Insights from Axion Dark Matter for the Field of Gravitational Wave Physics

University of Warsaw and National Centre for Nuclear Research
Warsaw, Poland
January 30, 2024

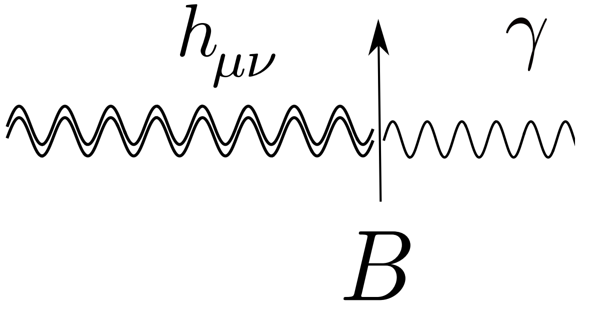
IFIC INSTITUT DE FÍSICA CORPUSCULAR
CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
VNIVERSITAT DE VALÈNCIA

Camilo García Cely

Based on PRL 129, 041101, hep-ph/2306.03125 and hep-ph/2402.xxxxxx
In collaboration with Valerie Domcke, Sung Mook Lee, Nicholas L. Rodd, and Andreas Ringwald

Outline

- How do people search for axion dark matter?
- Solar gravitational waves
- Detecting gravitational waves with axion haloscopes
- Conclusions



Axion dark matter versus gravitational waves

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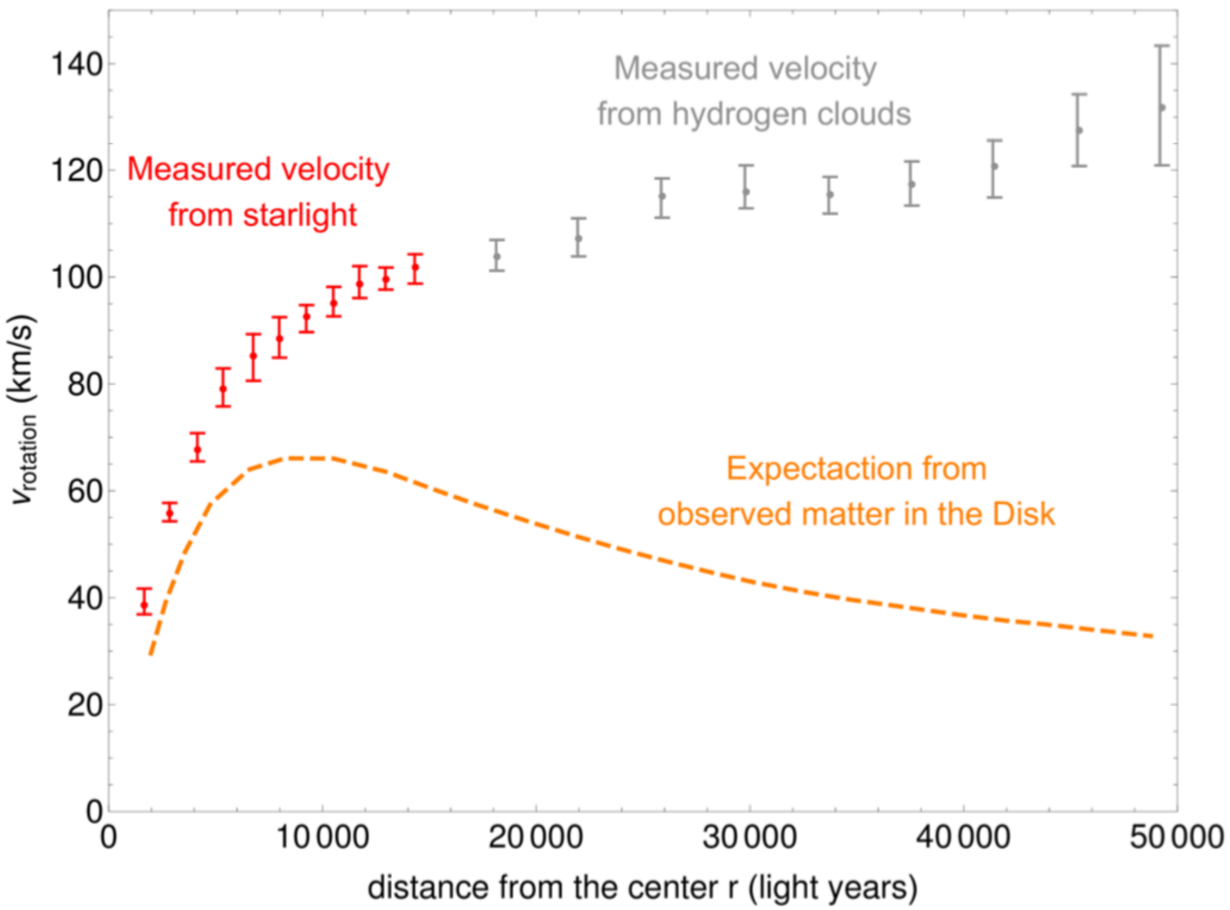
Dark Matter



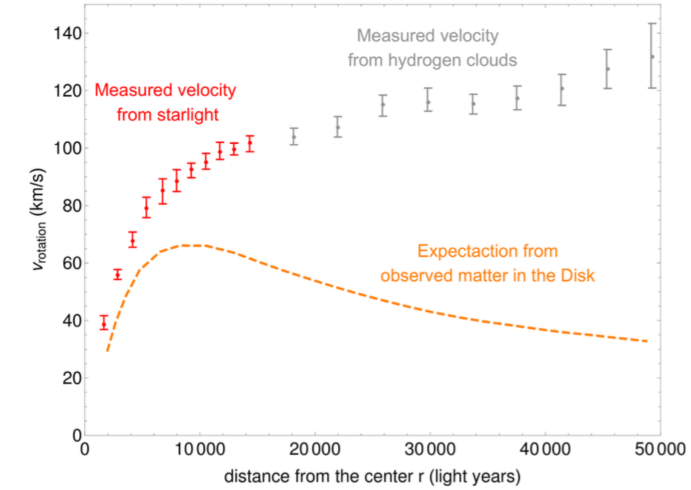
Triangulum Galaxy (M33)

There must be some ***matter that we don't see***
or Newton's Laws don't work in galaxies

Vera Rubin

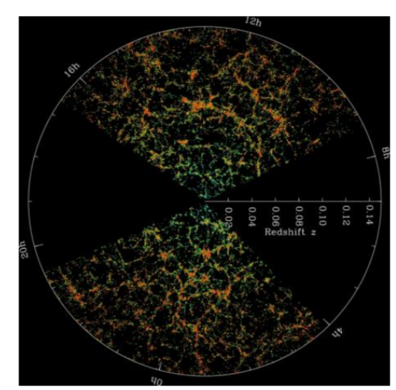
Collisionless Cold
Dark Matter

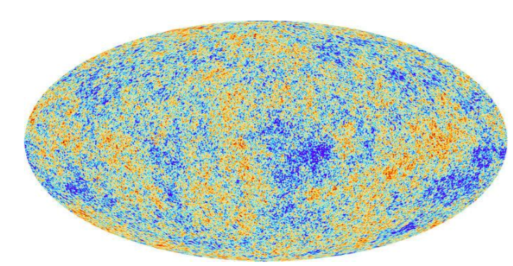
The dark matter hypothesis is remarkably simple and explain observations at many other scales



kpc

Mpc





Gpc

Velocity measurements

- Flat rotation curves of spiral galaxies
- Velocity dispersion of stars in giant elliptical and dwarf spheroidal galaxies
- Velocity dispersion of galaxies in clusters

Lensing

- Weak lensing by large-scale structure and cluster mergers
- Strong lensing by individual galaxies and clusters

Universe at large scales

- Abundance of clusters
- Large-scale distribution of galaxies
- Power spectrum of CMB anisotropies

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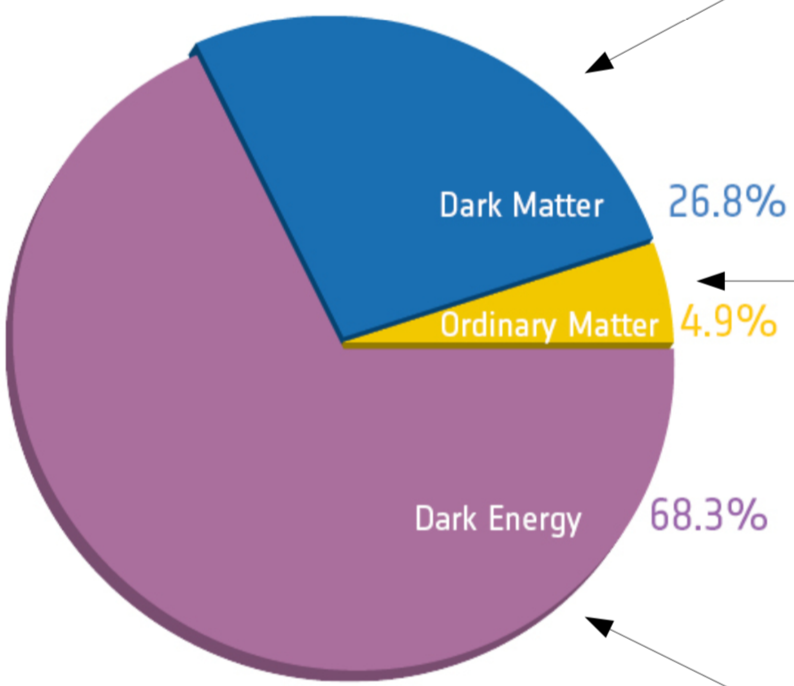
Collisionless Cold
Dark Matter

?



Standard Model stable particles:

Mostly protons, electrons,
neutrinos and photons

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

?

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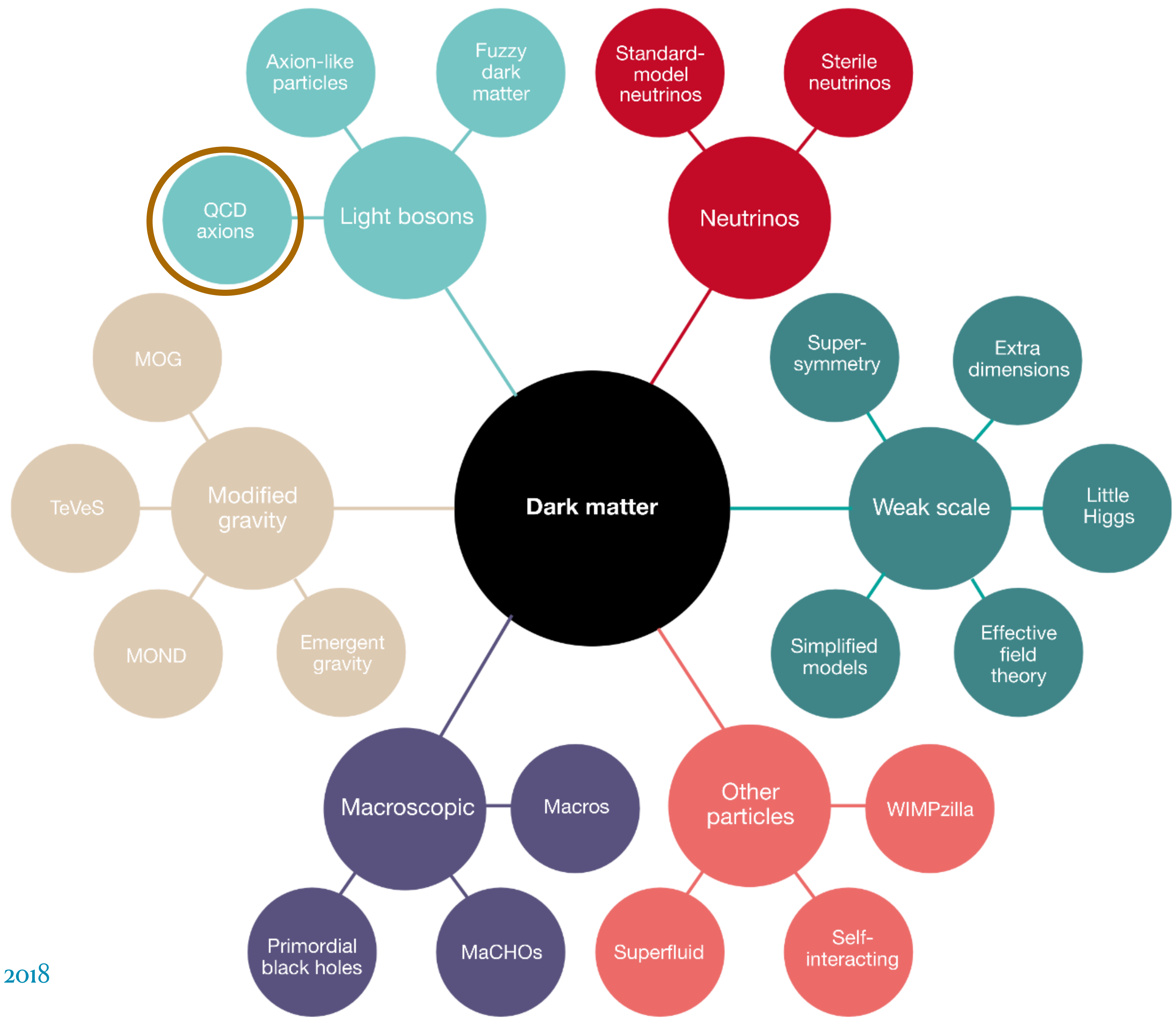
Collisionless Cold
Dark Matter



Bertone Tait, 2018

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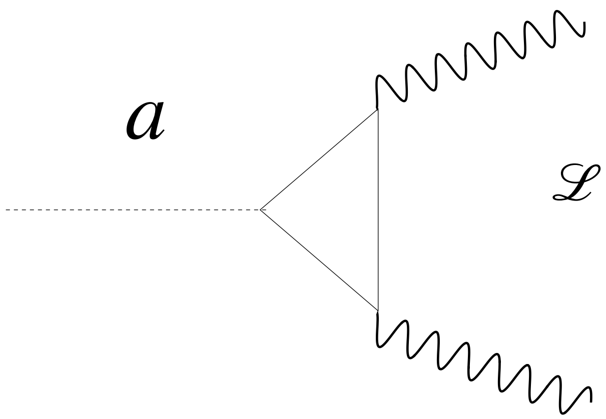
QCD axion as dark matter

- Pseudoscalar field



$$\mathscr{L} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- Solution to the strong CP problem

Peccei, Quinn 1977

- Excellent dark matter candidate

Weinberg, Wilczek 1978

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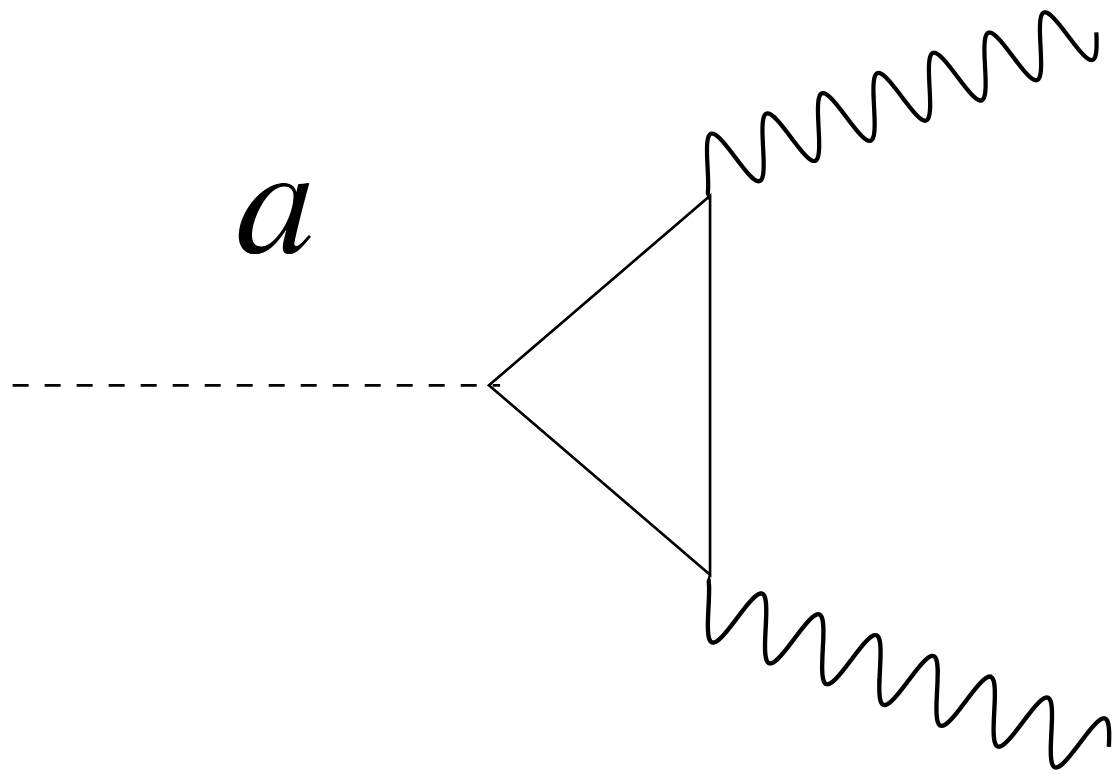
Axion electrodynamics



$$\mathscr{L} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

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Axion electrodynamics

Axions act as a source term to Maxwell's equations, effectively inducing an electromagnetic current.

$$\nabla \cdot \mathbf{B} = 0$$

Sikivie, 1983

$$\nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = j^0$$

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{j}$$

$$j^0 = -\, g_{a\gamma\gamma} \nabla a \cdot \mathbf{B} \qquad \mathbf{j} = g_{a\gamma\gamma} \left(\nabla a \times \mathbf{E} + \partial_t a \mathbf{B} \right)$$

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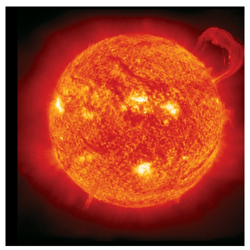
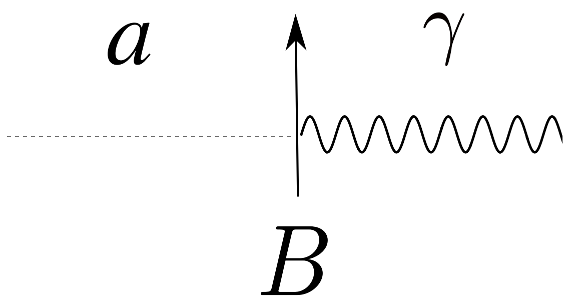
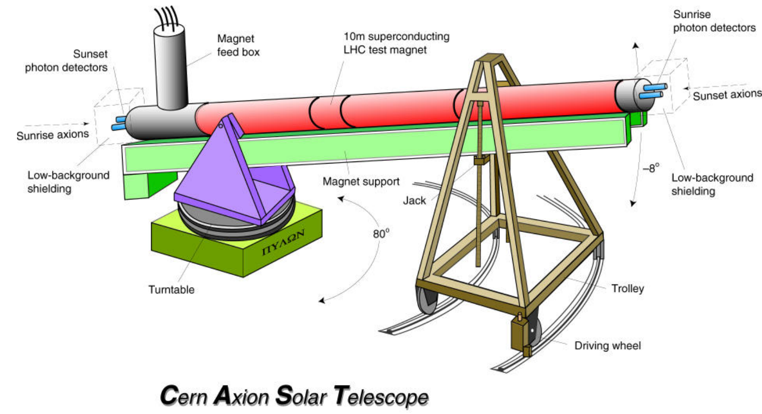
Axion electrodynamics

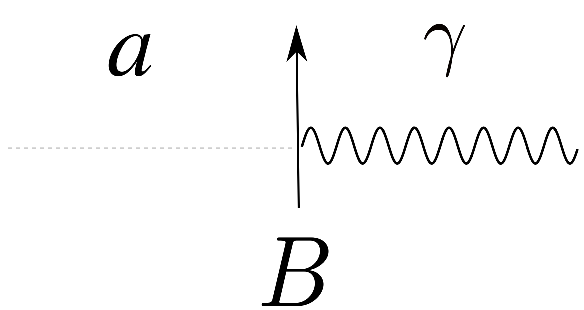
- Helioscopes (X rays)

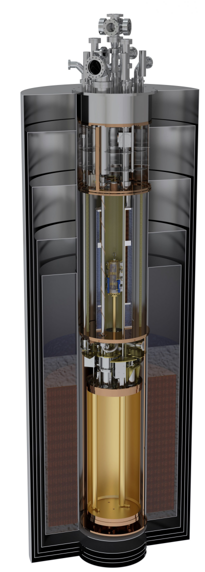





- CAST
- IAXO
- .....

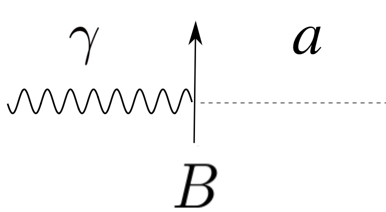
- Haloscopes (radio frequencies)

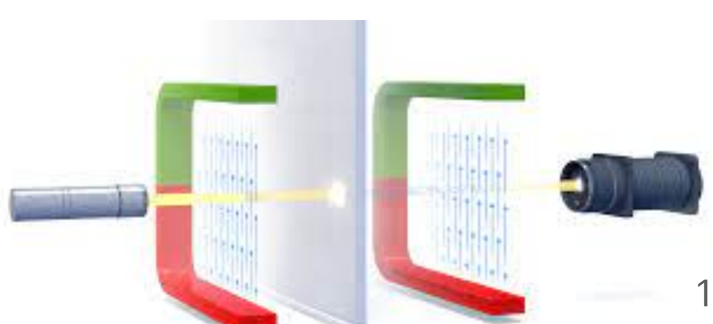
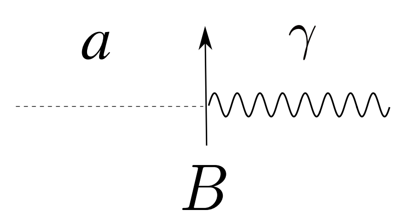





- microwave cavities
- MADMAX
- ADMX
- HAYSTAC
- ABRACADABRA
- Lumped element detectors
- ...

- Purely lab experiments



- Light shining through the walls
- OSCAR
- ALPS II
- ...

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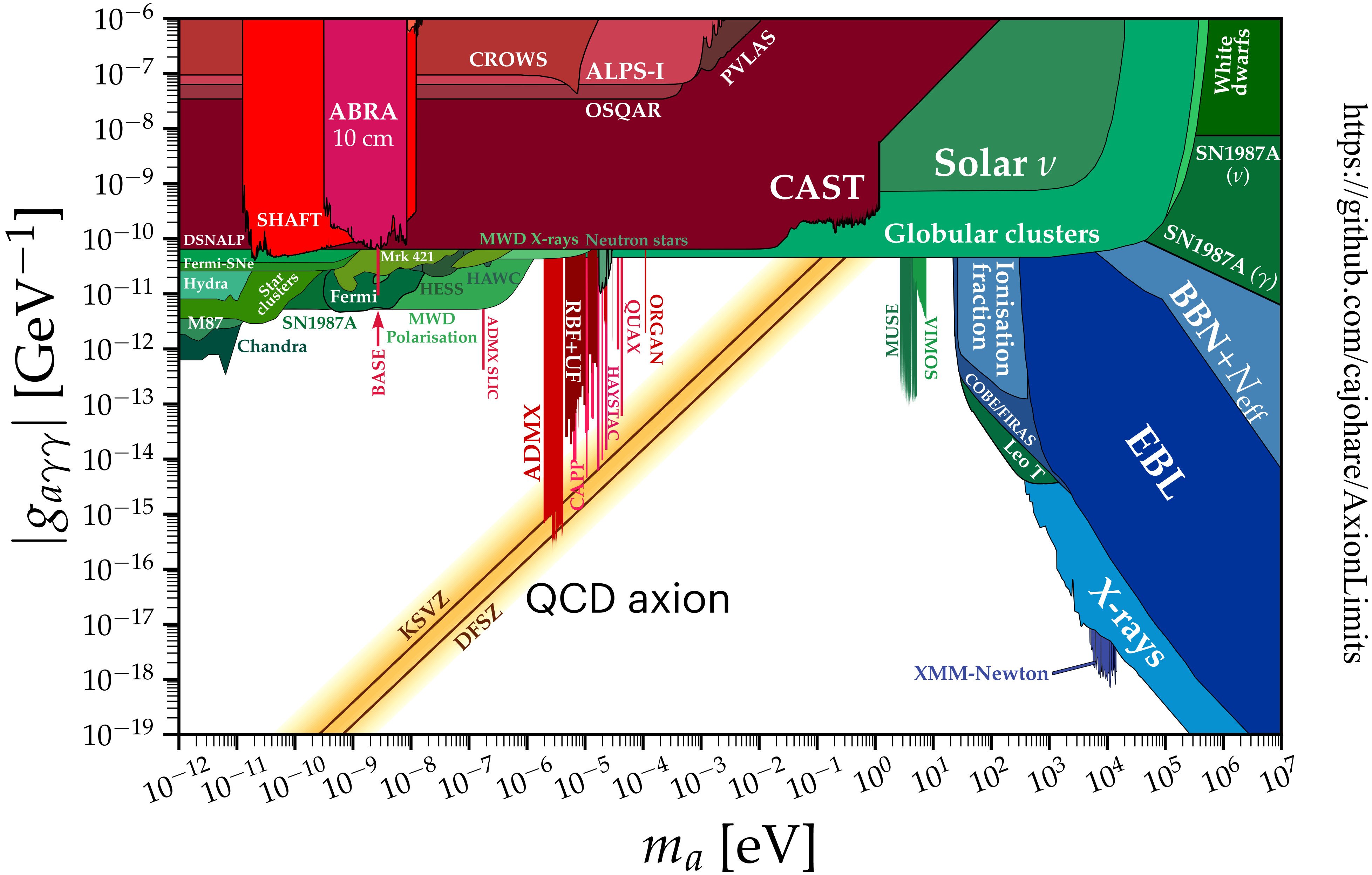
Axion electrodynamics



$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right)$$

DFSZ (Dine, Fischler, Srednicki, Zhitniskii)

axions couple to fermions $E/N = 8/3$

KSVZ (Kim, Shifman, Vainshetein, Zakharov)

axions couple to exotic heavy quarks only. $E/N = 0$

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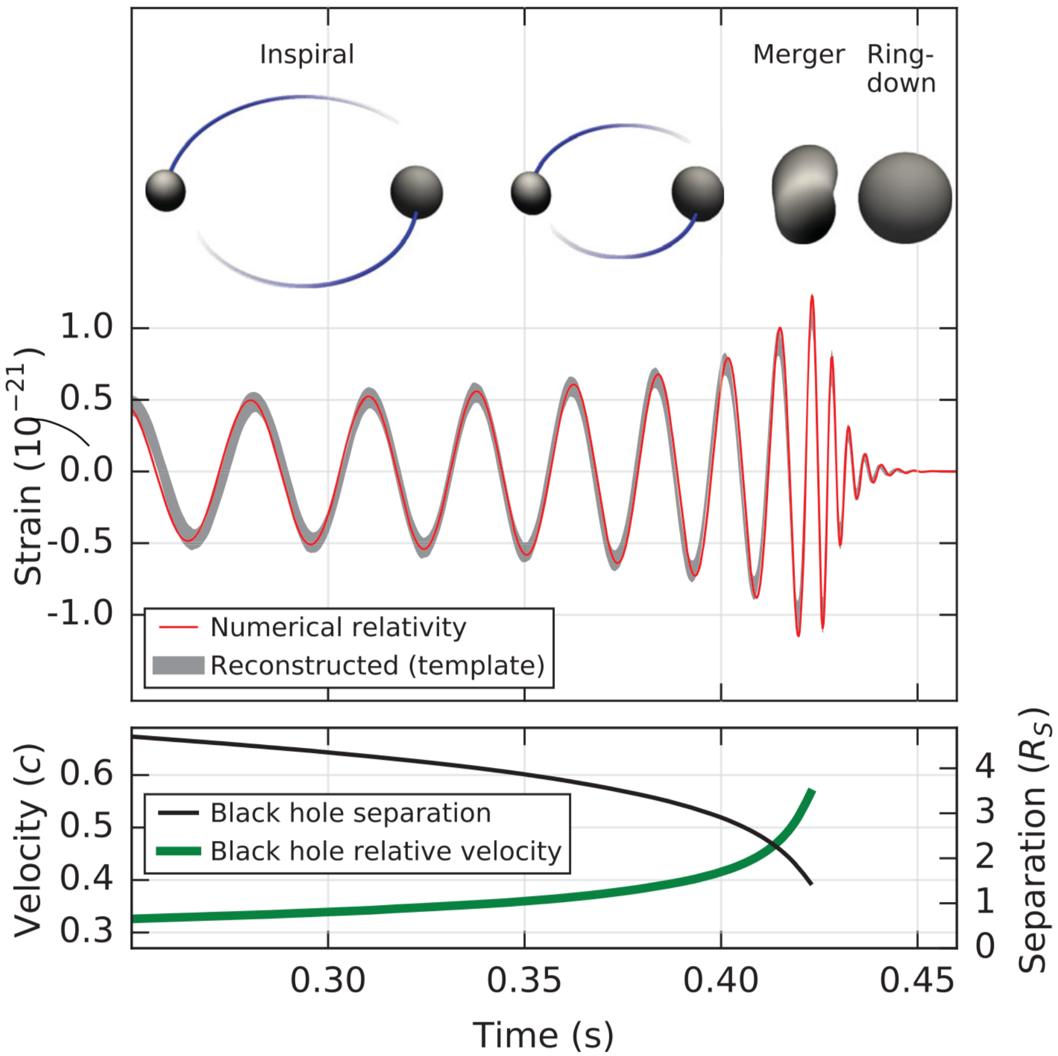
Gravitational Waves

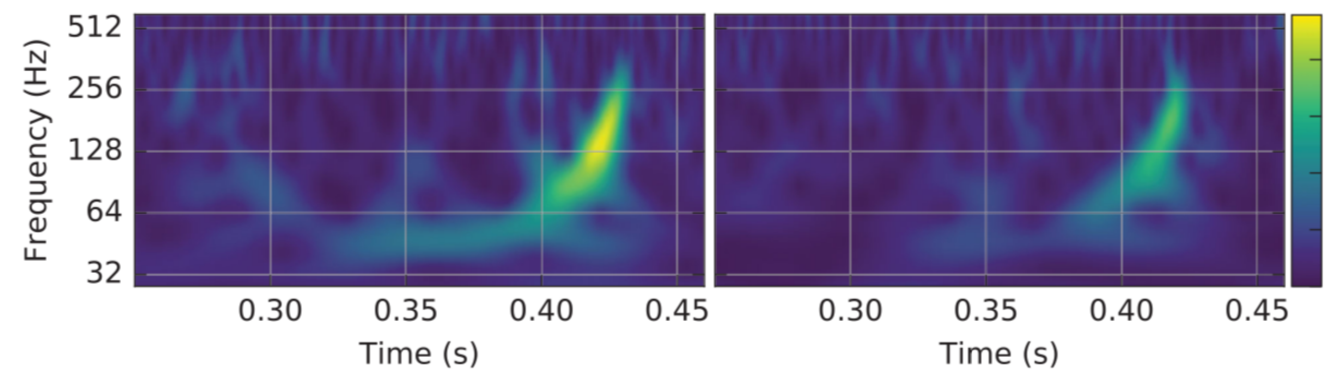
- Speculation by Poincaré (1905)
- Einstein provided a firm theoretical background for them (1916)



PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)



interferometers

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Revisiting Gertsenhstein's ideas

SOVIET PHYSICS JETP VOLUME 16, NUMBER 2 FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN and V. I. PUSTOVOĬT

Submitted to JETP editor March 3, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) 43, 605-607 (August, 1962)

It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocrystals is ten orders of magnitude worse than that estimated by Weber.[1] In the low frequency range it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.

Terrestrial interferometers

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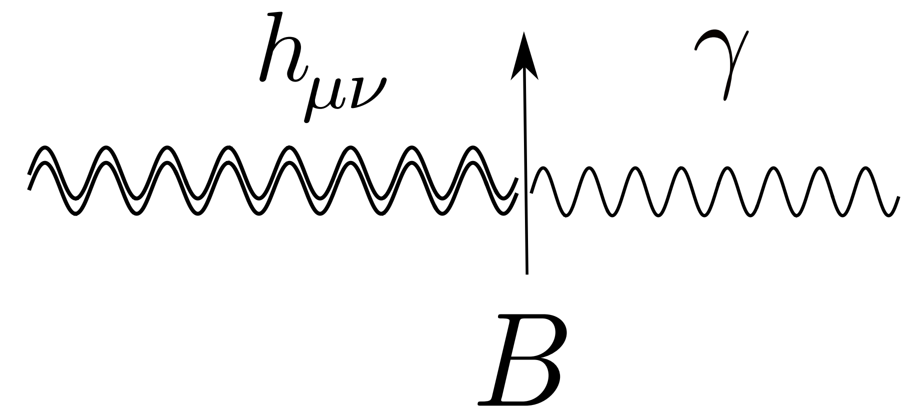
Revisiting Gertsenhstein's ideas

SOVIET PHYSICS JETP VOLUME 14, NUMBER 1 JANUARY, 1962



WAVE RESONANCE OF LIGHT AND GRAVITIONAL WAVES

M. E. GERTSENSHTEĬN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.

SOVIET PHYSICS JETP VOLUME 16, NUMBER 2 FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

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Terrestrial interferometers

The (inverse) Gertsenhstein Effect

- The conversion of gravitational waves into electromagnetic waves is a classical process. Its rate does not involve $\hbar$

$$P \sim G B^2 L^2$$

- Cosmological conversion

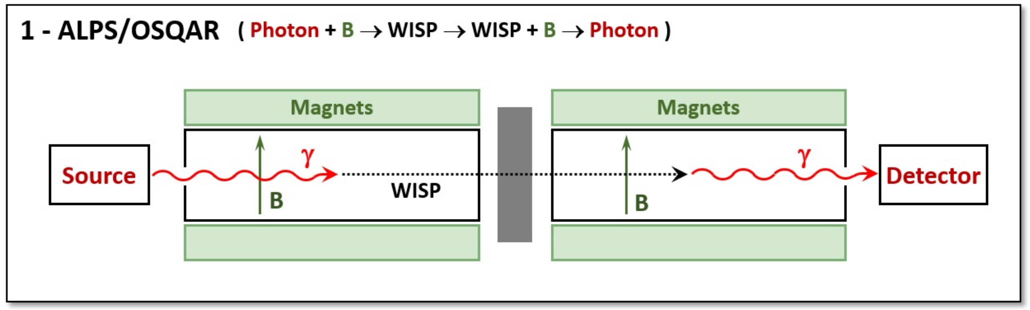
Potential of Radio Telescopes as High-Frequency Gravitational Wave Detectors

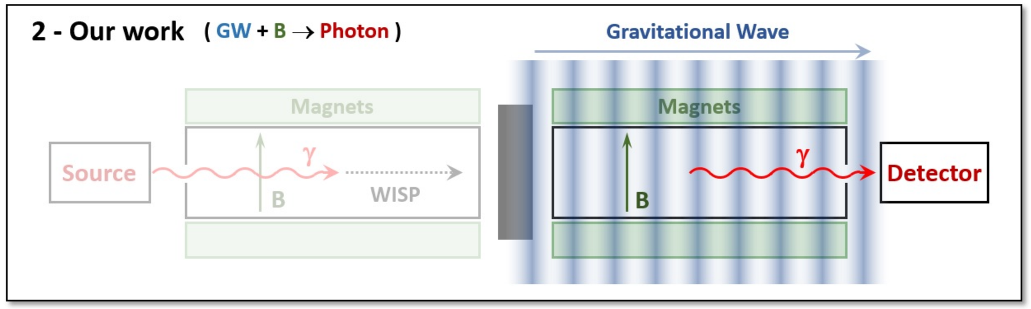
Valerie Domcke and Camilo Garcia-Cely
Phys. Rev. Lett. **126**, 021104 – Published 14 January 2021

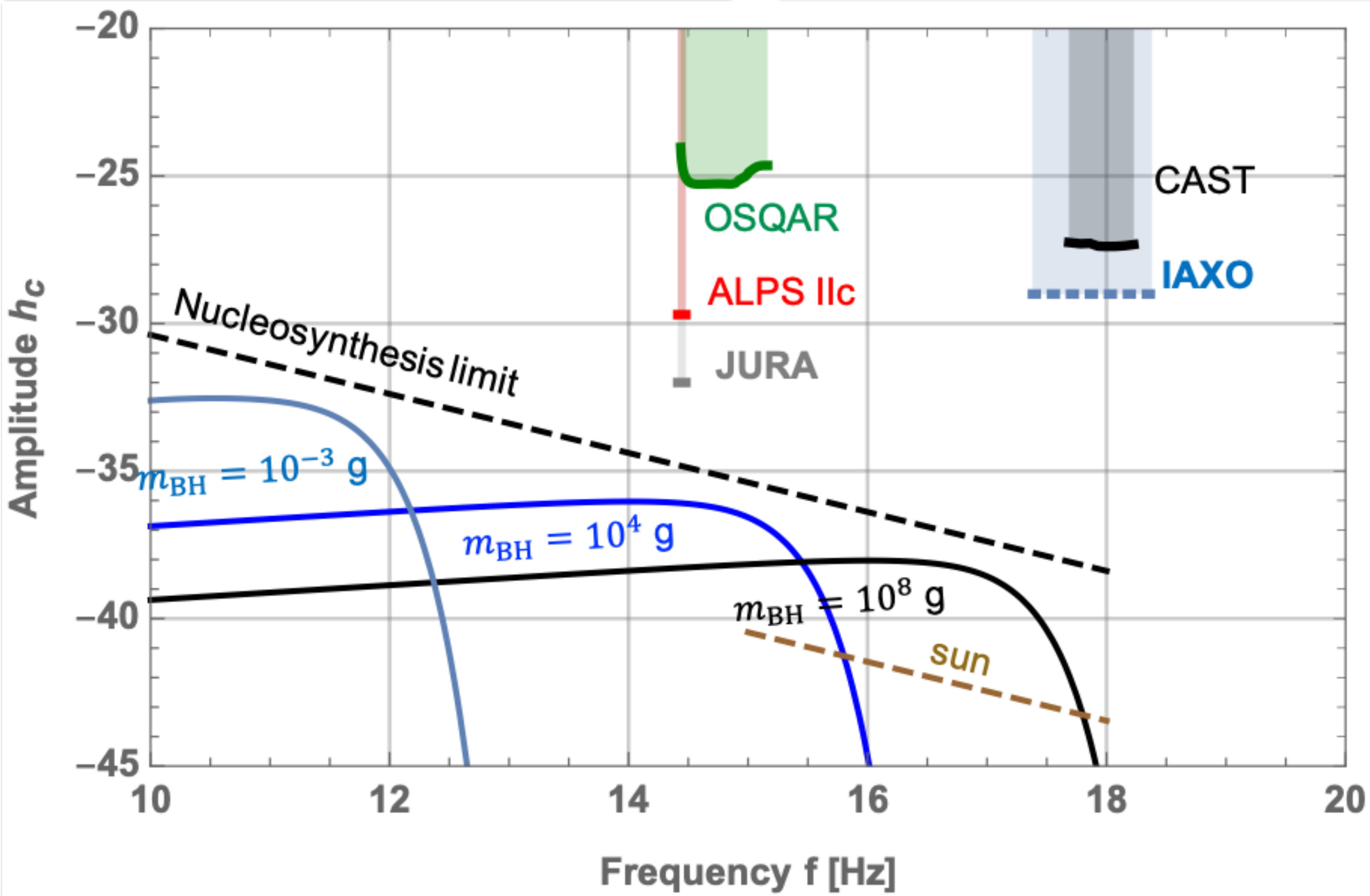
- The process is strictly analogous to axion dark matter conversion.

Raffelt, Stodolski'89

The (inverse) Gertsenhstein Effect







A. Ejlli, D. Ejlli, A. M. Cruise, G. Pisano & H. Grote

The European Physical Journal C **79**, Article number: 1032 (2019)

High-frequency gravitational waves

Part of a collection:

Gravitational Waves

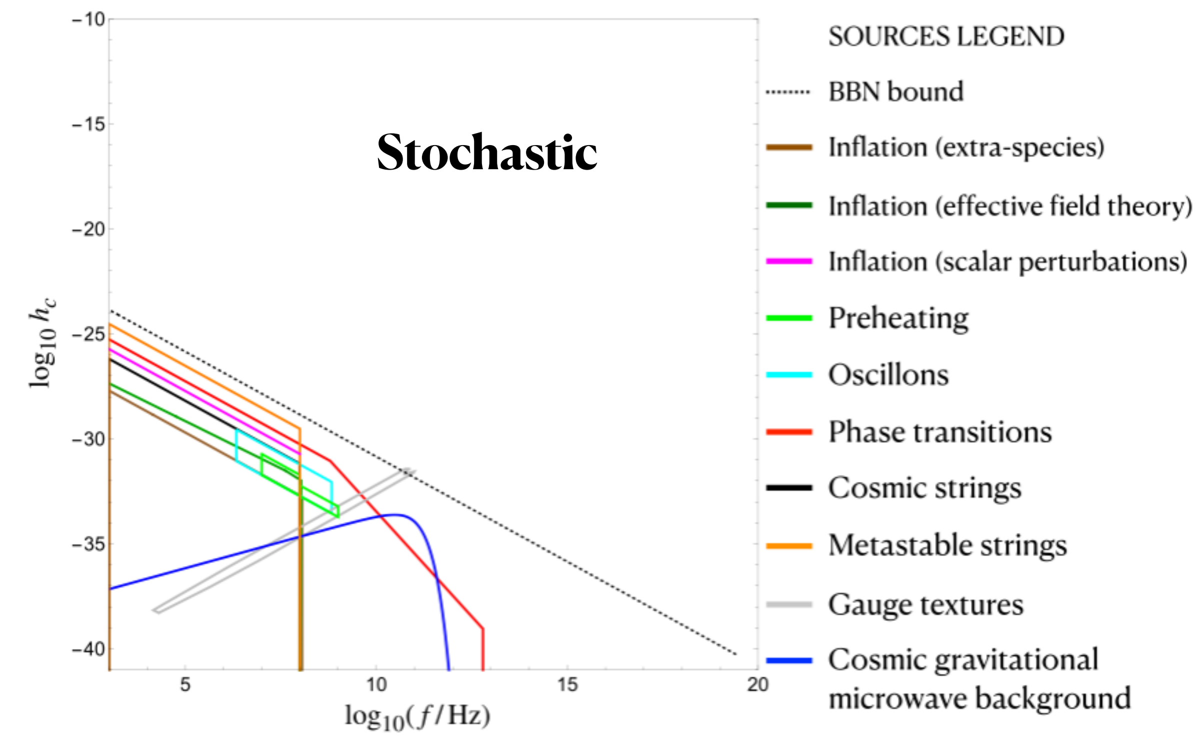
Review Article | Open Access | Published: 06 December 2021

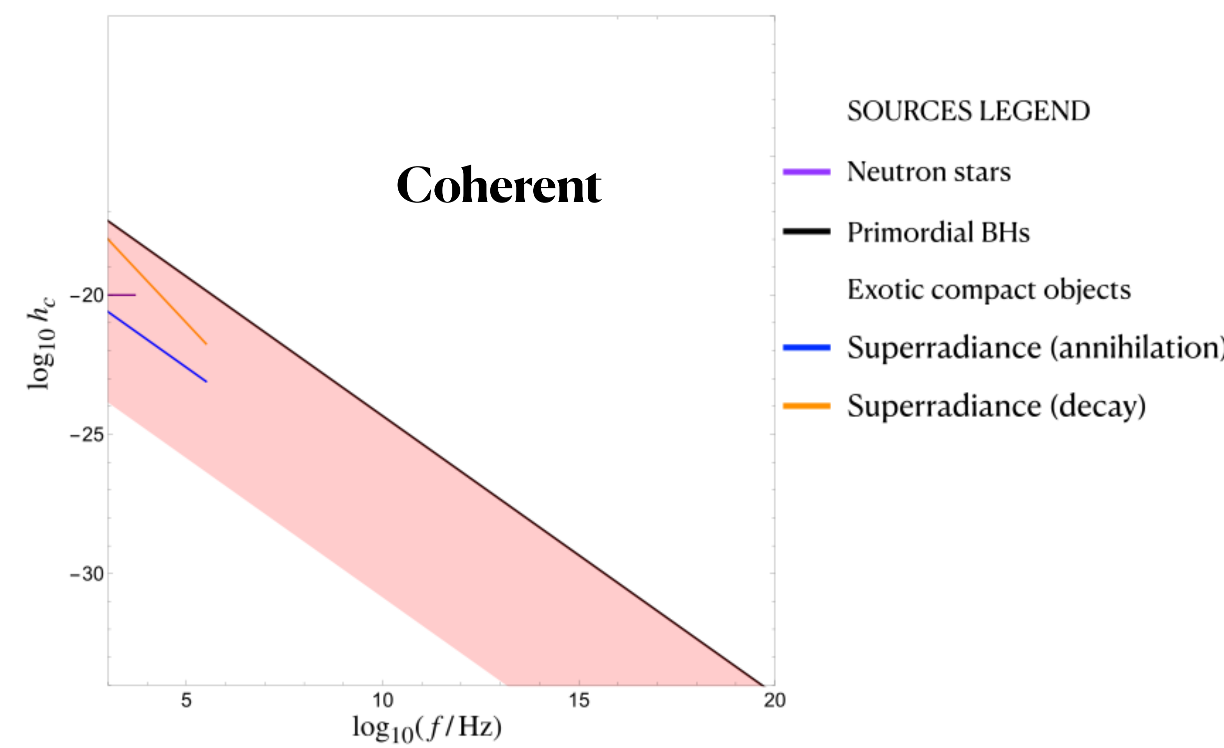
Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

Nancy Aggarwal, Odylio D. Aguiar, Andreas Bauswein, Giancarlo Cella, Sebastian Clesse, Adrian Michael Cruise, Valerie Domcke, Daniel G. Figueroa, Andrew Geraci, Maxim Goryachev, Hartmut Grote, Mark Hindmarsh, Francesco Muia, Nikhil Mukund, David Ottaway, Marco Peloso, Fernando Quevedo, Angelo Ricciardone, Jessica Steinlechner, Sebastian Steinlechner, Sichun Sun, Michael E. Tobar, Francisco Torrenti, Caner Ünal & Graham White

Living Reviews in Relativity **24**, Article number: 4 (2021) | Cite this article

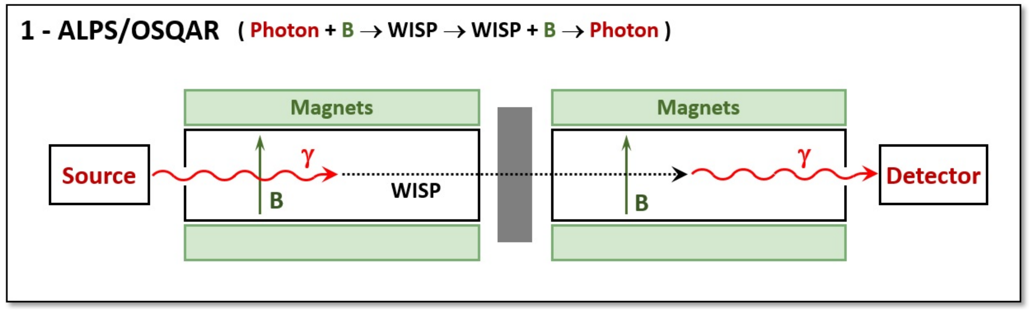
A growing community is seriously considering the search of high frequency gravitational waves

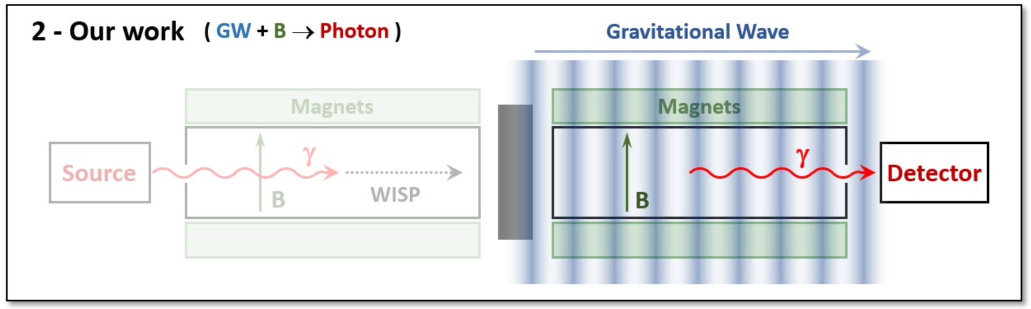


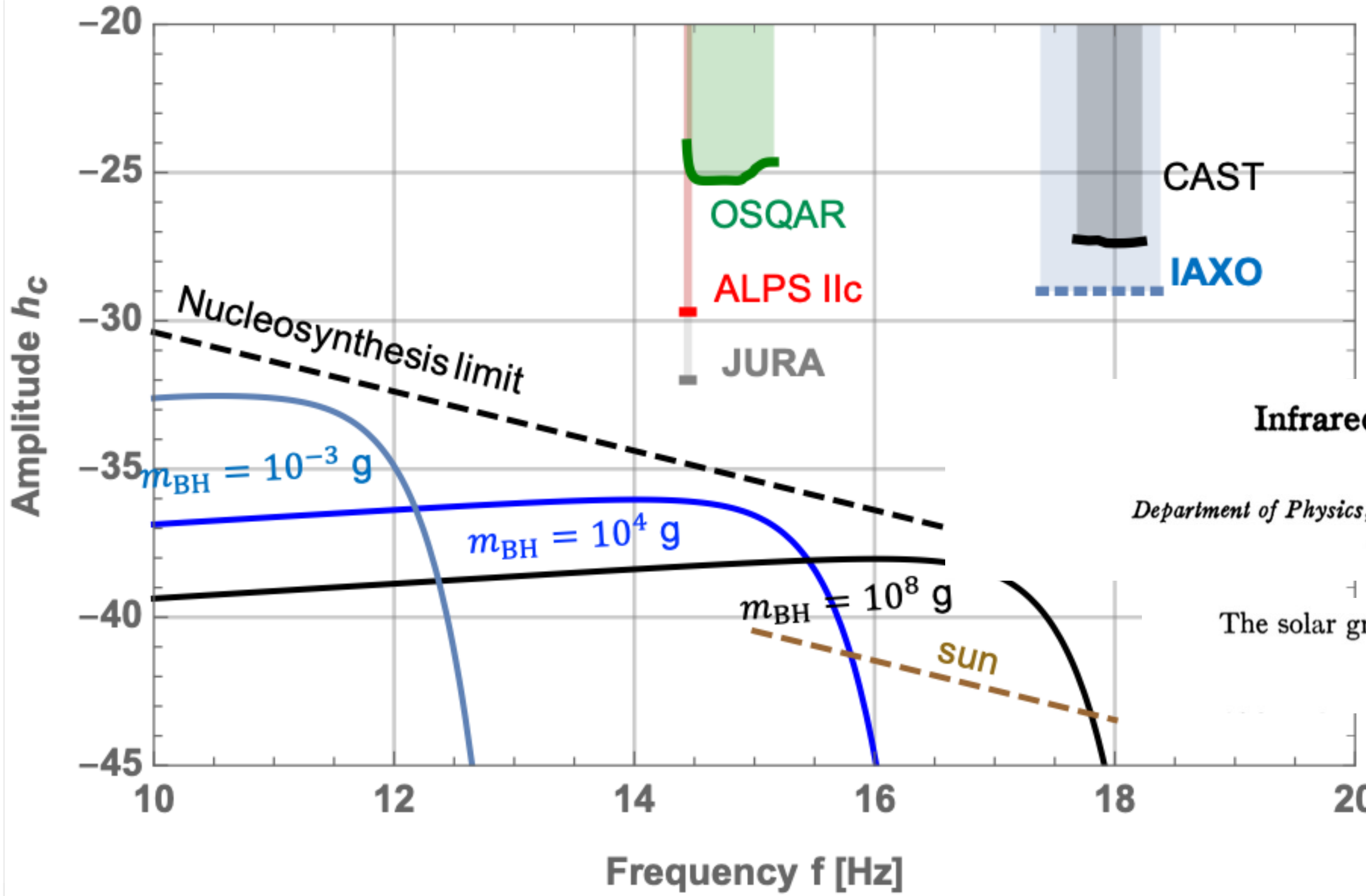


Solar gravitational waves

The (inverse) Gertsenhstein Effect







Infrared Photons and Gravitons*

STEVEN WEINBERG†
Department of Physics, University of California, Berkeley, California

(Received 1 June 1965)

The solar gravitational radiation power is then

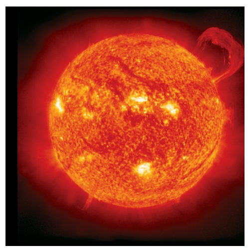
$$P_{\odot} \simeq 6 \times 10^{14} \text{ erg/sec}. \quad (4.24)$$

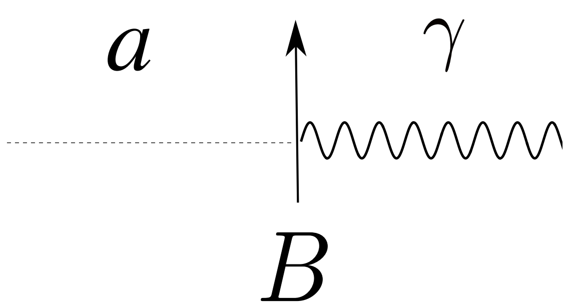
A. Ejlli, D. Ejlli, A. M. Cruise, G. Pisano & H. Grote

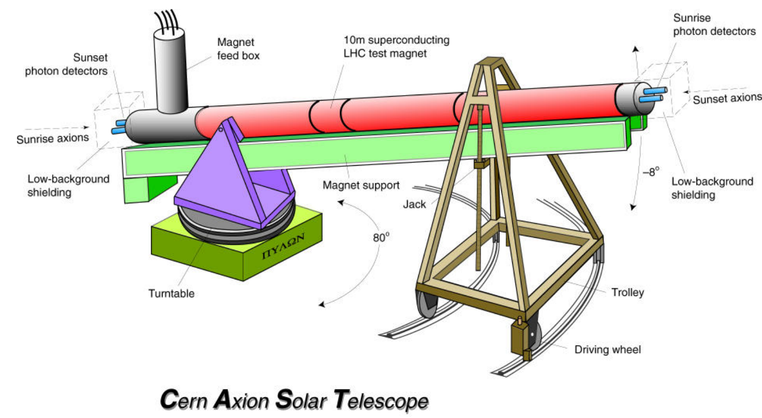
The European Physical Journal C **79**, Article number: 1032 (2019)

Solar axions

- Helioscopes (X rays)







- CAST
- IAXO
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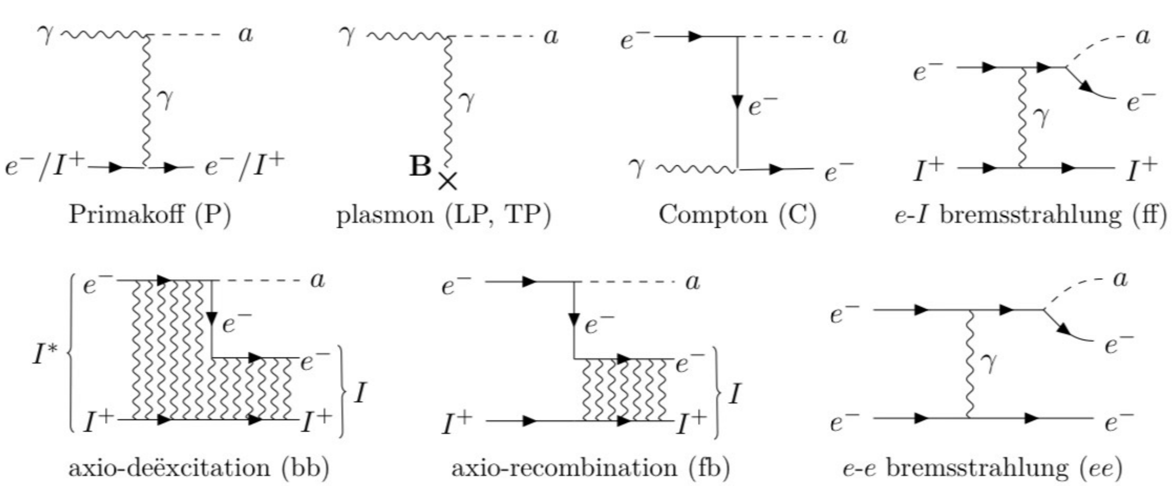

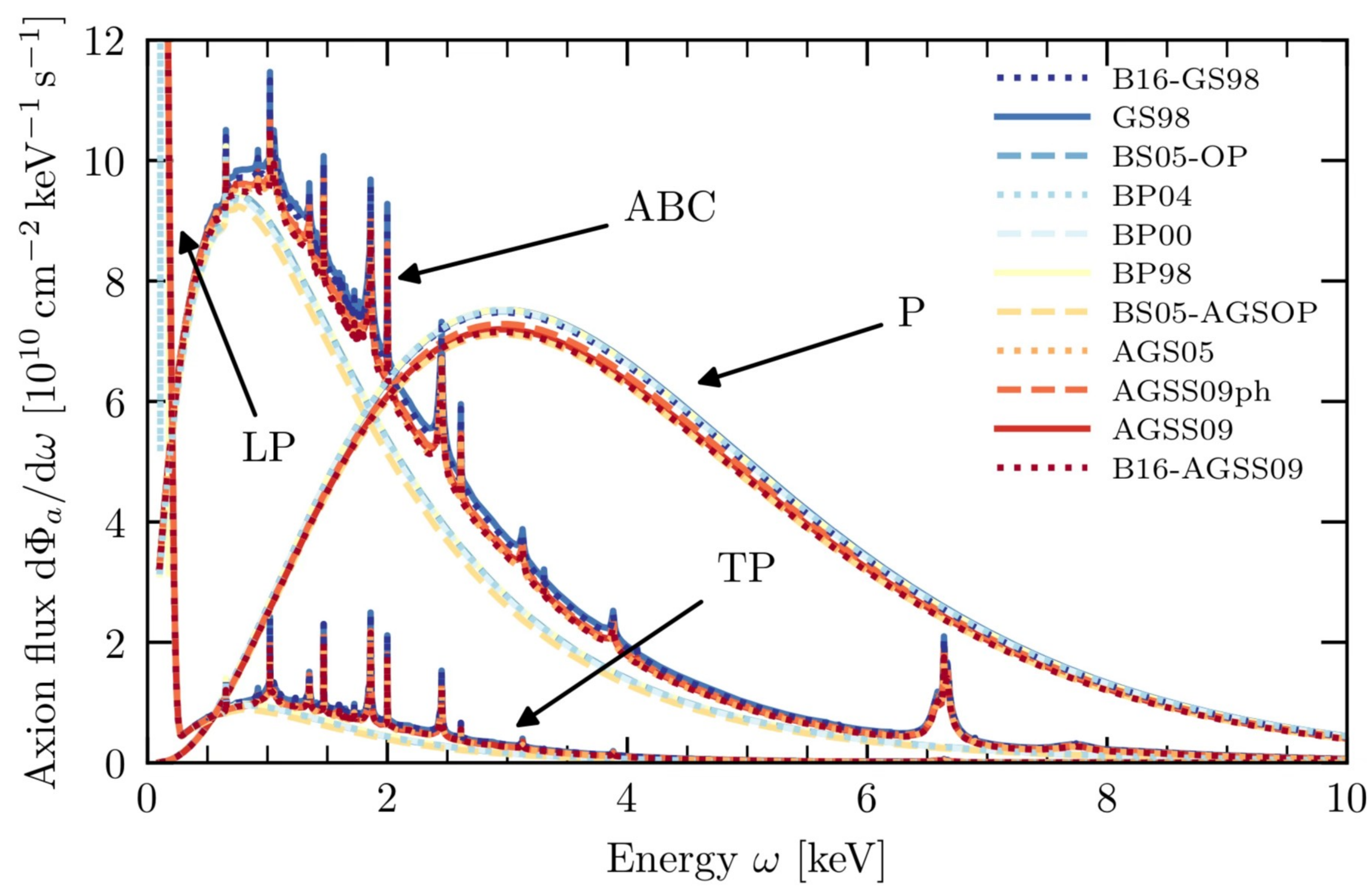

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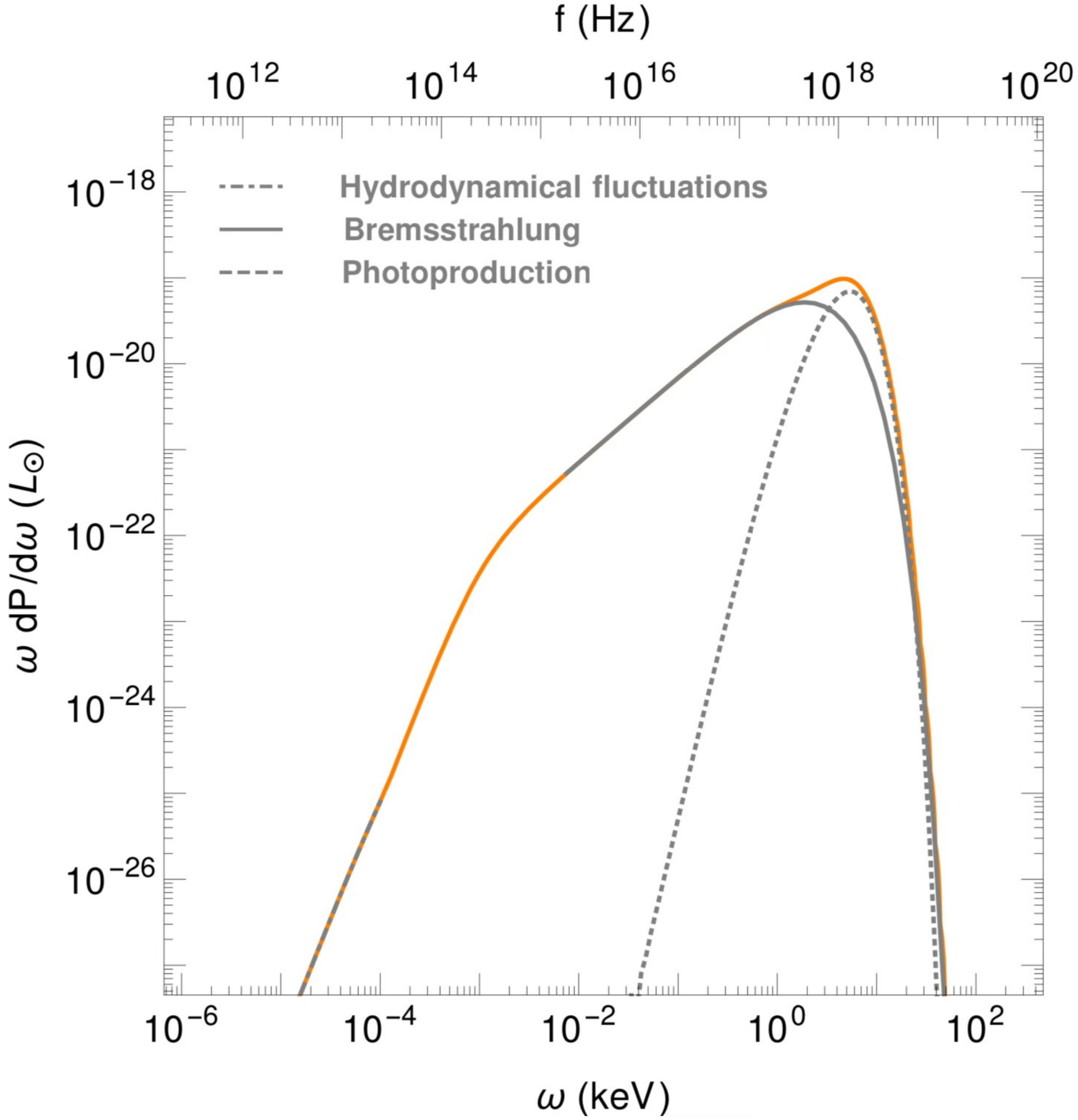
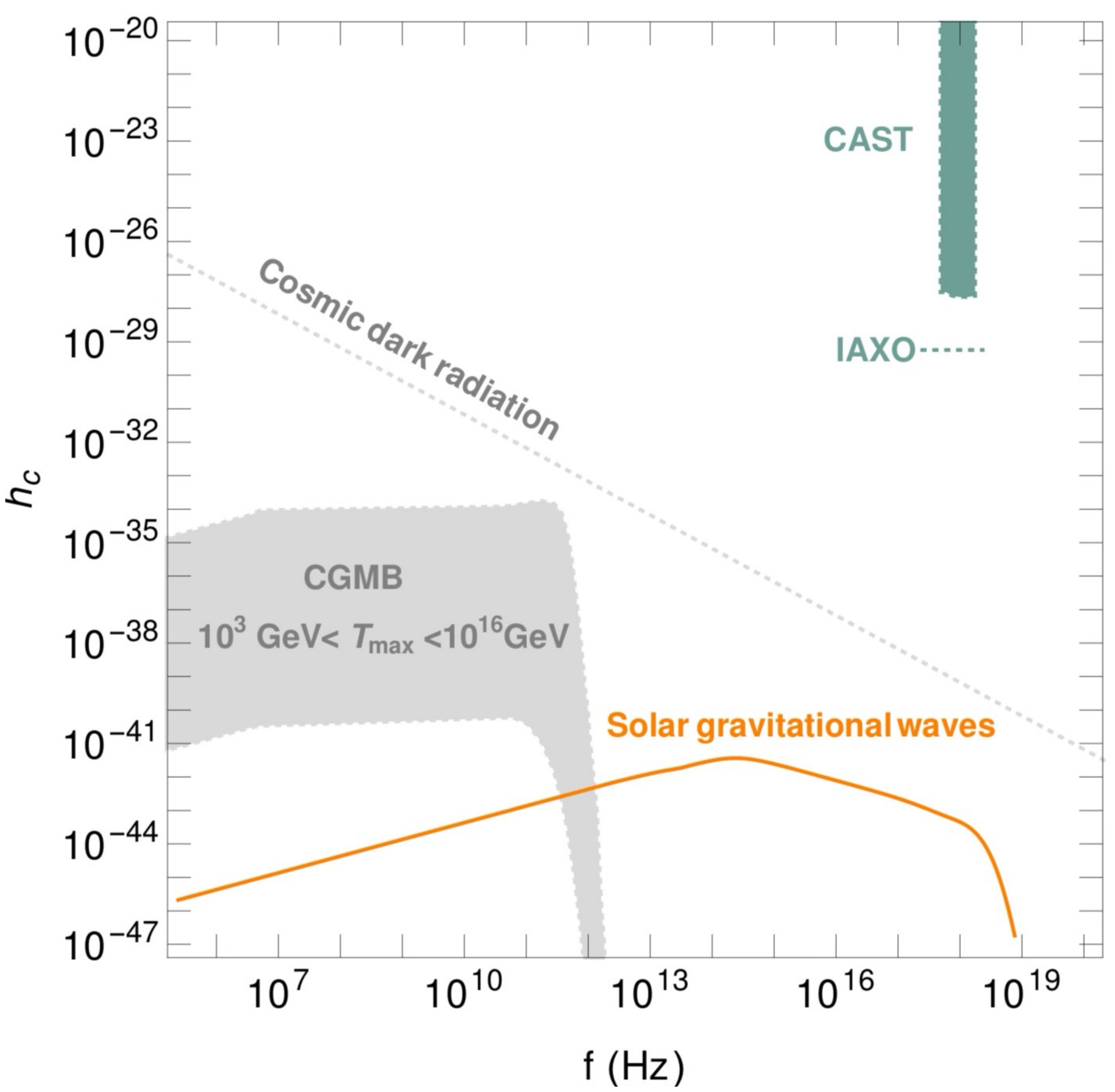
Solar gravitational waves



CGC, Ringwald **PRELIMINARY**

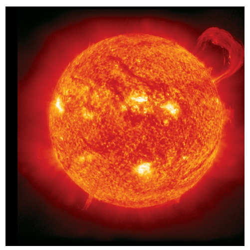


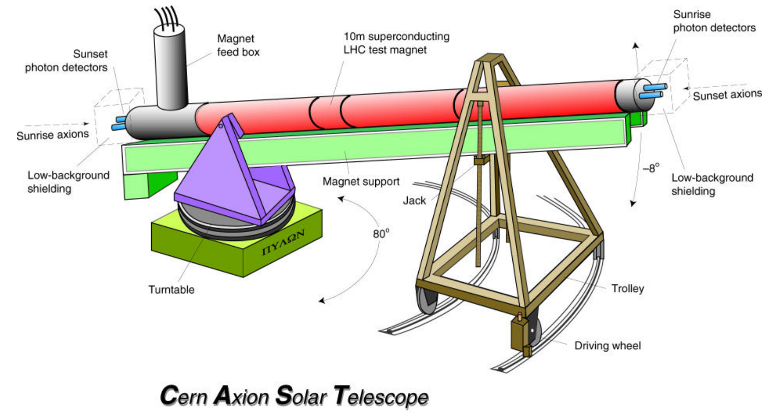
Haloscopes based on lumped-element detectors

Camilo García Cely, University of Valencia

Many possibilities

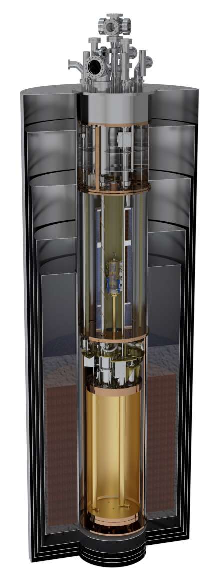
- Helioscopes (X rays)





- CAST
- IAXO
- .....

- Haloscopes (radio frequencies)



- microwave cavities
- MADMAX
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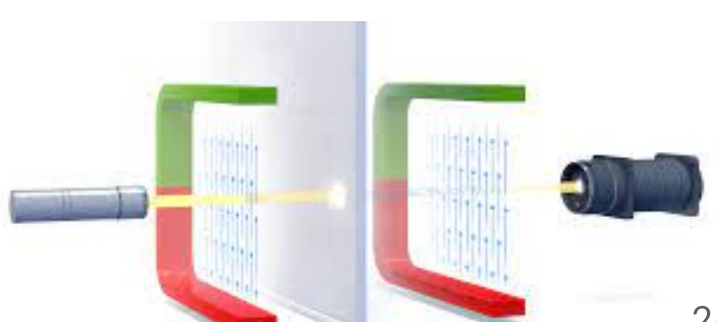
- ABRACADABRA
- Lumped element detectors
- ...

- Purely lab experiments



- Light shining through the walls
- OSCAR
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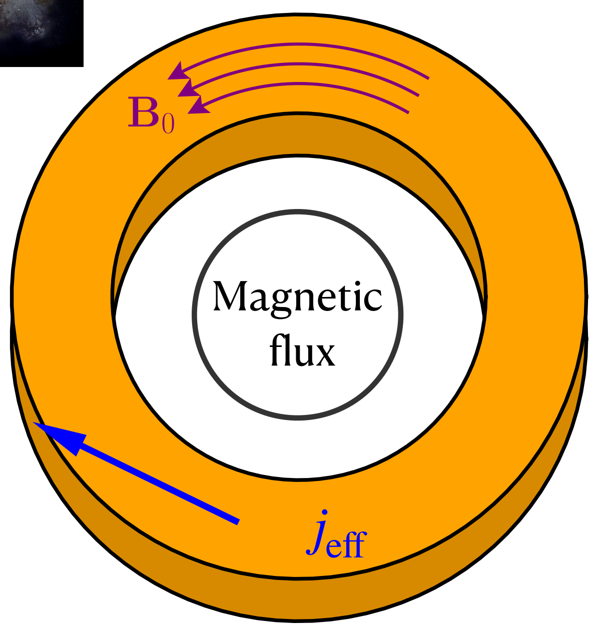
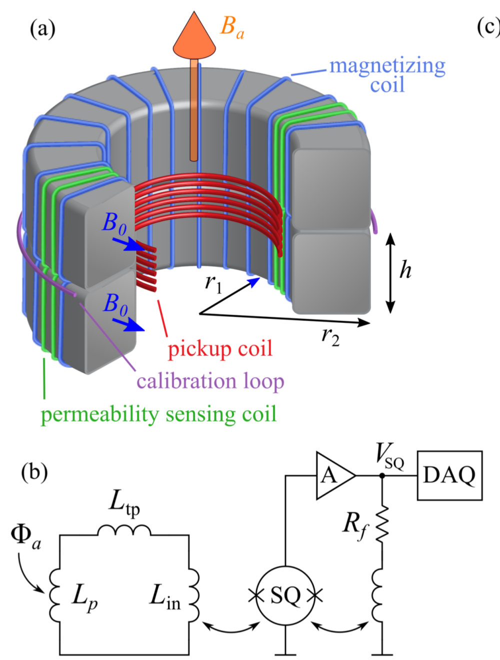
- ...

Haloscopes based on lumped-element detectors



$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \underbrace{g_{a\gamma\gamma}\, \partial_t a\, \mathbf{B_0}}_{j_{\text{eff}}}$$



(c) SHAFT

physics https://doi.org/

Search for axion-like dark matter with ferromagnets

Alexander V. Gramolin[1], Deniz Aybas[1,2], Dorian Johnson[1], Janos Adam[1] and Alexander O. Sushkov[1,2,3]

PRL 117, 141801 (2016) PHYSICAL REVIEW LETTERS week ending 30 SEPTEMBER 2016

Broadband and Resonant Approaches to Axion Dark Matter Detection

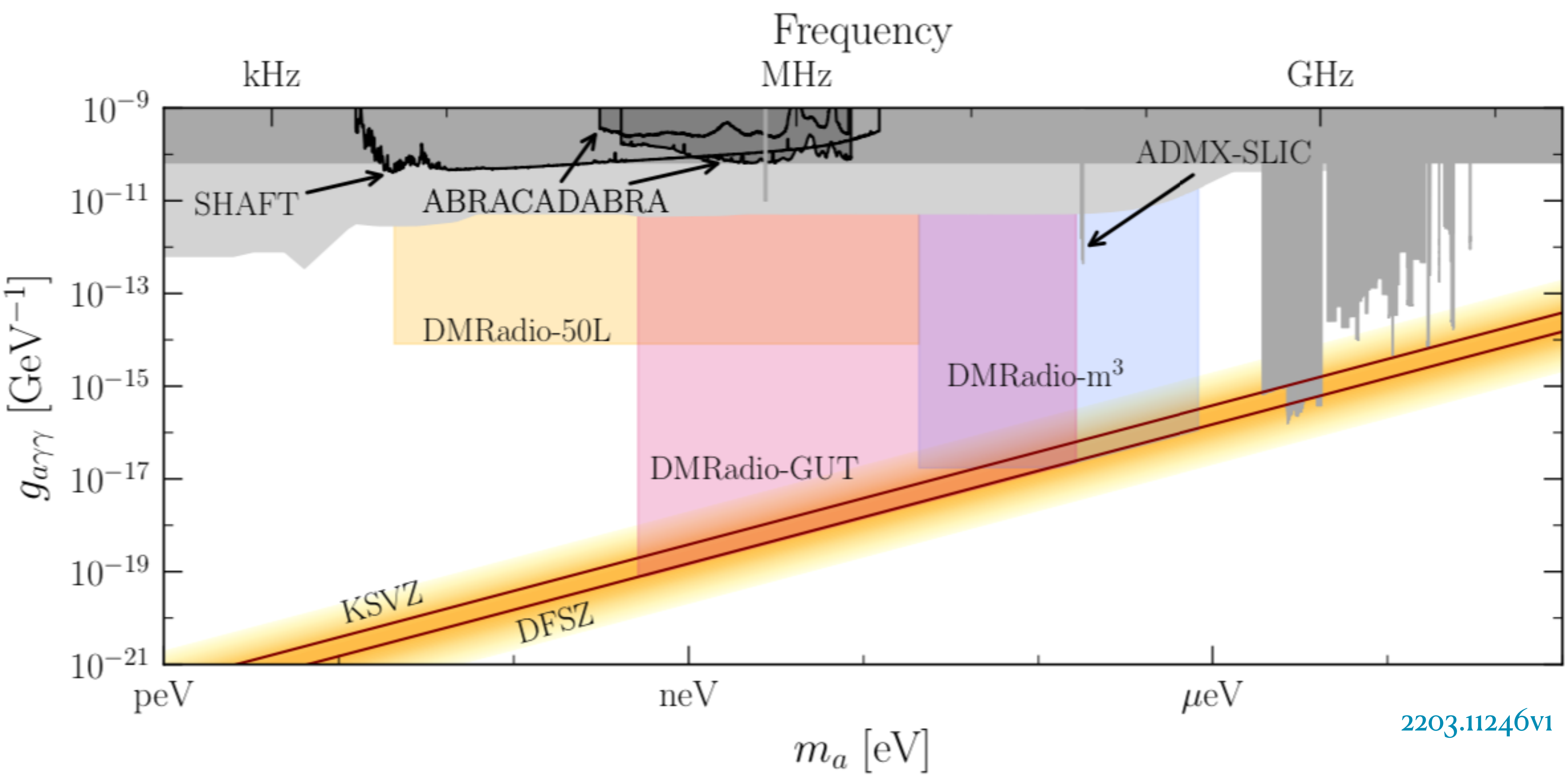
Yonatan Kahn,[1,*] Benjamin R. Safdi,[2,†] and Jesse Thaler[2,‡]
[1]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[2]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Received 3 March 2016; published 30 September 2016)

The electromagnetic fields produced by the axion drive a current through a pickup coil

Camilo García Cely, University of Valencia

DMRadio program

Searches at frequencies lower than those achieved with conventional cavity haloscopes.



2203.11246v1

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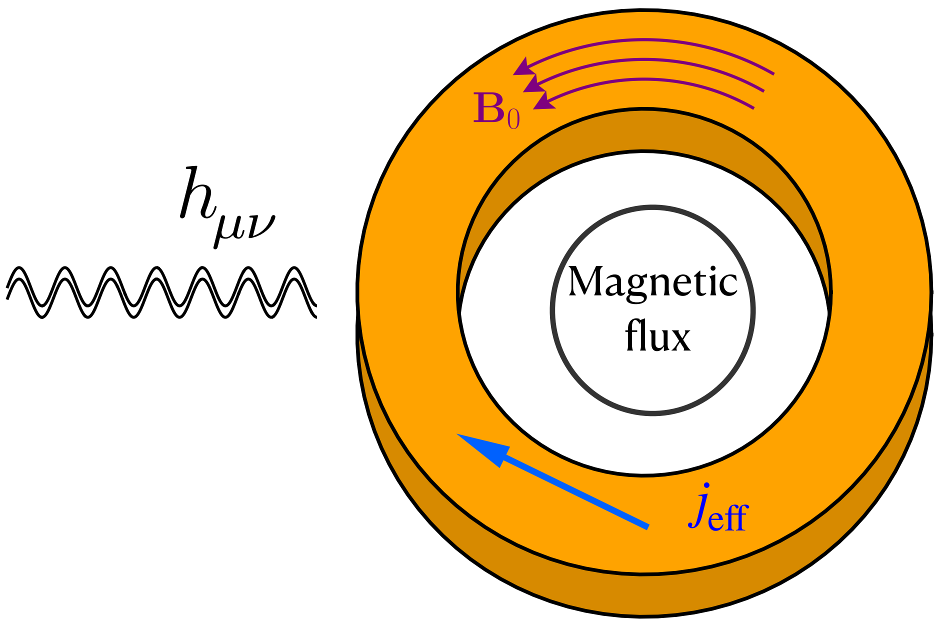
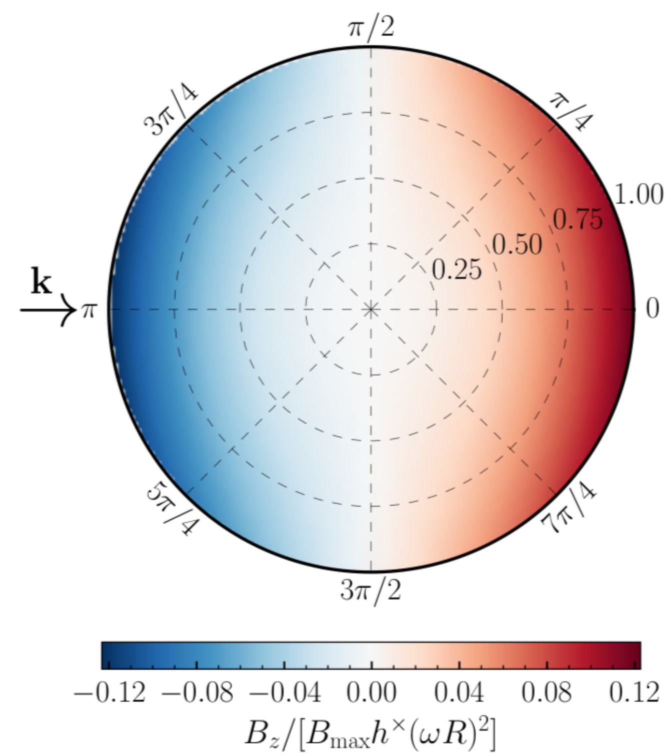
Haloscopes based on lumped-element detectors



Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



$$\Phi \approx \frac{\mathrm{i}e^{-\mathrm{i}\omega t}}{16\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 R a(a+2R) s_{\theta_h}^2$$

Only one polarization

Suppression at small frequencies

$$\Phi_{\text{axions}} \approx e^{-\mathrm{i}\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_{\max} \pi r^2 R$$

The sensitivity scaling with the volume is faster than for axions

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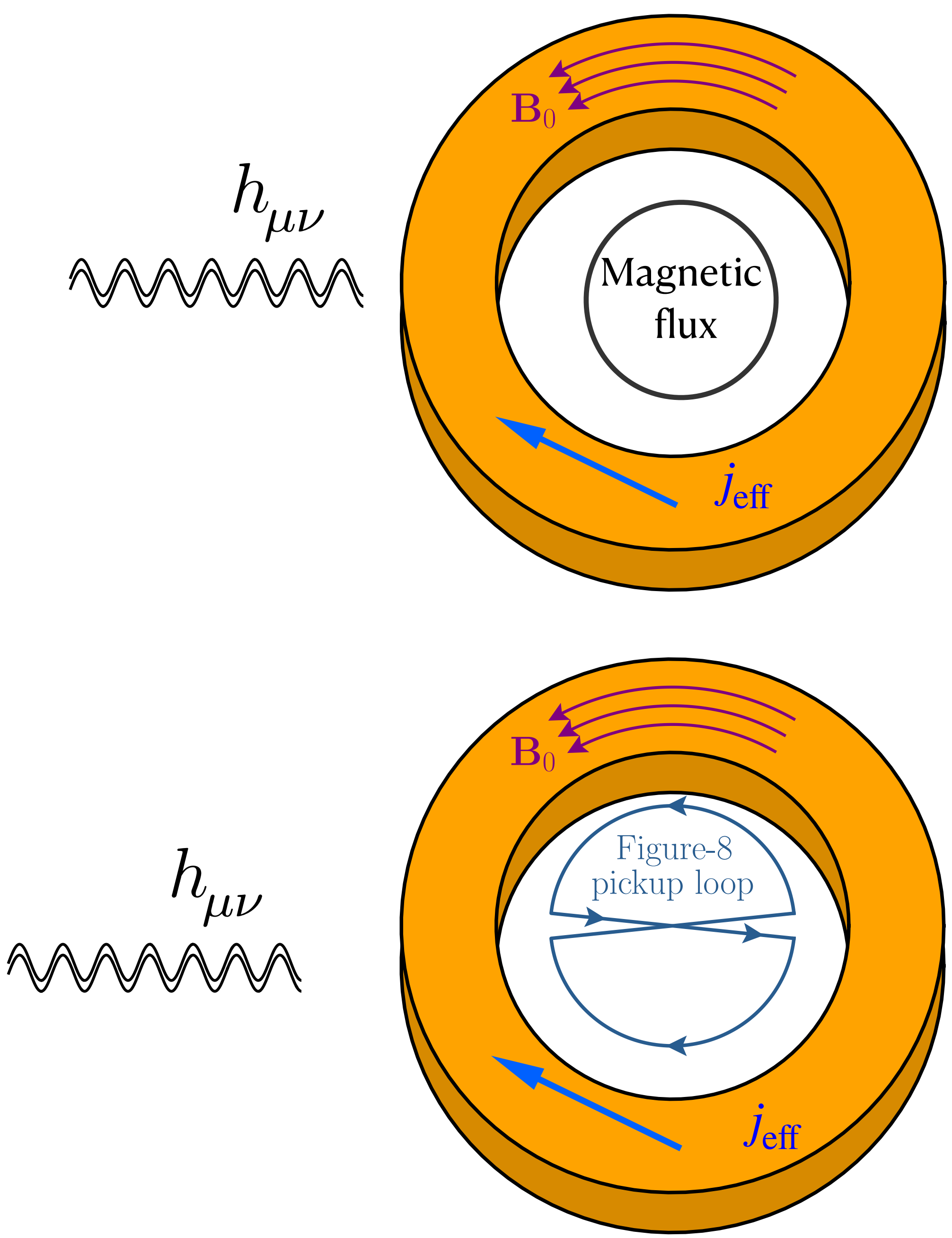
Toroidal magnetic fields



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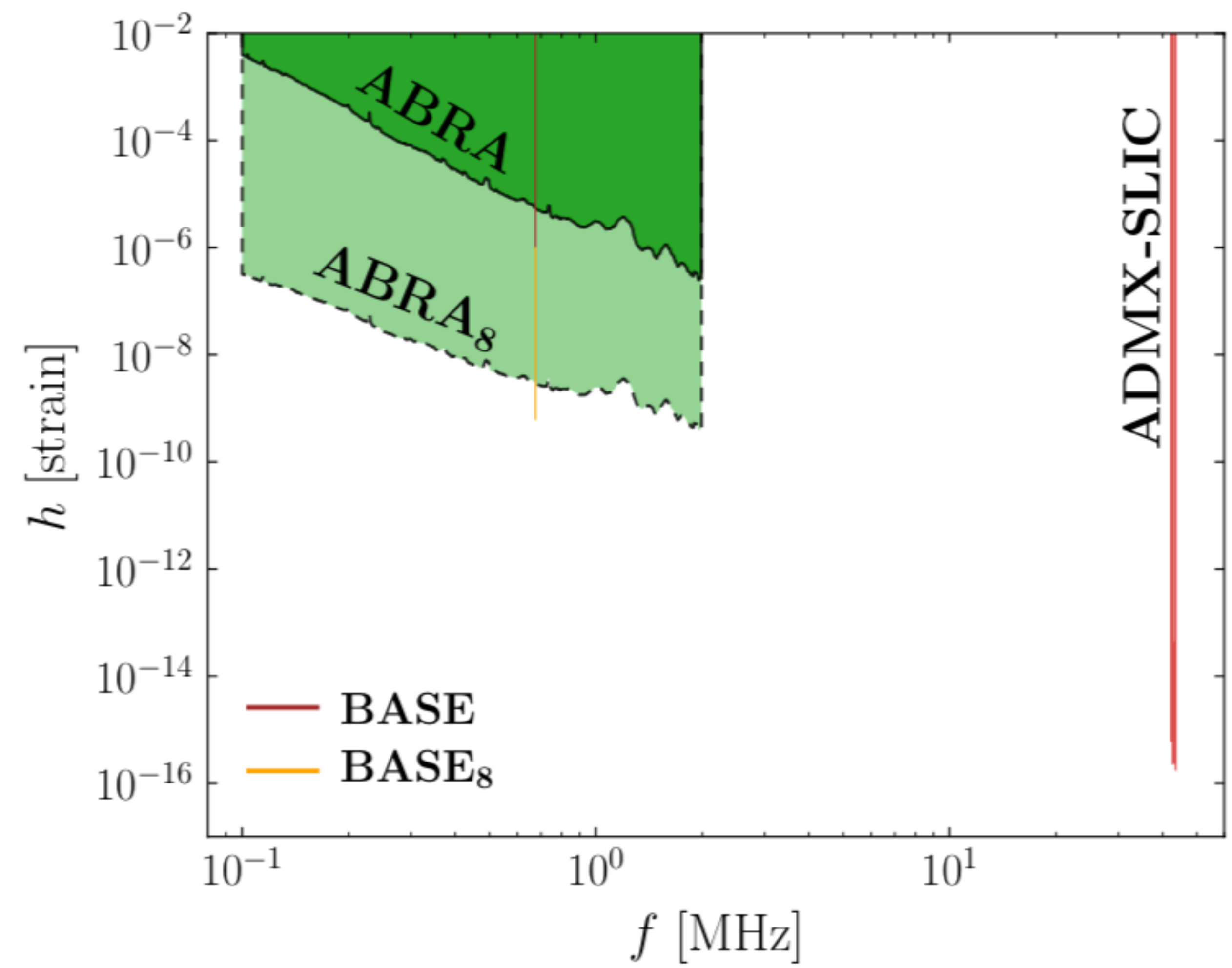
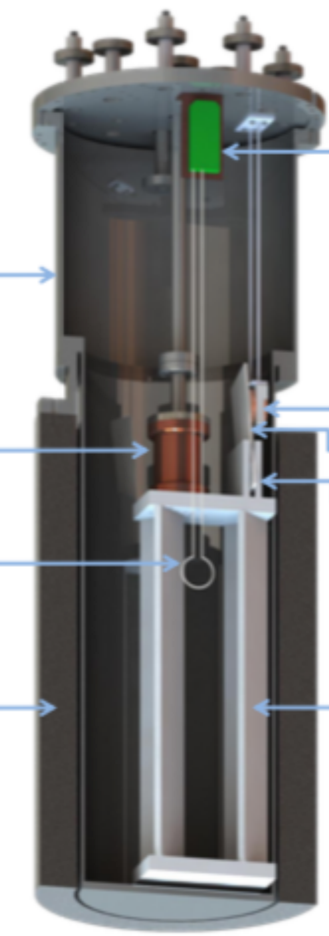


Solenoidal configurations

Domcke, CGC, Lee, Rodd, 2023

ADMX SLIC: Results from a Superconducting *LC* Circuit Investigating Cold Axions

N. Crisosto, P. Sikivie, N. S. Sullivan, D. B. Tanner, J. Yang, and G. Rybka
Phys. Rev. Lett. **124**, 241101 – Published 17 June 2020



BASE

Constraints on the Coupling between Axionlike Dark Matter and Photons Using an Antiproton Superconducting Tuned Detection Circuit in a Cryogenic Penning Trap

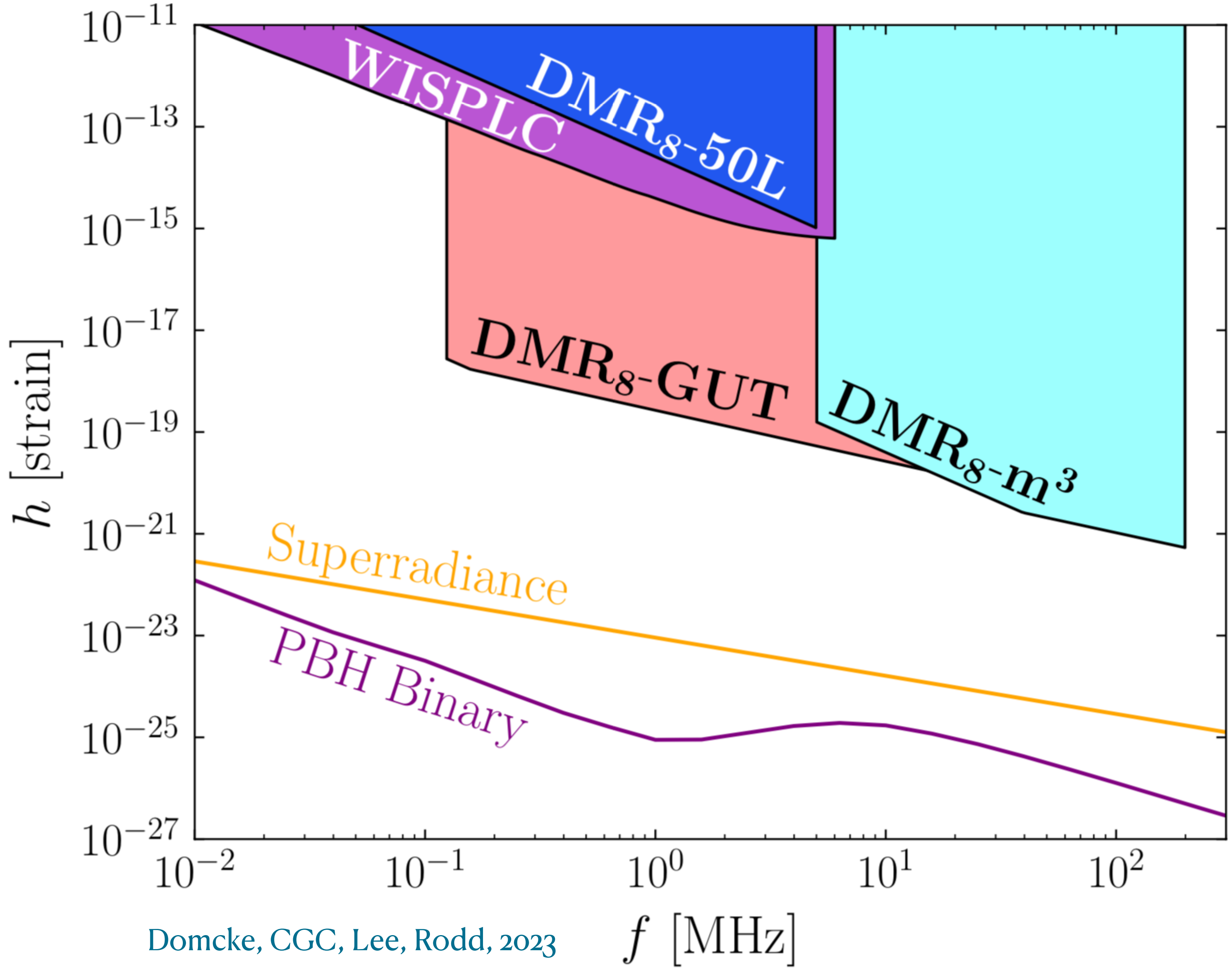
Jack A. Devlin, Matthias J. Borchert, Stefan Erlewein, Markus Fleck, James A. Harrington, Barbara Latacz, Jan Warncke, Elise Wursten, Matthew A. Bohman, Andreas H. Mooser, Christian Smorra, Markus Wiesinger, Christian Will, Klaus Blaum, Yasuyuki Matsuda, Christian Ospelkaus, Wolfgang Quint, Jochen Walz, Yasunori Yamazaki, and Stefan Ulmer
Phys. Rev. Lett. **126**, 041301 – Published 25 January 2021

WISPLC

Search for dark matter with an LC circuit

Zhongyue Zhang (张钟月), Dieter Horns, and Oindrila Ghosh
Phys. Rev. D **106**, 023003 – Published 5 July 2022

Haloscopes based on lumped-element detectors



Domcke, CGC, Lee, Rodd, 2023

Selection rules

Domcke, CGC, Lee, Rodd, 2023

Write down the detector response matrix for a wave coming from an arbitrary direction, and impose **cylindrical symmetry** for both external magnetic field and loop:

Selection Rule 1: For an instrument with azimuthal symmetry, $\Phi_h \propto h^+$ at $\mathcal{O}[(\omega L)^2]$

Selection Rule 2: For an instrument with azimuthal symmetry, the flux is proportional to either h^+ or $h^\times$, but not both. This holds to all orders in (ωL).

Selection Rule 3: For an instrument with full cylindrical symmetry, Φ_h will contain only even or odd powers of ω.

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Novel effects

Effective magnetization and polarization

$$j^{\mu}_{\text{eff}} = \left(-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P}\right)$$

$$\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$$

McAllister et al, 1803.07755
Tobar et al, 1809.01654
Ouellet et al, 1809.10709

$$P_i = -h_{ij}E_j + \frac{1}{2}hE_i + h_{00}E_i - \epsilon_{ijk}h_{0j}B_k$$

$$M_i = -h_{ij}B_j - \frac{1}{2}hB_i + h_{jj}B_i + \epsilon_{ijk}h_{0j}E_k$$

Domcke, CGC, Rodd, 2202.00695

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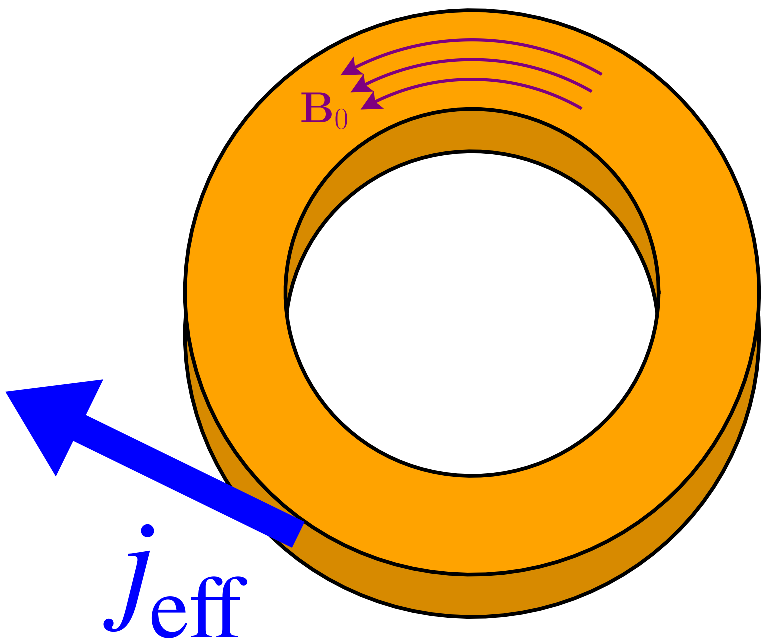
Non-zero *effective* surface currents

Domcke, CGC, Lee, Rodd, 2023



At the interface of two bodies with different values of the magnetisation vector M, Maxwell's equations predict a surface current proportional to $n \times \Delta M$

For axions this happens to vanish, but that is not the case of GWs

Sizeable effects. This should also be relevant for cavities

Conclusions

The techniques developed for detecting axion dark matter could potentially be used to discover new sources of gravitational waves.

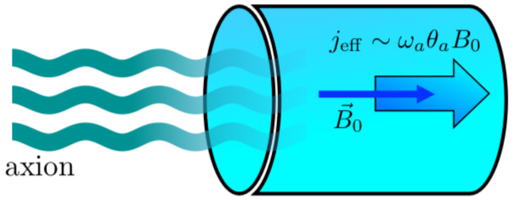
Different experimental proposals have coalesced on a strain sensitivity of 10^{-22} for MHz GWs, still orders of magnitude away from signals of the early Universe.

Lots of room for improvement because experiments are not optimized for gravitational wave searches.

Indeed, theoretical studies indicate that selection rules limit the detectability of gravitational waves in highly symmetric detectors.

Simple modifications of readout (such as the figure-8 pickup loop) can overcome this limitation

Haloscopes based on microwave cavities



It resonates when the axion frequency matches one of the eigenmode frequencies

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\partial_t + \omega_n^2\right) e_n(t) = -\frac{\int_{V_{\rm cav}} d^3\mathbf{x}\mathbf{E}_n^* \cdot \partial_t \boldsymbol{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3\mathbf{x} \left|\mathbf{E}_n\right|^2}$$

Eigenmodes $\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t)\mathbf{E}_n(\mathbf{x})$

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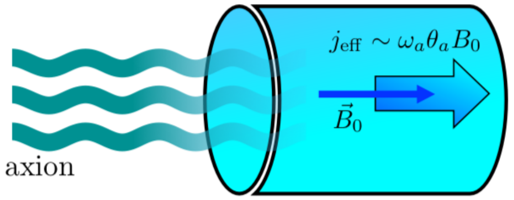
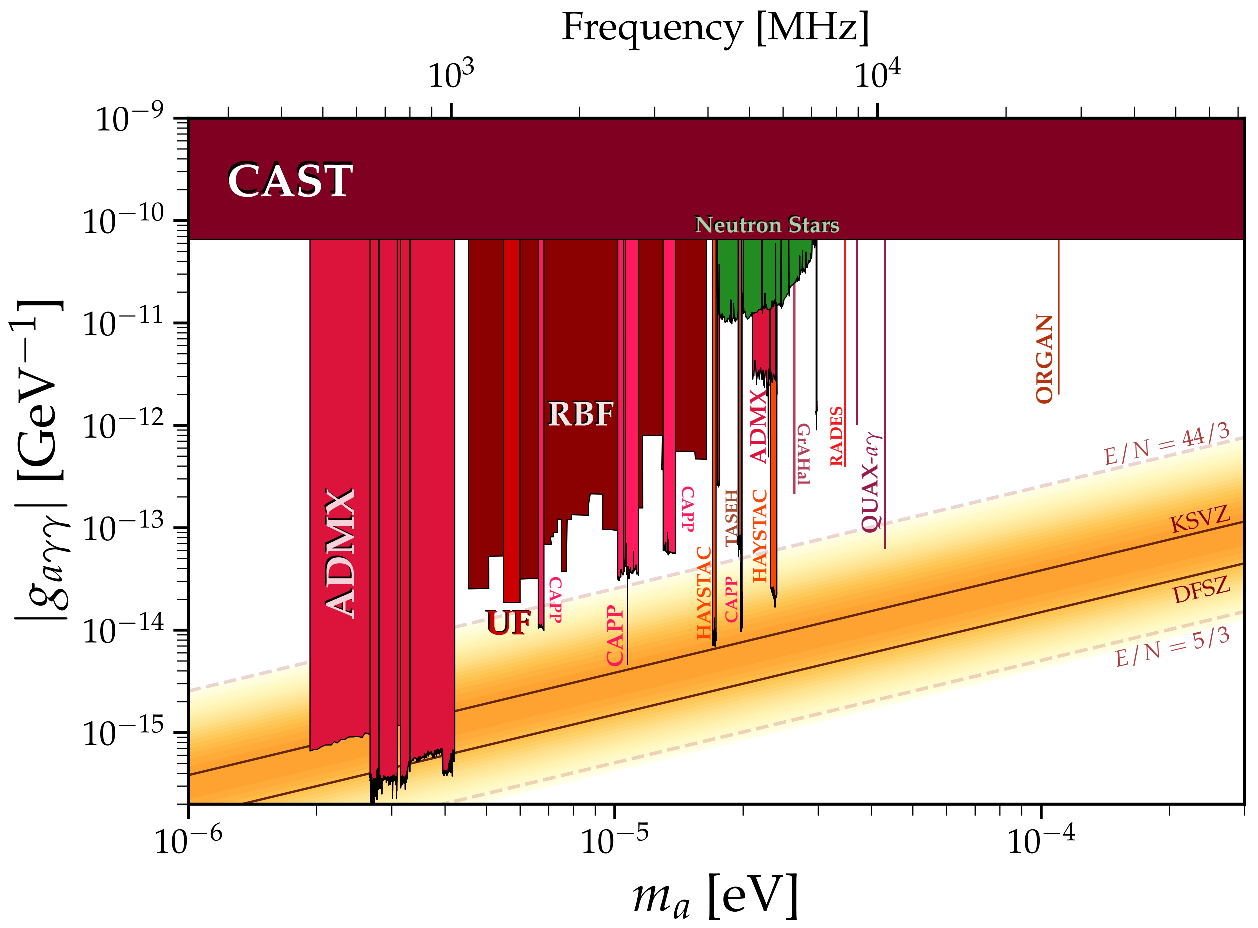
Haloscopes based on microwave cavities





https://github.com/cajohare/AxionLimits

It resonates when the axion frequency matches one of the eigenmode frequencies

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\partial_t + \omega_n^2\right) e_n(t) = -\frac{\int_{V_{\rm cav}} d^3\mathbf{x}\,\mathbf{E}_n^* \cdot \partial_t \boldsymbol{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3\mathbf{x}\,\left|\mathbf{E}_n\right|^2}$$

Eigenmodes $$\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t)\mathbf{E}_n(\mathbf{x})$$

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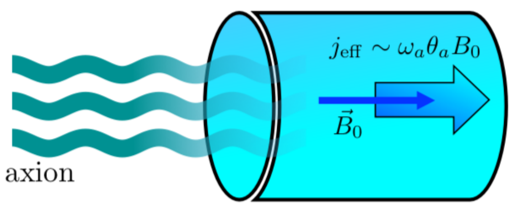
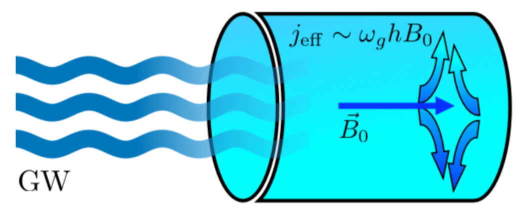
Haloscopes based on microwave cavities





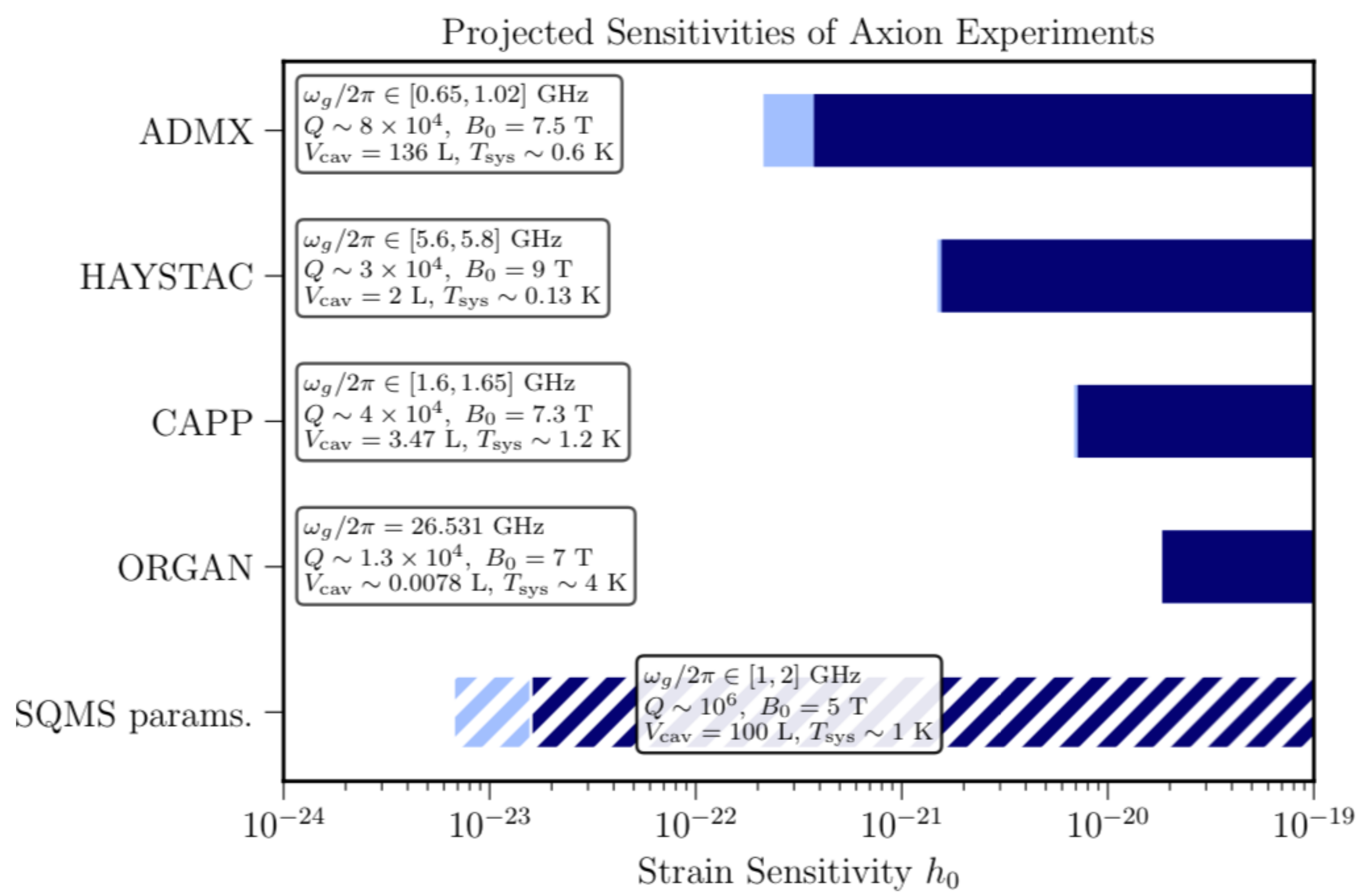
It resonates when the GW frequency matches one of the eigenmode frequencies

Detecting planetary-mass primordial black holes with resonant electromagnetic gravitational-wave detectors

Nicolas Herman, André Fűzfa, Léonard Lehoucq, and Sébastien Clesse
Phys. Rev. D **104**, 023524 – Published 19 July 2021

Detecting high-frequency gravitational waves with microwave cavities

Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo, Sebastian A. R. Ellis, Roni Harnik, Yonatan Kahn, and Jan Schütte-Engel
Phys. Rev. D **105**, 116011 – Published 17 June 2022

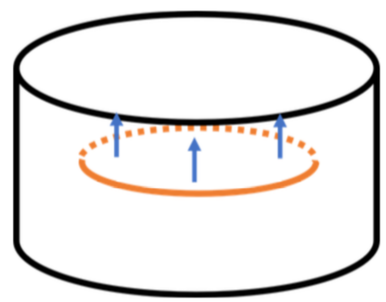
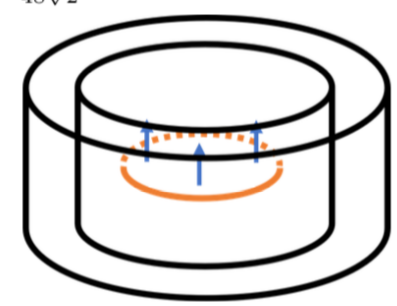
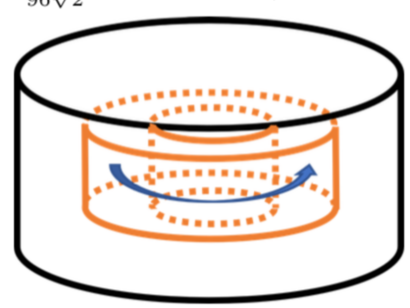
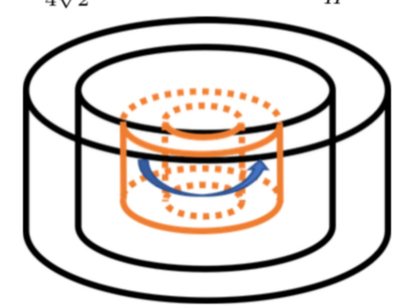
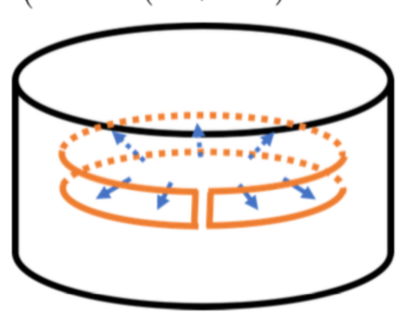
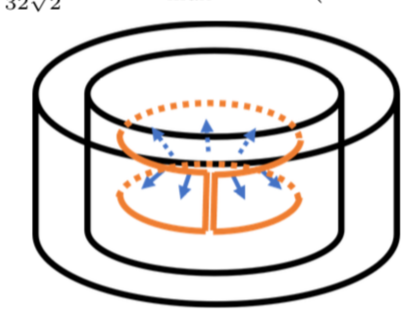


Impact of the geometry

Type of external field

Domcke, CGC, Lee, Rodd, 2023

Pickup loop orientation

	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$	h^+, n even $\Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^+ \omega^2 B_0 s^2_{\theta_h} \pi r^2 \left(11 r^2 + 14 R^2 + 16 R^2 \ln \frac{R}{H}\right)$ 	$h^\times$, n odd $\Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{\mathrm{i} e^{-i\omega t}}{48\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 a R (a + 2R) s^2_{\theta_h}$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$	$h^\times$, n odd $\Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{\mathrm{i} e^{-i\omega t}}{96\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 l (12 R^2 - 5 r^2) s^2_{\theta_h}$ 	h^+, n even $\Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3 e^{-i\omega t}}{4\sqrt{2}} h^+ \omega^2 B_{\max} \frac{\pi r^2 a R l (a+2R)}{H^2} s^2_{\theta_h}$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$	h^+, n odd $\Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{\mathrm{i} e^{-i\omega t}}{96\sqrt{2}} h^+ B_0 \omega^3 c_{\theta_h} s^2_{\theta_h}$ $\times \pi r^2 l \left(3 l^2 - 22 (r^2 + 2R^2) - 36 R^2 \ln \frac{R}{H}\right)$ 	$h^\times$, n even $\Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{e^{-i\omega t}}{32\sqrt{2}} h^\times \omega^4 B_{\max} \pi r^2 a R l (a + 2R) c_{\theta_h} s^2_{\theta_h}$ 

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Proper detector frame

The coordinate system closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922
Manasse and Misner, 1963
Ni and Zimmermamn, 1978

- Coordinates given by ideal rigid rulers

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \text{ for } dx^\mu = (0, dr\,\hat{\mathbf{r}})$$

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- The gravitational wave acts as a Newtonian force.

If negligible, the static fields applied in experiments remain static in the presence of GWs.

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- The gravitational wave acts as a Newtonian force.

  If negligible, the static fields applied in experiments remain static in the presence of GWs.

- Crucial for haloscopes

Berlin et al 2022

Excitation of mechanical modes

The proper detector frame closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922
Manasse and Misner, 1963
Ni and Zimmermamn, 1978

- Coordinates given by ideal rigid rulers

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \ \text{ for } \ dx^\mu = (0, dr\,\hat{\mathbf{r}})$$

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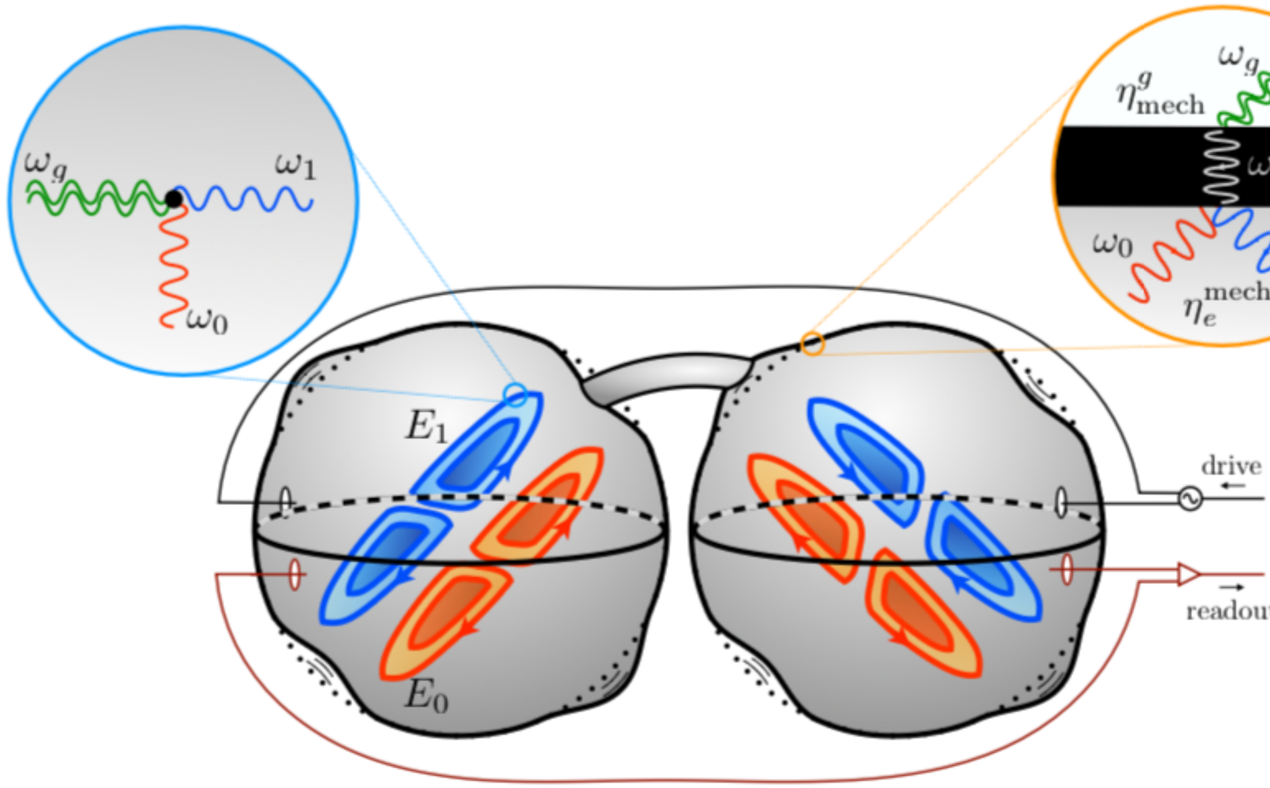
Berlin et al 2022

Excitation of mechanical modes



- The gravitational wave acts as a Newtonian force.
  If not negligible, coupling of the mechanical modes can play an important role (this is certainly the case at frequencies above the first mechanical resonance)
- This can enhance the sensitivity

Berlin et al 2022