$ and $b_A = -1$, α is the flavor

BSM WD cooling

$L_\mu - L_\tau$

$$\mathcal{Q}_L = \frac{G_F^2}{96\pi^4\alpha}\int_0^\infty dq \sum_\alpha (C_V^{\alpha,\text{SM+BSM}}(q))^2 \, q^2 \, Z_l(q) \times \left(\omega_l(q)^2 - q^2\right)^2 \omega_l(q)^2 \, n_B(\omega_l(q)) \tag{59}$$

$$\mathcal{Q}_T = \frac{G_F^2}{48\pi^4\alpha}\int_0^\infty dq \sum_\alpha (C_V^{\alpha,\text{SM+BSM}}(q))^2 \, q^2 \, Z_t(q) \times \left(\omega_t(q)^2 - q^2\right)^3 n_B(\omega_t(q)) \tag{60}$$

$$\mathcal{Q}_A = \frac{G_F^2}{48\pi^4\alpha}\int_0^\infty dq \sum_\alpha (C_A^{\alpha,\text{SM+BSM}}(q))^2 \, q^2 \, Z_t(q) \tag{61}$$

$$\times \left(\omega_t(q)^2 - q^2\right) \Pi_A\left(\omega_t(q), q\right)^2 \; n_B(\omega_t(q))$$

BSM WD cooling

We are interested in measuring:

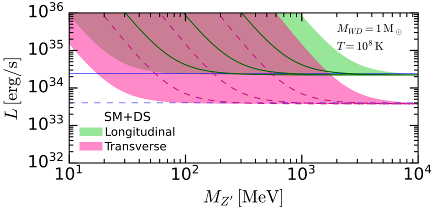
$$F_{\mathrm{DS}} = \frac{\mathcal{L}_{\mathrm{DS+SM}} - \mathcal{L}_{\mathrm{SM}}}{\mathcal{L}_{\mathrm{SM}}} \tag{62}$$

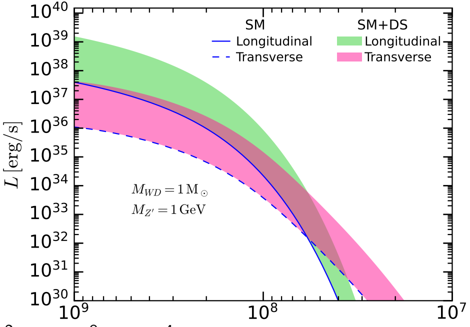
Data of WD computations:

- $M_{\mathrm{WD}} = 1\ M_{\odot}$
- $T_{\mathrm{WD}} = 10^8$ K
- Degenerate and classical limits

BSM WD cooling

Three Portal Model: results

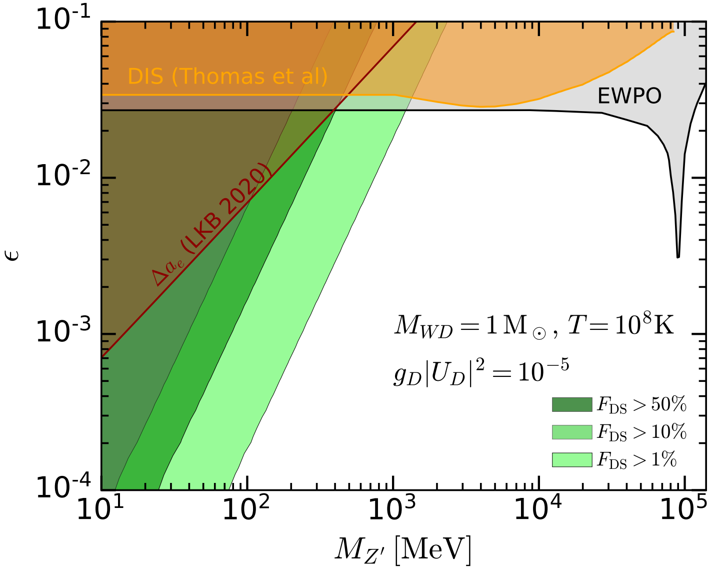




where $\epsilon g_D |U_D|^2 = [10^{-8} - 10^{-4}]$

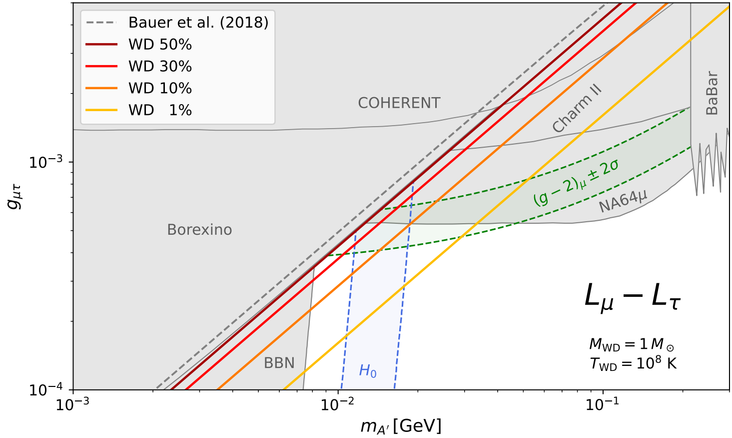
BSM WD cooling

Three Portal Model: results



BSM WD cooling

$L_\mu - L_\tau$: results



BSM WD cooling

$L_\mu - L_\tau$: results

Heavy case ($m_{A'}^2 \gg Q^2$):

$$F_{\rm DS} = \sum_\alpha \left(C_V^{\alpha,\text{SM+BSM}} \right)^2 / \sum_\alpha \left(C_V^{\alpha,\text{SM}} \right)^2 - 1$$
$$\simeq 1.50 \times 10^{17} \left(\frac{g_{\mu\tau}}{m_{A'}/1\ \mathrm{MeV}} \right)^4$$
$$- 1.66 \times 10^5 \left(\frac{g_{\mu\tau}}{m_{A'}/1\ \mathrm{MeV}} \right)^2 \quad (63)$$

Ultra light A'

$m_{A'}^2 \ll Q^2 \to$ propagator needs to consider $\Pi_{A'}^{\mu\nu}$: since $(d_A^e)^2/(d_V^e)^2 \sim \mathcal{O}(10^{-3})$, it is in terms of the photon self-energy:

$$\Pi_{A'}^{\mu\nu}(Q) \simeq \frac{(d_V^e)^2 + (d_A^e)^2}{4\pi\alpha}\,\Pi_\gamma^{\mu\nu}(Q) \equiv r_{\rm BSM}\,\Pi_\gamma^{\mu\nu}(Q) \tag{64}$$

$$\Pi_{A'}^{\mu\nu} = F_{A'}\,P_L^{\mu\nu} + G_{A'}\,P_T^{\mu\nu} \tag{65}$$

$$D_{A'}^{\mu\nu} = \frac{-i\,g^{\mu\lambda}}{Q^2 - m_{A'}^2 - F_{A'}}\;P_{L\lambda}^{\nu} + \frac{-i\,g^{\mu\lambda}}{Q^2 - m_{A'}^2 - G_{A'}}\;P_{T\lambda}^{\nu} \tag{66}$$

$$F_{A'} \equiv r_{\rm BSM}\,\frac{Q^2}{q^2}\,\Pi_L^\gamma \tag{67}$$

$$G_{A'} \equiv r_{\rm BSM}\,\Pi_T^\gamma \tag{68}$$

Ultra light A'

$m_{A'}^2 \ll Q^2$:

$$\sum_\alpha (C_V^{\alpha,\text{SM+BSM}}(q))^2 = \frac{d_e^V}{G_F^2\,(q_r^2)^2}\Bigg(d_e^V\,(6\,(d_e^A)^2 + g_{\mu\tau}^2) + \sqrt{2}\,G_F\,q_r^2\Big[2\,d_e^A \sum_\alpha C_V^{\alpha,\text{SM}} + g_{\mu\tau}\,(C_V^{\mu,\text{SM}} - C_V^{\tau,\text{SM}})\Big]\Bigg)$$

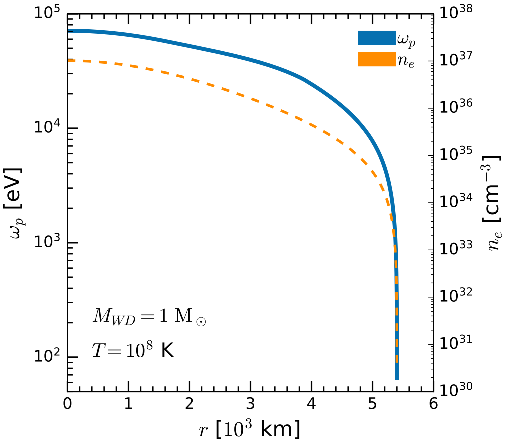
$$\sum_\alpha (C_A^{\alpha,\text{SM+BSM}}(q))^2 = \frac{d_e^A}{G_F^2(q_r^2)^2}\Bigg(6\,(d_e^A)^3 - \sqrt{2}\,G_F\,g_{\mu\tau}\,(C_A^{\mu,\text{SM}} - C_A^{\tau,\text{SM}})\,q_r^2 + G_F\,d_e^A\Big[g_{\mu\tau}^2 - 2\sqrt{2}\,G_F\,q_r^2 \sum_\alpha C_A^{\alpha,\text{SM}}\Big]\Bigg) \tag{69}$$

where $q_r^2 \equiv (1 - r_{\text{BSM}})\,Q^2$

Resonant A'

Region where $m_{A'} \sim \omega_p$:

$$\omega_p^2 = \frac{4\pi}{\alpha} \int_0^\infty dp \frac{k^2}{E_k} \left(1 - \frac{1}{3} v^2\right) \left[f_e(E_k) + f_{\bar{e}}(E_k)\right] \quad (70)$$



Resonant A'

Propagator exhibit poles, but:

- A' self-energy still important
- A' self-energy at $T = 0$ non-negligible: imaginary part

Breit-Wigner propagator:

$$G^{\mu\nu}_{\mathrm{BW}}(Q^2) = \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{Q^2 - m^2 - \mathrm{Re}(F) - i\,\mathrm{Im}(F)} P^{\nu}_{L\lambda} + \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{Q^2 - m^2 - \mathrm{Re}(G) - i\,\mathrm{Im}(G)} P^{\nu}_{T\lambda} \tag{71}$$

Resonant A'

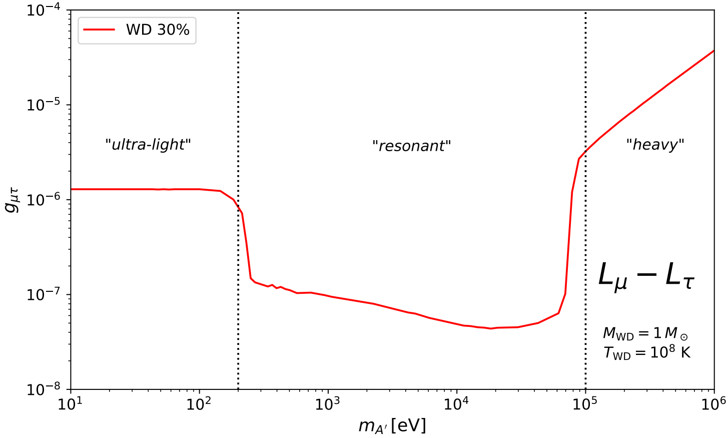
At $T = 0$, we consider the self-energy of light neutrinos after $\overline{MS}$-renormalization:

$$\bar{\Pi}^{\mu\nu}_{A'}(Q^2) = -\frac{\left(k^{\alpha}_{\nu}\right)^2}{4\pi^2} Q^2 g^{\mu\nu} \int_0^1 dx\, x\,(1-x) \times \log\left(\frac{m_\alpha^2}{m_\alpha^2 - x(1-x)Q^2}\right) \tag{72}$$

such that:

$$\begin{aligned}\mathrm{Im}(\bar{\Pi}^{\mu\nu}_{A'})(Q^2) &= \frac{\left(k^{\alpha}_{\nu}\right)^2}{24\pi} Q^2 g^{\mu\nu} \\ &= \frac{\left(k^{\alpha}_{\nu}\right)^2}{24\pi} \frac{(\omega_l^2 - q^2)^2}{q^2} P_L^{\mu\nu} \\ &\quad - \frac{\left(k^{\alpha}_{\nu}\right)^2}{24\pi} (\omega_t^2 - q^2) P_T^{\mu\nu}\end{aligned} \tag{73}$$

Whole range of masses A'



Conclusions

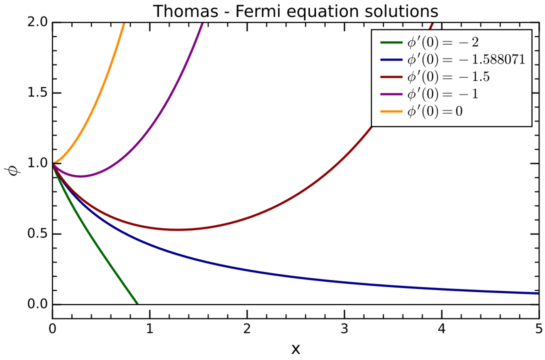
- WDs are an interesting place to search for physics BSM and dark sectors
- Searching for new physics in WDs can be a good training for further searches in neutron stars or supernovae.
- We still need to link some of the predictions to experimental observables in order to have proper constraints.

A1. Thomas - Fermi model

- Constant energy of the cell: $E_F = -eV(r) + \frac{p_F^2(r)}{2m_e}$
- The density is: $n_e = \frac{p_F^3}{3\pi^2} = \frac{1}{3\pi^2}\left(2m_e\left[E_F + eV(r)\right]\right)^{3/2}$
- Poisson's eq.: $\nabla^2 V = 4\pi e(n_e - \delta^{(3)}(0))$, where: $\lim_{r\to 0} rV(r) = Ze$ and $\lim_{r\to r_0} \frac{dV}{dr} = 0$
- Change of variables: $r = \left(\frac{9\pi^2}{128Z}\right)^{1/3} a_0\ x = \mu x$ and $E_F + eV(r) = \frac{Ze^2\phi(x)}{r}$

$$\frac{d^2\phi}{dx^2} = \frac{\phi^{3/2}}{x^{1/2}},\ \phi(0) = 1,\ \phi'(x_0) = \phi(x_0)/x_0 \tag{74}$$

A1. Thomas - Fermi model



$$P = \frac{1}{10\pi}\frac{Z^2 e^2}{\mu^4}\left[\frac{\phi(x_0)}{x_0}\right]^{5/2}$$
$$\rho = \frac{3A\overline{m_N}}{4\pi\mu^3 x_0^3} \quad (75)$$

A2. Other limits on A'

- **BBN**: At masses below $\mathcal{O}(10)$ MeV the dark photon A' contributes significantly to the heating of the neutrino gas in the early universe leading to a too large number of neutrino degrees of freedom, $\Delta N_{\rm eff}$, during BBN.
- **NA64**μ: by using a missing energy-momentum technique with a high energy muon beam.
- **Borexino**: from the measurement of the ^{7}Be solar neutrino flux, masses of $m_{A'} \sim 10$ MeV are excluded for $g_{\mu\tau} \sim 0.0005$.
- **BaBar**: from resonance searches in four-muon production, high masses excluded.
- **COHERENT**: from measurements of coherent elastic neutrino-nucleus scattering (CEνNS) with a CsI[Na] target, high couplings excluded.
- **CHARM-II**: from the search for neutrino trident production, for masses ~ 100 MeV.