NNU·南京师范大学

NANJING NORMAL UNIVERSITY

Gravitational waves from first order phase transitions

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(Nanjing Normal University)

Seminar: National Centre for Nuclear Research (Warsaw)

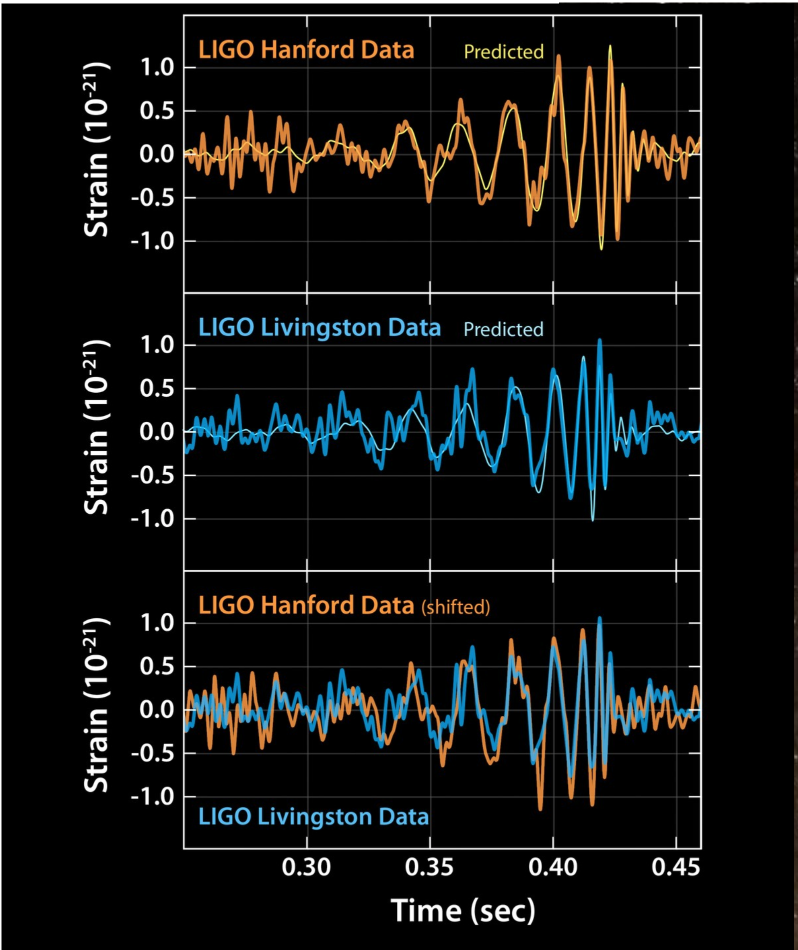
Outline

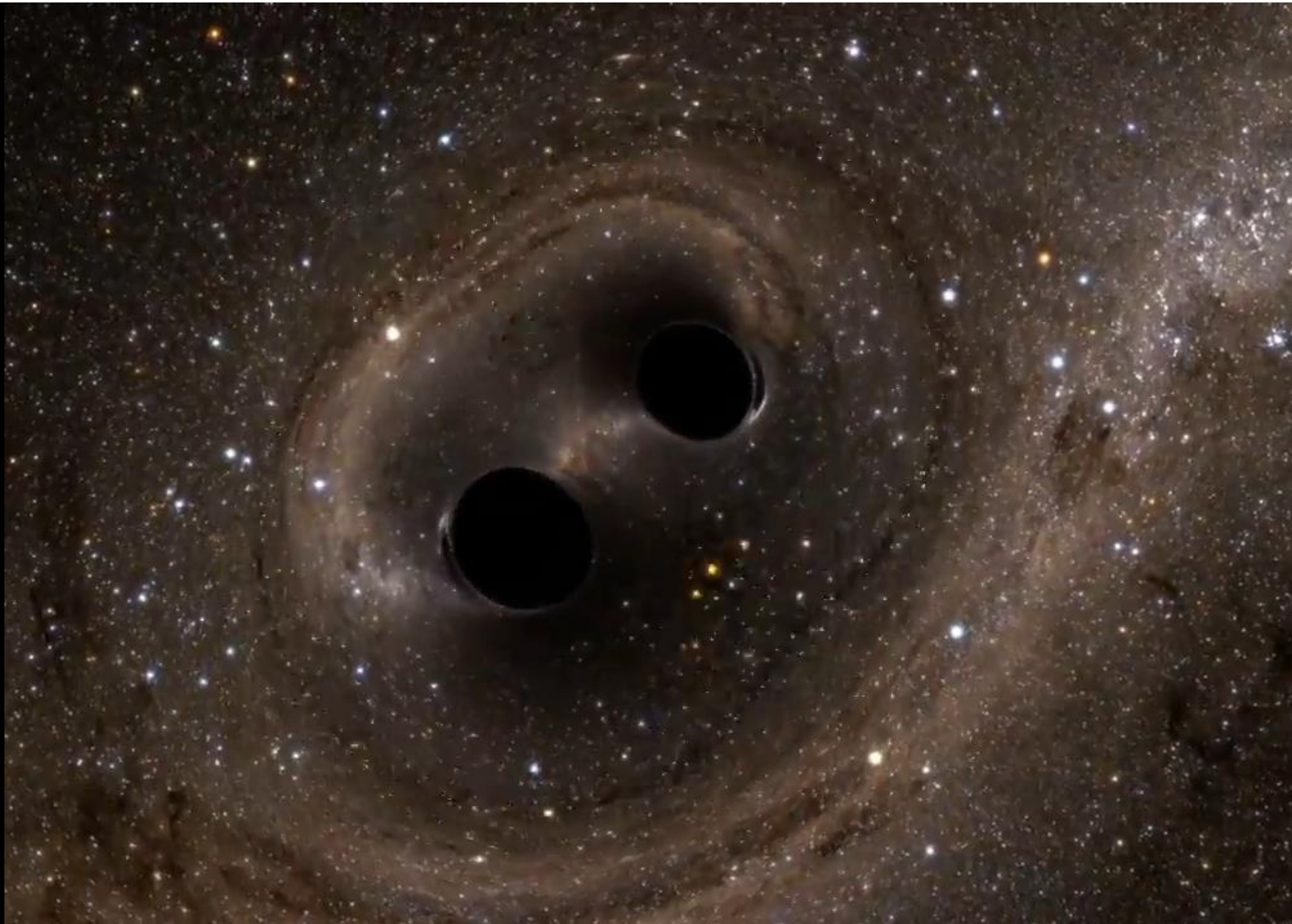
- General overview
  - Motivation: gravitational waves and phase transitions
  - What are first order phase transitions?
  - How FOPTs proceed through bubble nucleation and bubble growth
- Exciting results:
  - First LIGO constraints on a well motivated Pati-Salam GUT
  - Nangrav signal of a SGWB and supercooled phase transitions
- Calculations in depth: state-of-the-art vs approximations
  - Completion criteria and temperature dependence
  - Duration of the phase transition and length scales
  - Kinetic energy fraction and ‘strength’ of the transition

Event GW150914 https://www.ligo.org/detections/GW150914.php

GWs signal detected by the LIGO-Virgo Consortium on 14/09/2015.

The GWs were produced by two coalescing black holes.





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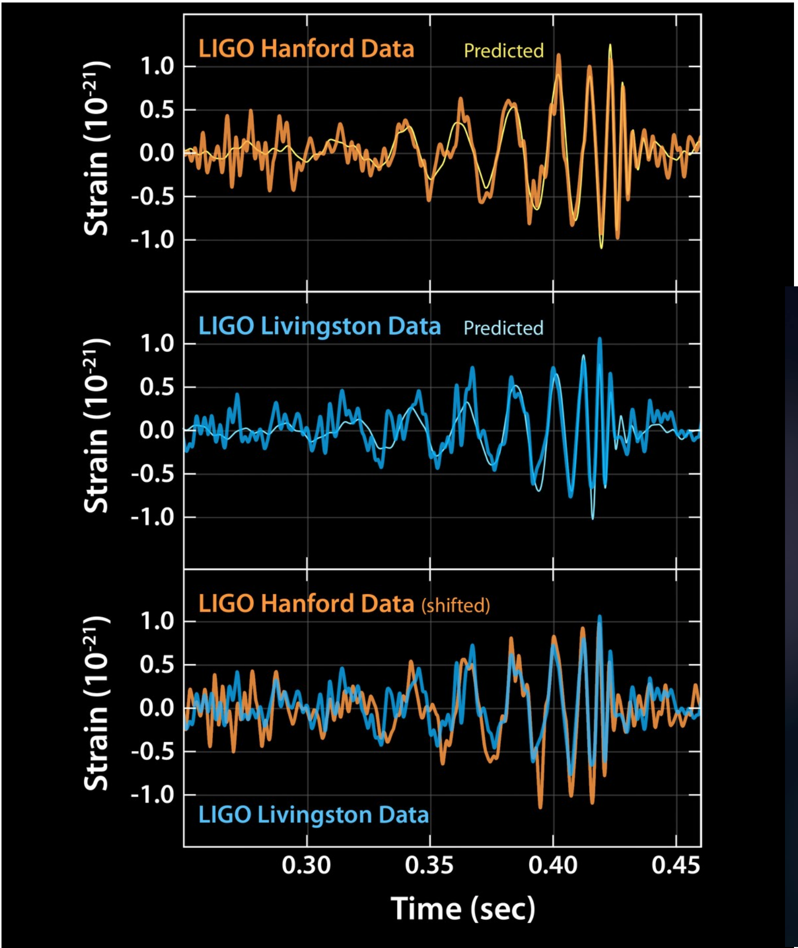
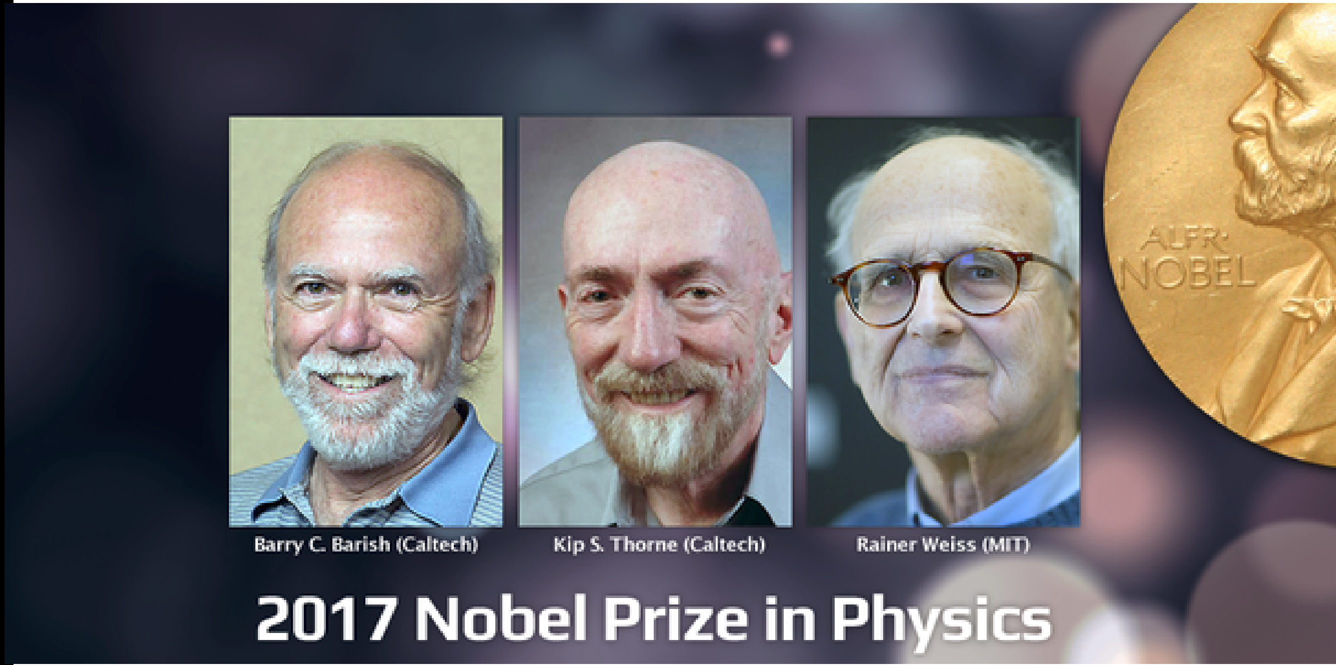
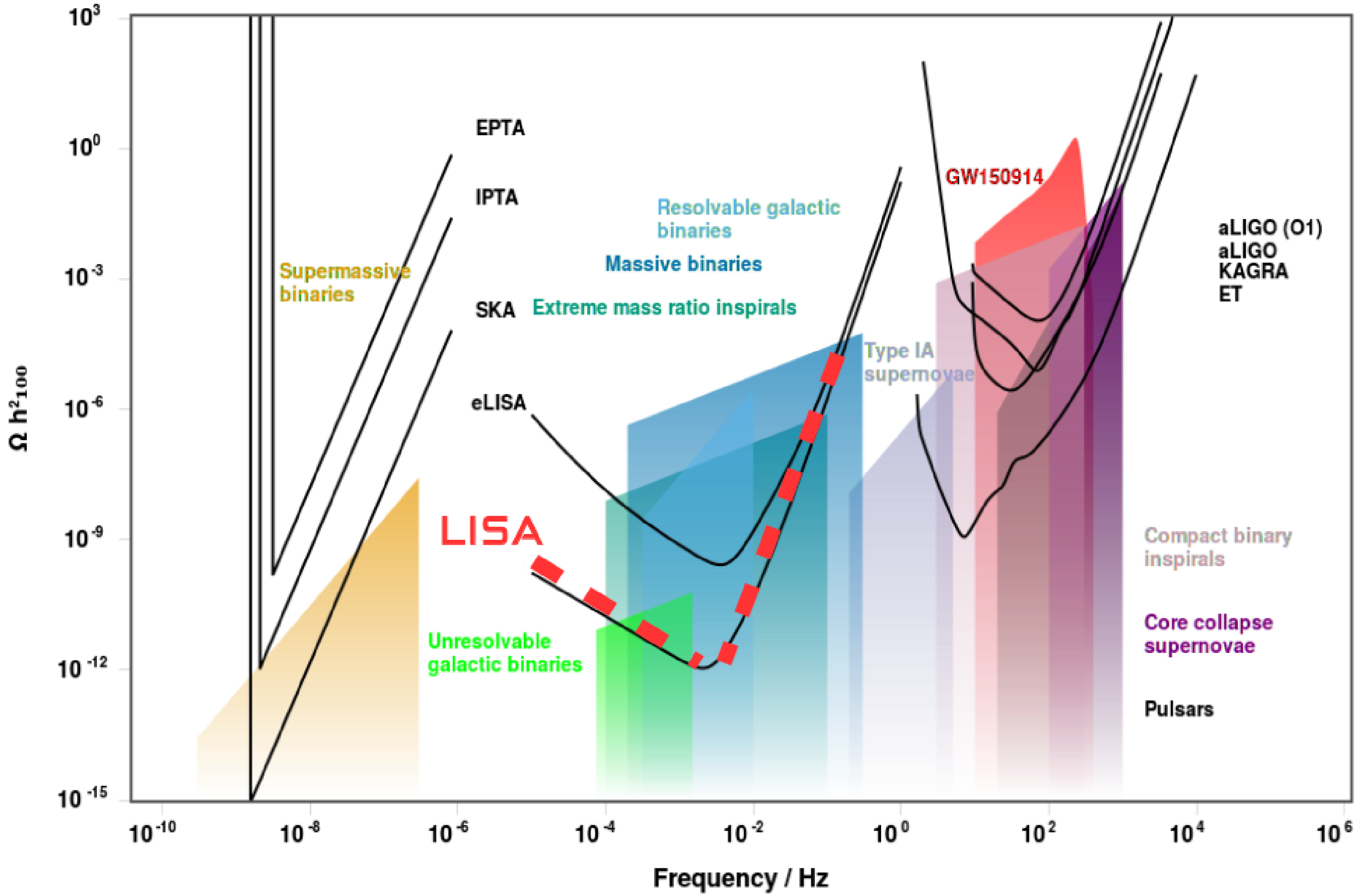


2017 Nobel Prize for physics



Now there are many existing or planned GW experiments covering a wide range of frequencies



[C. Moore, R. Cole, C. Berry, GWplotter]

This has opened up
a whole new way to explore astro-physics

But it is also a huge opportunity
for particle physics and cosmology

Highly energetic events
in the early universe

stochastic gravitational wave
background (SGWB)

Highly energetic events in the early universe $\longrightarrow$ stochastic gravitational wave background (SGWB)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Large fluctuations in the energy density

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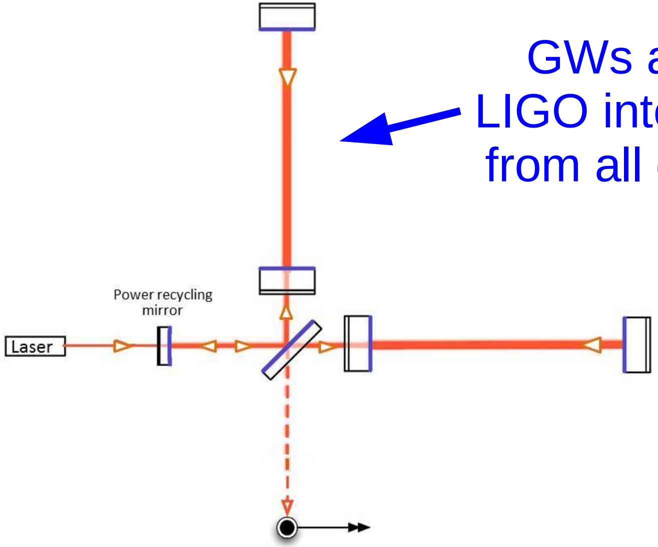

GWs arrive at LIGO inteferometer from all directions

https://www.ligo.caltech.edu/page/ligos-ifo

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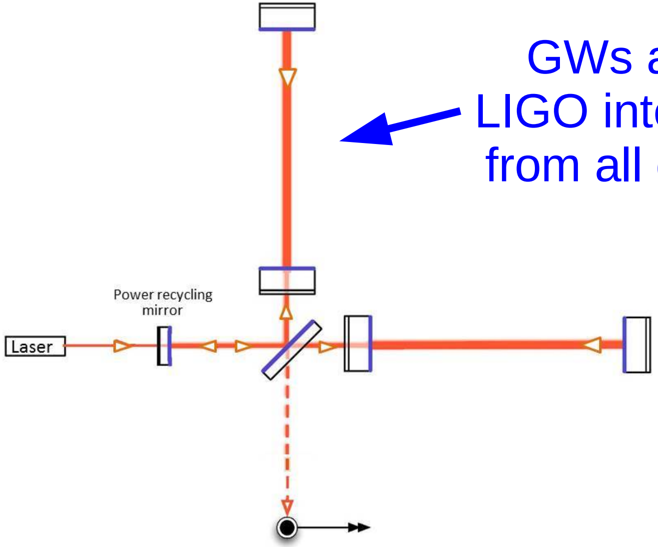

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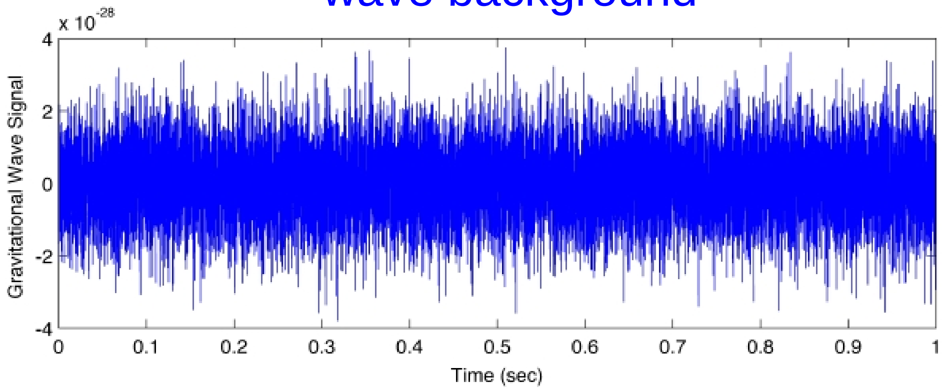

https://www.ligo.org/science/GW-Stochastic.php

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Peak frequency (after redshifting)

$$f_{GW}^{\rm peak} \sim 20\,T \frac{\mu Hz}{(100\,{\rm GeV})}$$

$$T \approx M$$

Planned GW experiments can probe phase transtions at a wide range of energies!

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- Electroweak (EW) phase transition
  - Higgs mechanism: massless $\longrightarrow$ massive
- QCD phase transitions
  - Quark-gluon plasma $\longrightarrow$ gas of confined Hadrons
  - Chiral symmetry breaking (left and right handed particles)

In SM of particle physics these are cross-overs

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- If first order it may have an EW baryogenesis explanation of the matter anti-matter asymmetry we observe
- More freedom for modifying SM to make this a first order phase transition with observable GWs

*or cross-over transition

Cosmological Phase Transitions

Phase transitions are also predicted in many ideas, e.g.

- Electroweak (EW) phase transition
- QCD phase transitions
- Grand Unified theories
- String inspired models
- Left-right symmetric models
- Gauge extensions of the standard model of particle physics

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I will discuss first order cosmological phase transitions
and their gravitational wave predictions in general

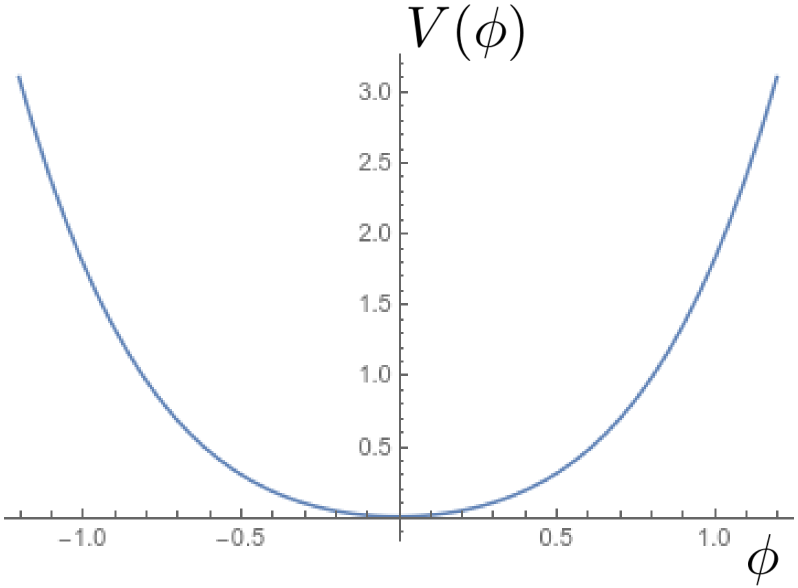
What do I mean by

First Order Phase Transition?

Higgs mechanism

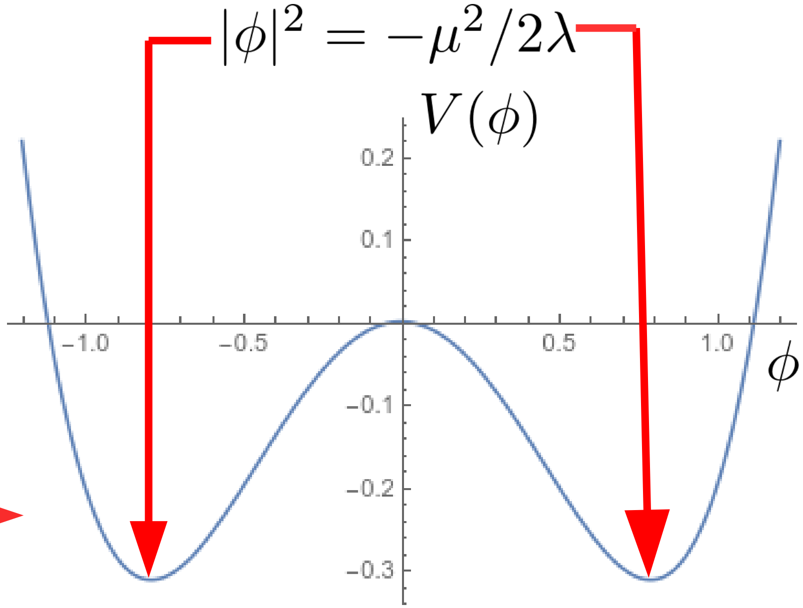
$$V(\phi) = \mu^2(\phi\phi^*) + \lambda(\phi\phi^*)^2$$

If $\mu^2 > 0$



If $\mu^2 < 0$

Mexican Hat Potential



This is really just the zero temperature shape of this Higgs potential

Electroweak Phase Transitions

Finite temperature potential: $\mathrm{V} = \mathrm{V}(\phi, T) = \mu^2|\phi|^2 + \lambda|\phi|^4 + V_T$

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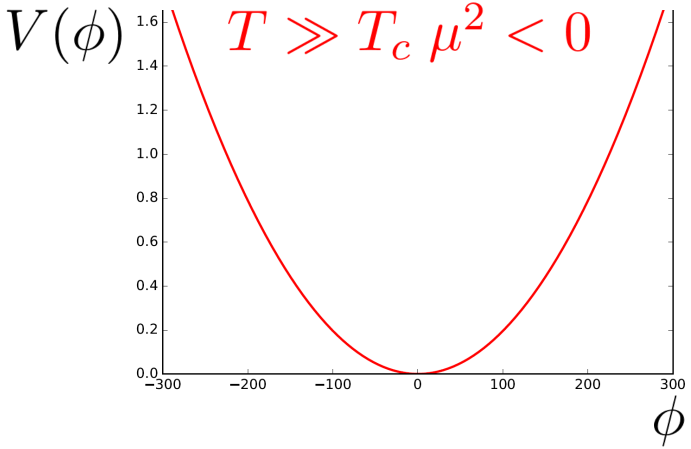
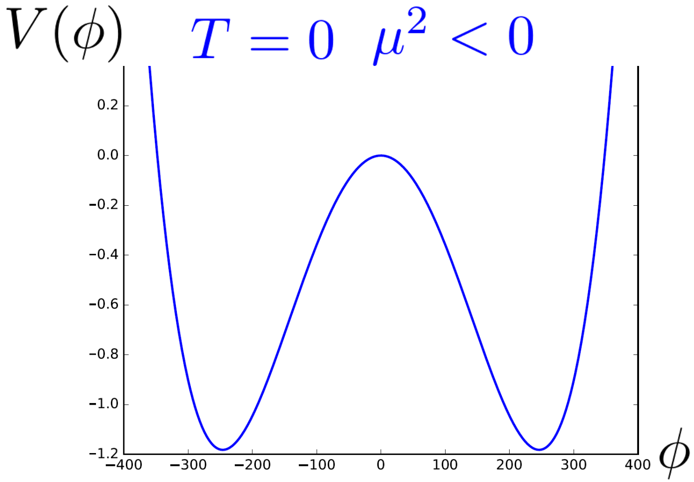
$T \gg \phi \;\; \mu^2 < 0 \Rightarrow V_T \sim T^2|\phi|^2$ (High temperature expansion)





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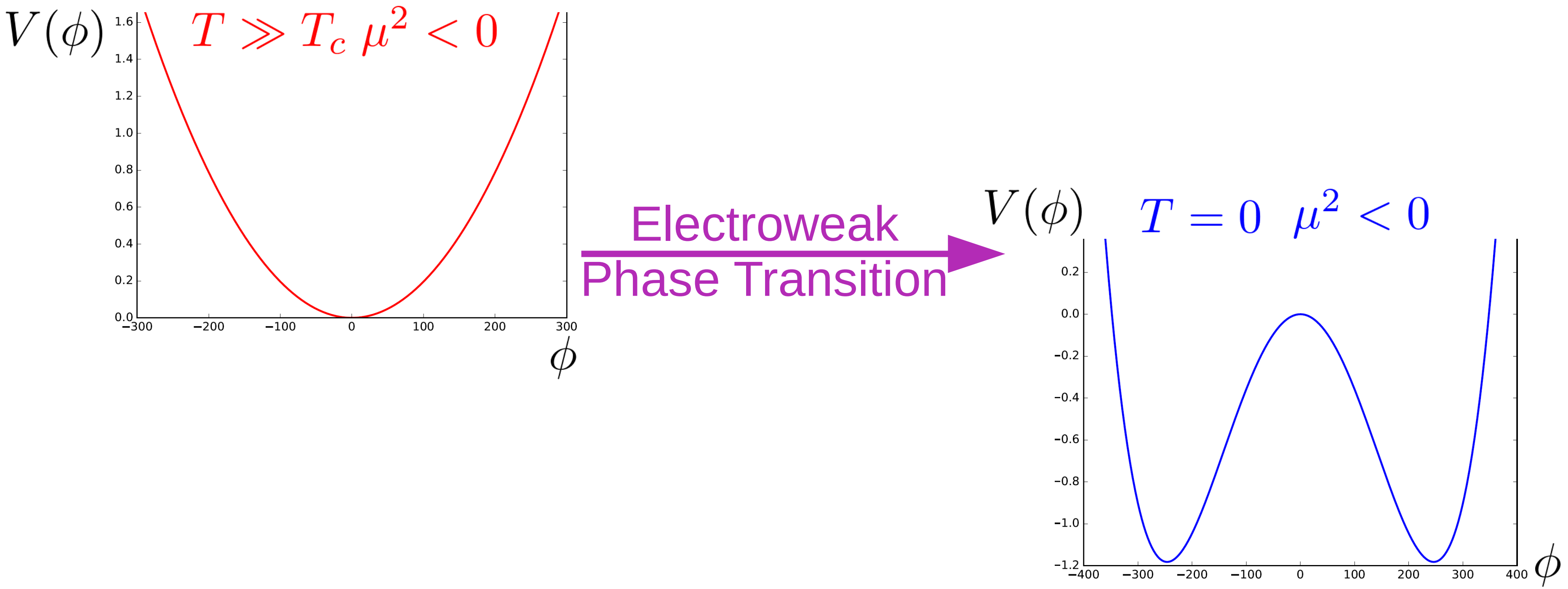
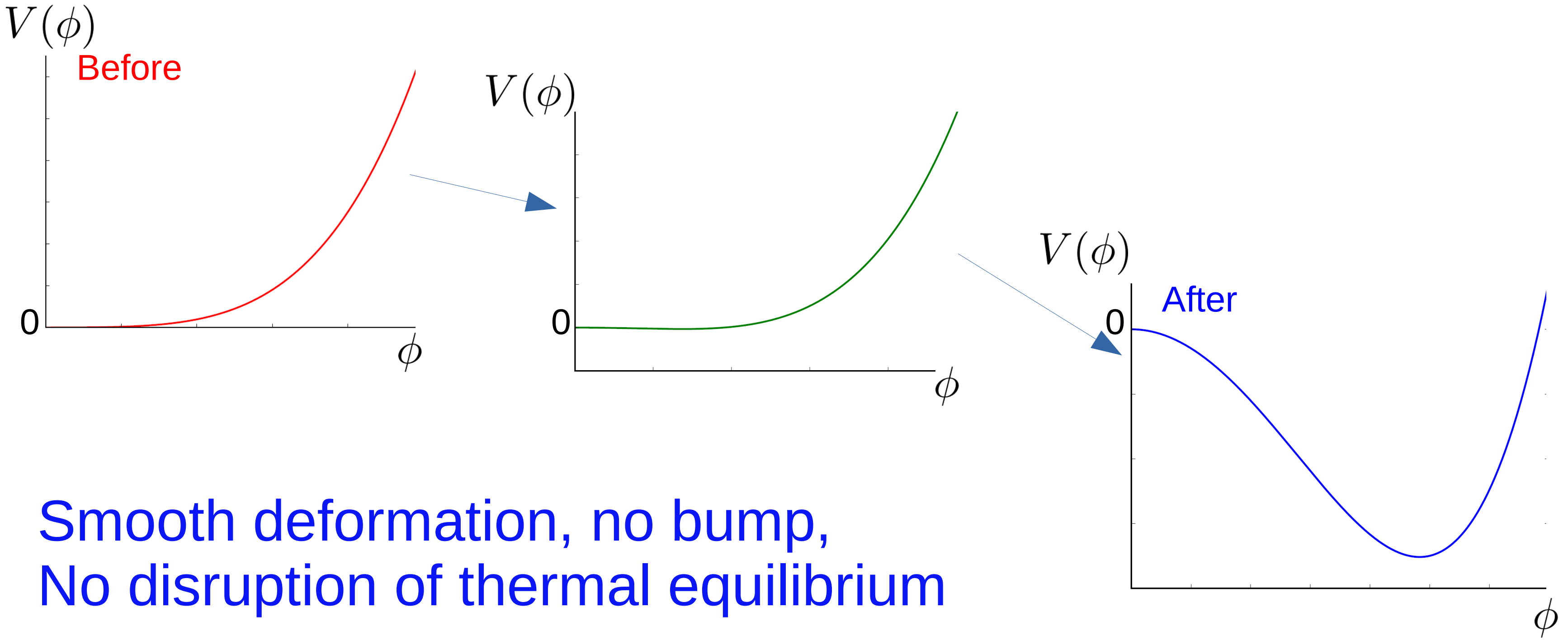


Electroweak Phase Transitions

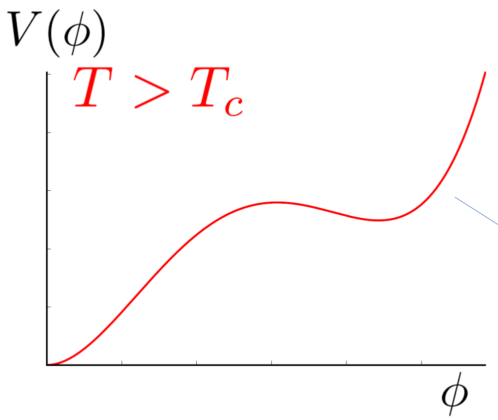
Finite temperature potential: $\mathrm{V} = \mathrm{V}(\phi, T)$

The phase transition can proceed in different ways, e.g.

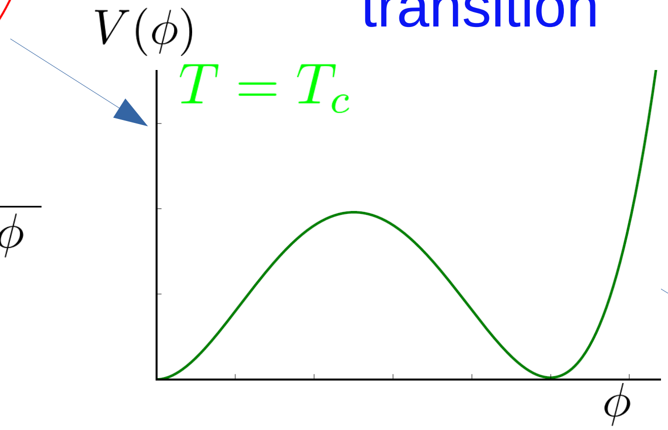


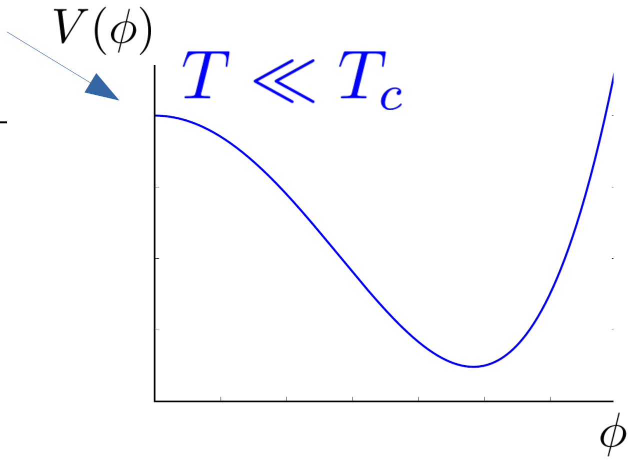
Smooth deformation, no bump,
No disruption of thermal equilibrium

First Order Phase Transitions



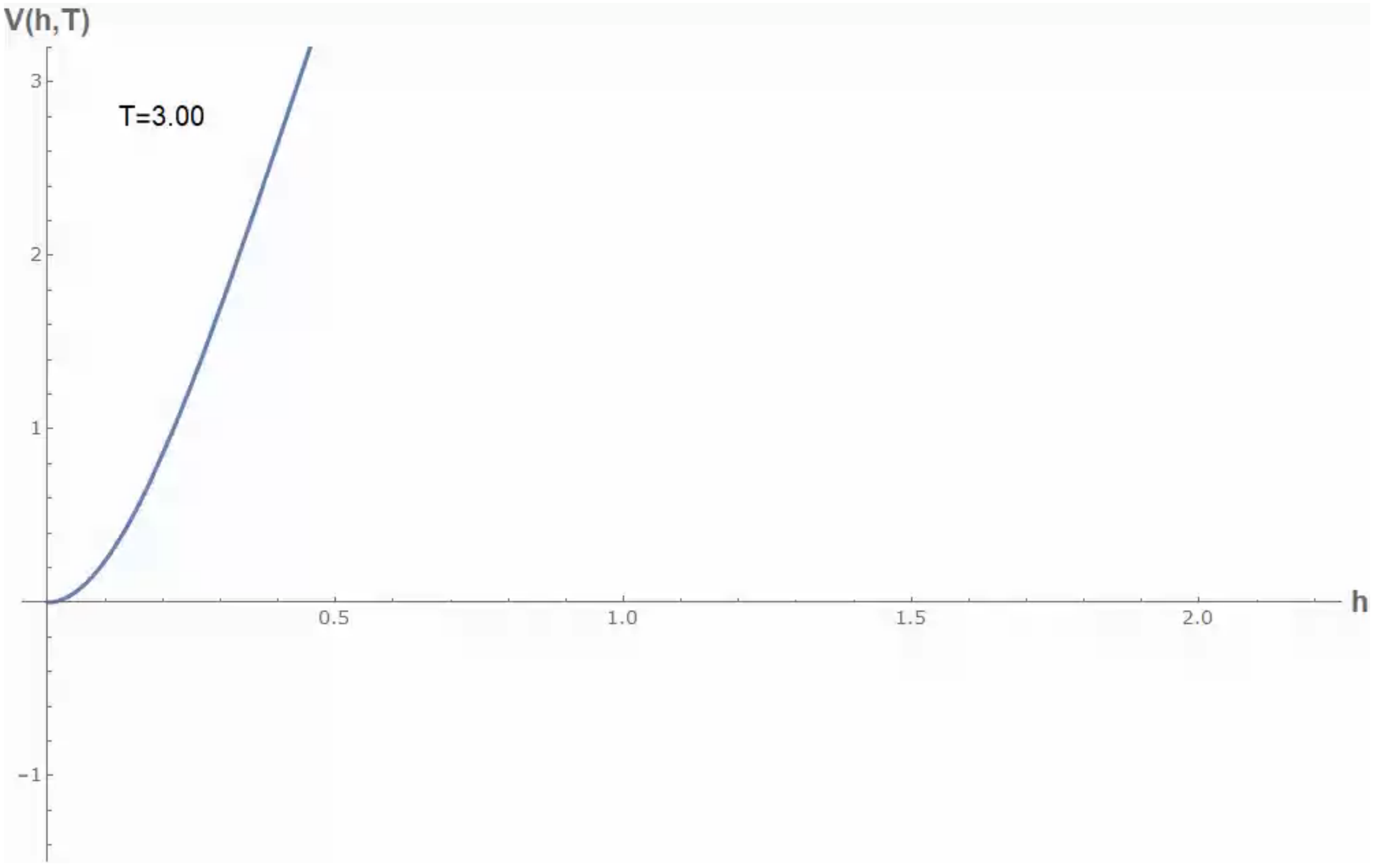
In a first order phase transition a barrier seperates the two phases during the phase transition





$$T_c = \text{critical temperature}$$
$$\equiv \text{Temperature where } V \text{ at minima are degenate}$$

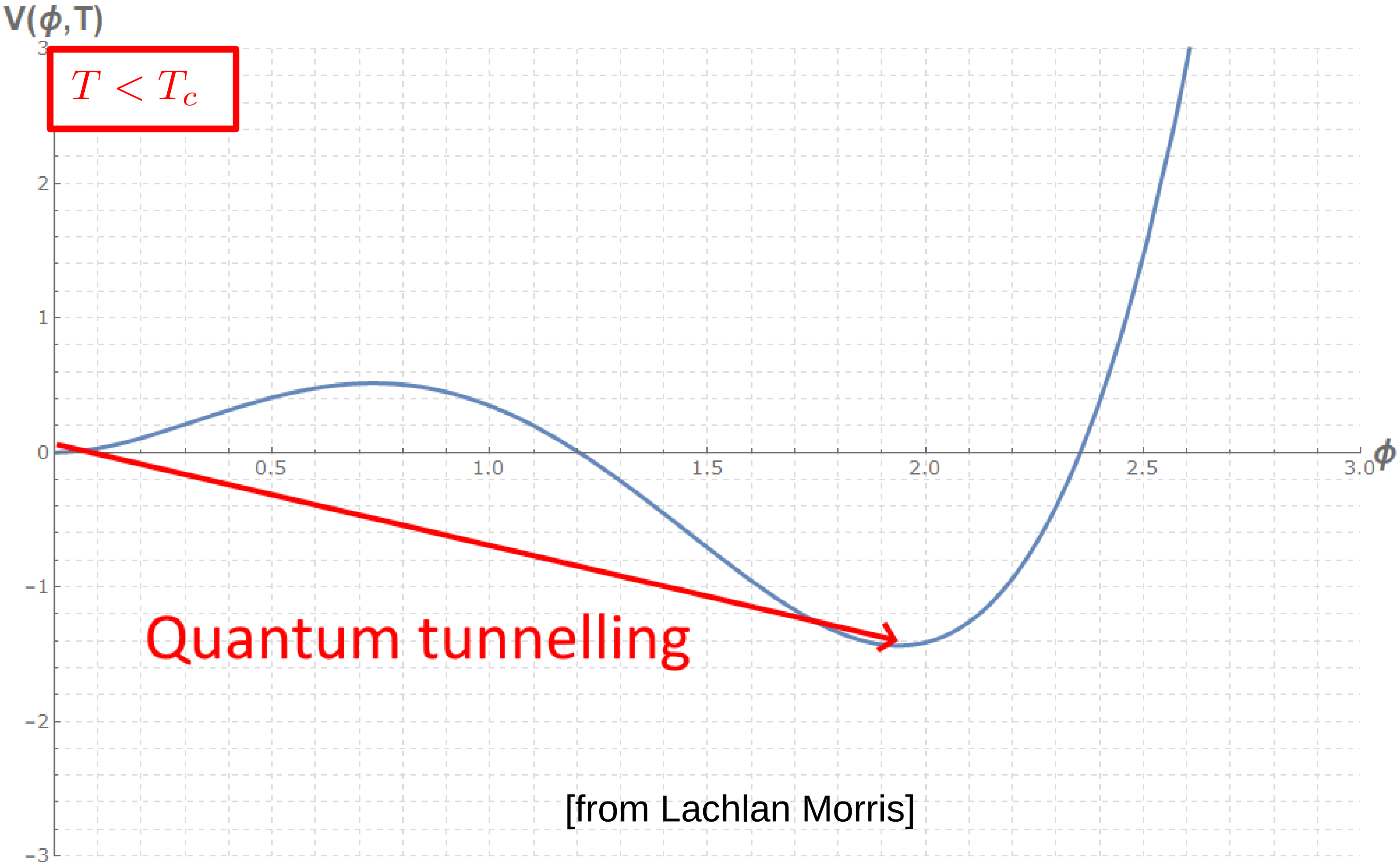
Temperature evolution



[Gif from Lachlan Morris]

Quantum tunnelling

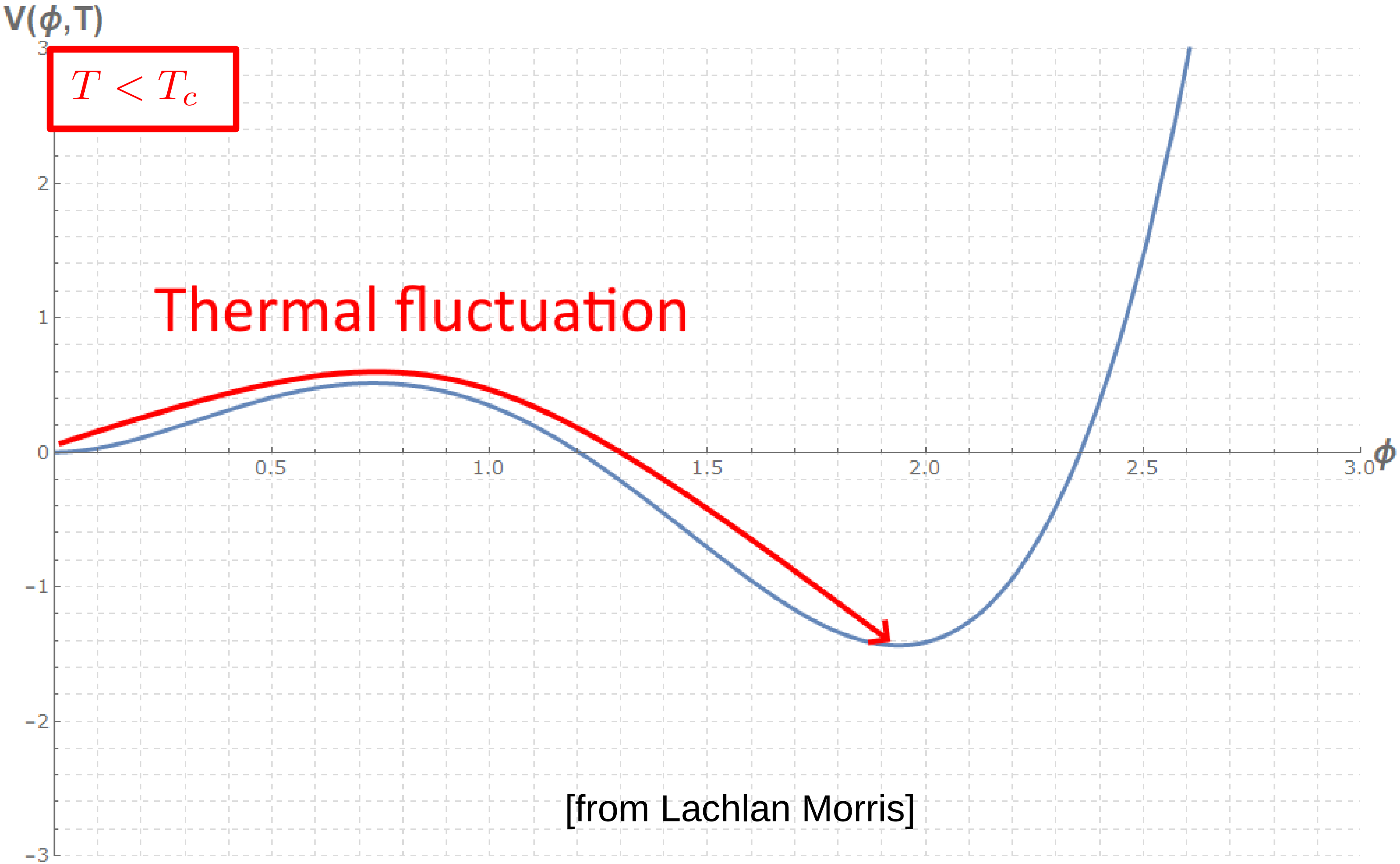
Quantum tunneling through the barrier is now possible



[from Lachlan Morris]

Thermal fluctuation

Thermal fluctuations over the barrier are also possible



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First Order Phase Transitions and Bubble Nucleation

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When they become possible

quantum tunneling or thermal fluctuations over the barrier

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At any point where this happens a bubble of the new phase will form

Bubble nucleation

Bubbles of the new phase form at random locations

[image: from Lachlan Morris]

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The bubbles that already formed grow in size

while more bubbles nucleate

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[image: from Lachlan Morris]

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And more and more of the space is
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Until almost all the space is in the
true vacuum

Gravitational Waves
from
first order phase transitions

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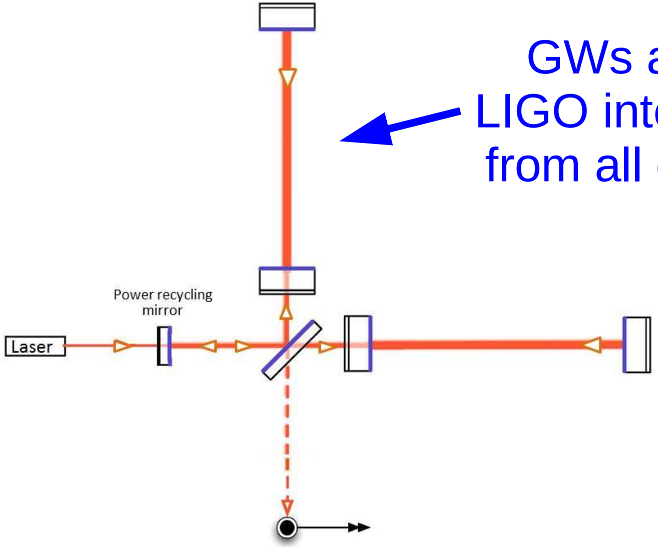
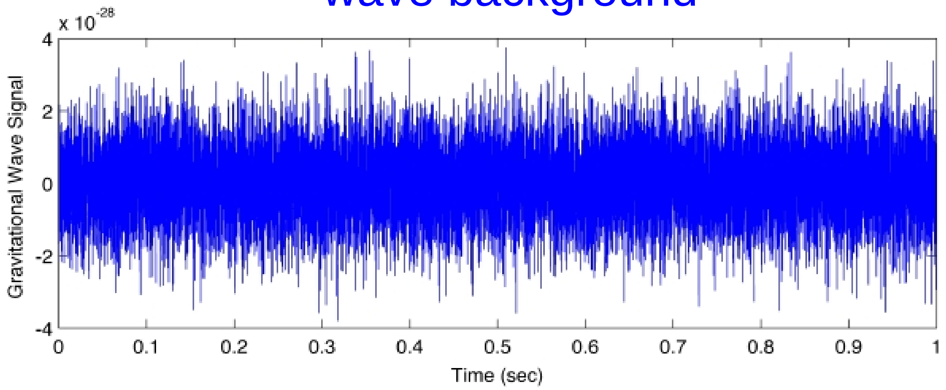


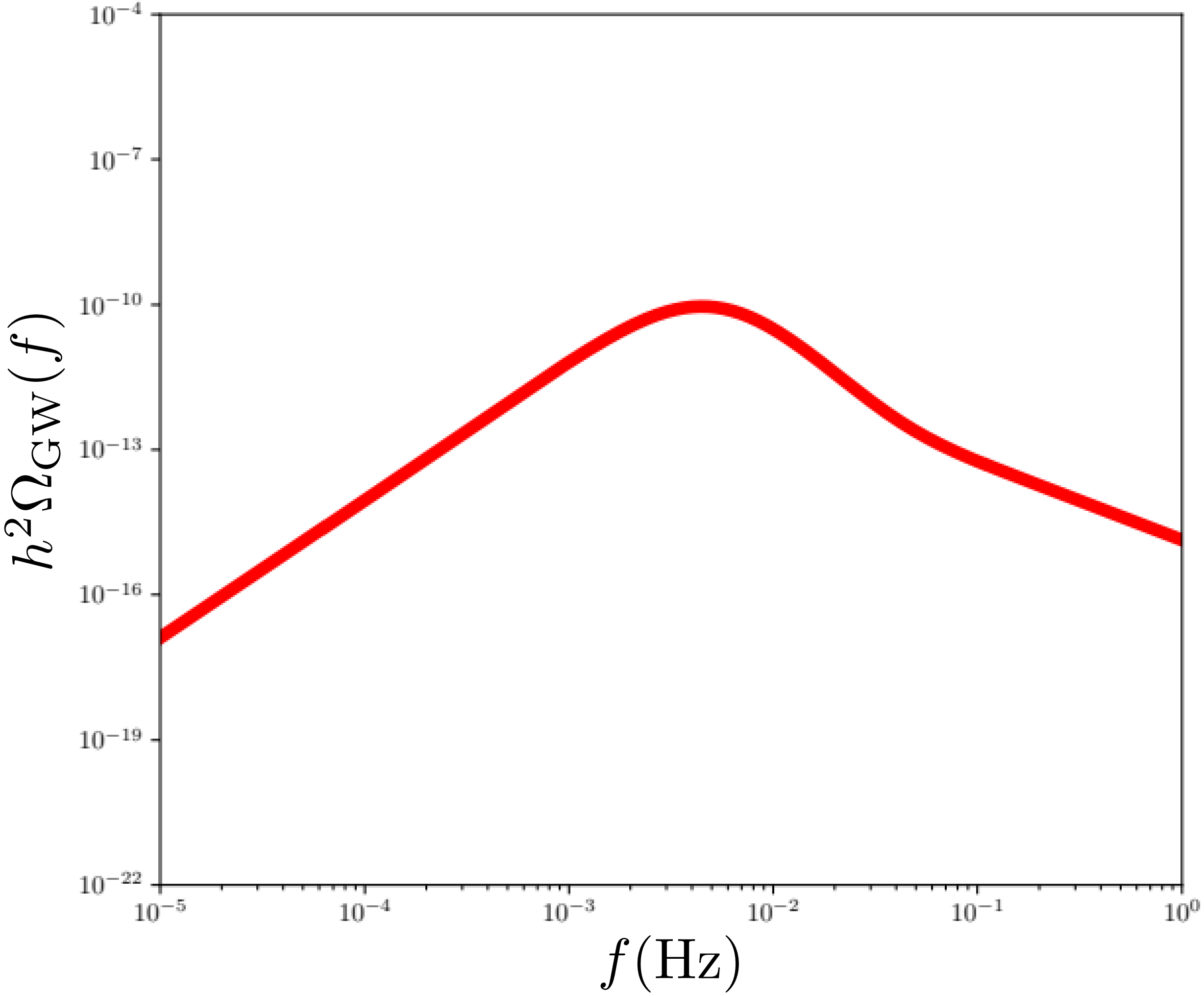
https://www.ligo.caltech.edu/page/ligos-ifo

Stochastic gravitational wave background



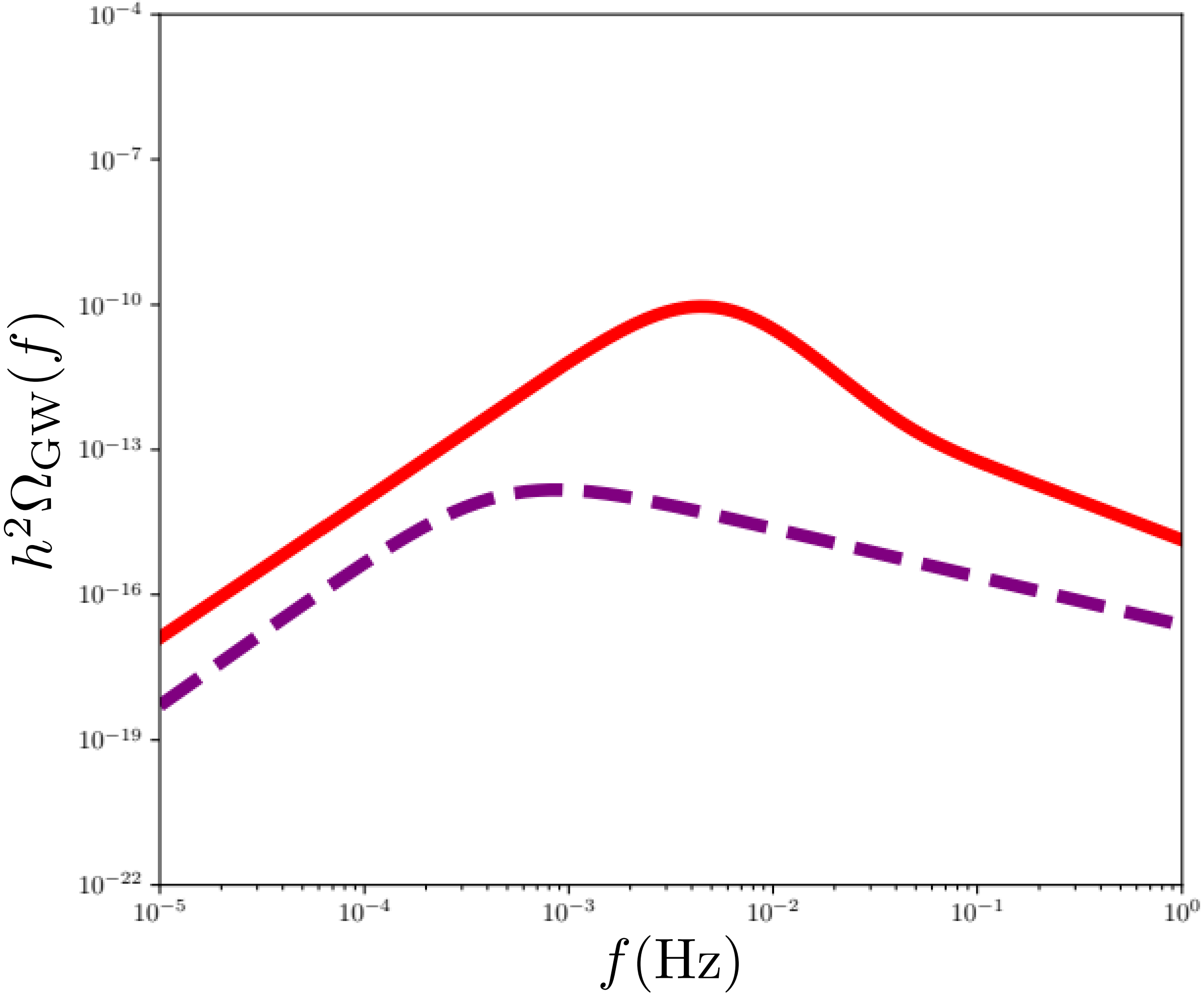
https://www.ligo.org/science/GW-Stochastic.php

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln f}$$



The peak amplitide varies with the frequency

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_{\rm tot}} \frac{d\rho_{\rm GW}}{d\ln f} \qquad h^2\Omega_{\rm GW} = h^2\Omega_{\rm coll} + \ldots$$



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The signal has several contributions

1) the collision of bubbles – which breaks their spherical symmetry.

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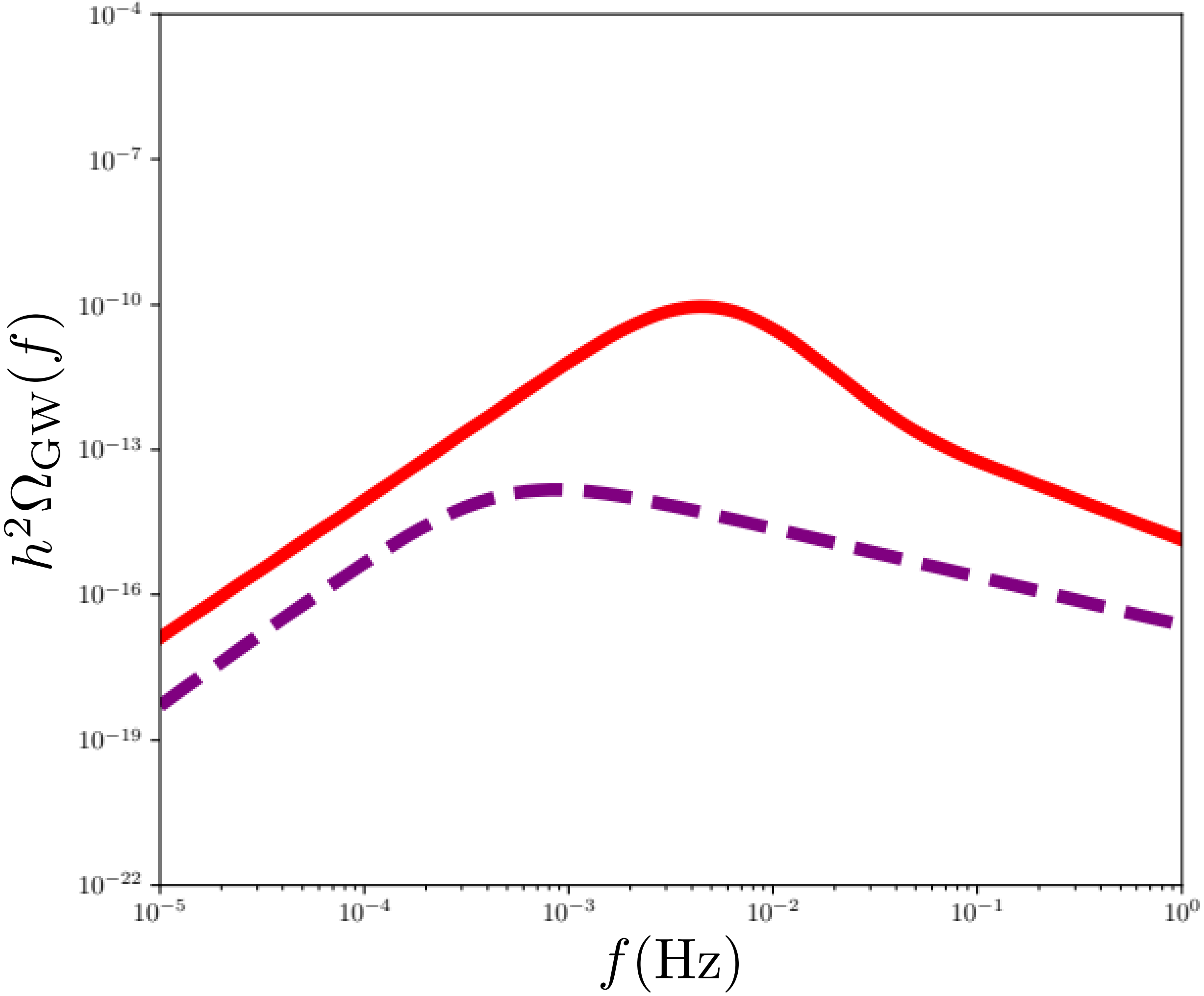

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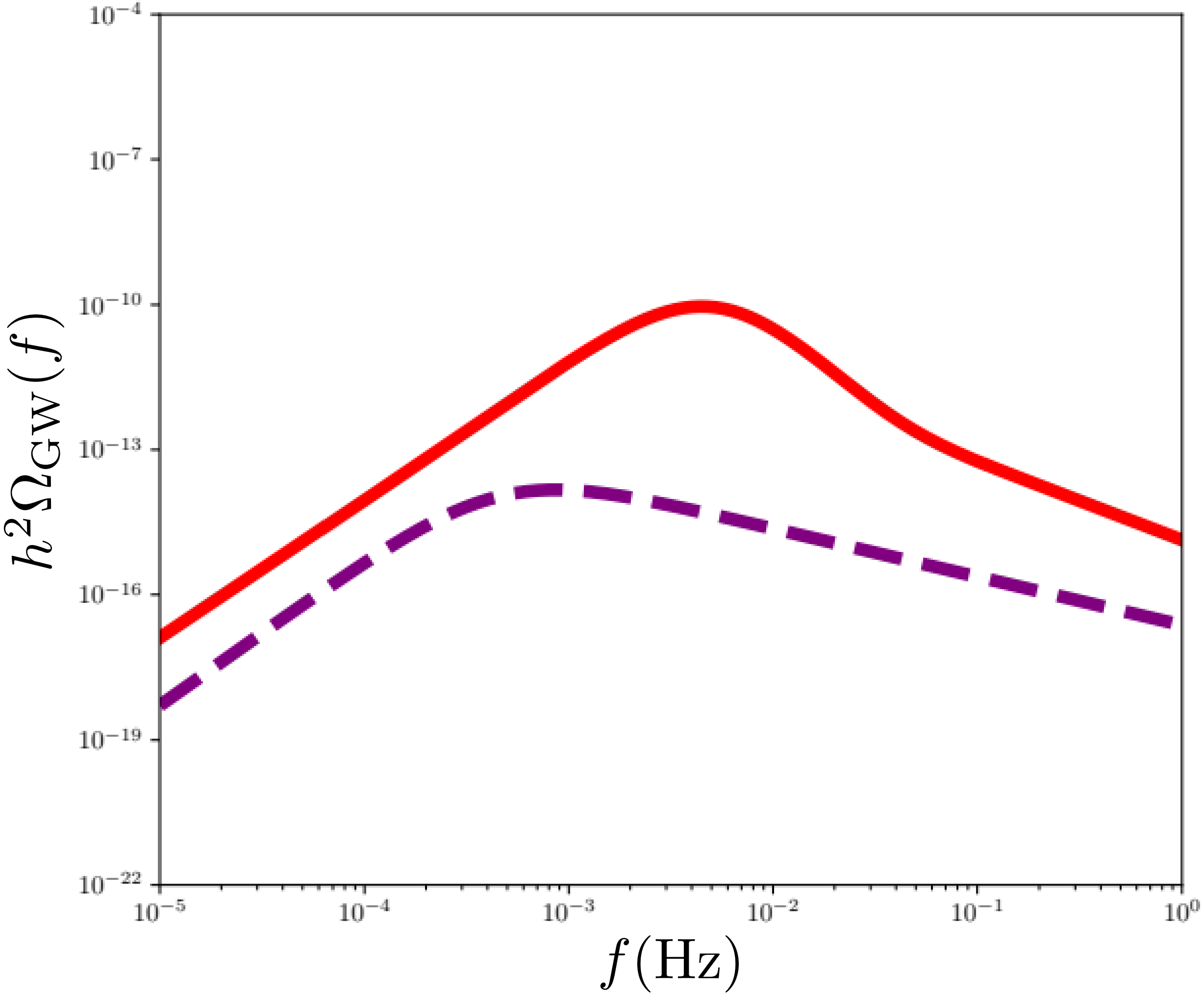

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Exceptions: very low temp PTs, Secluded/dark sectors PTs

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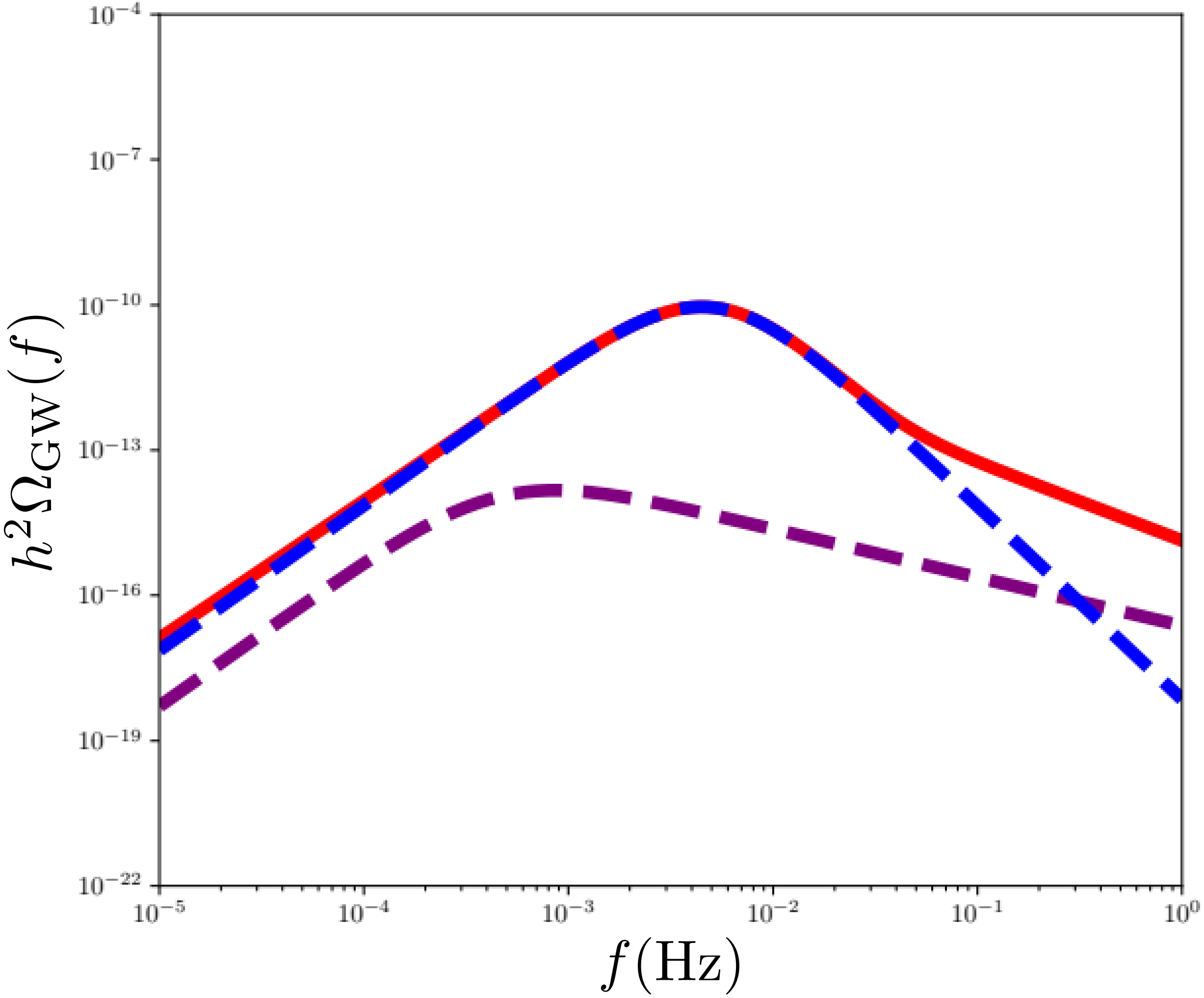

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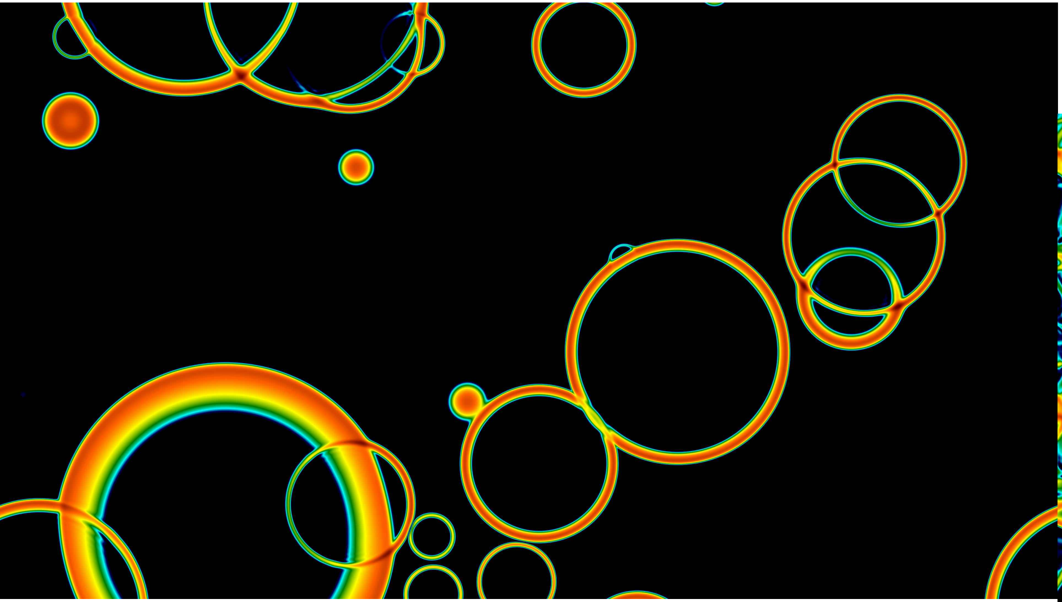
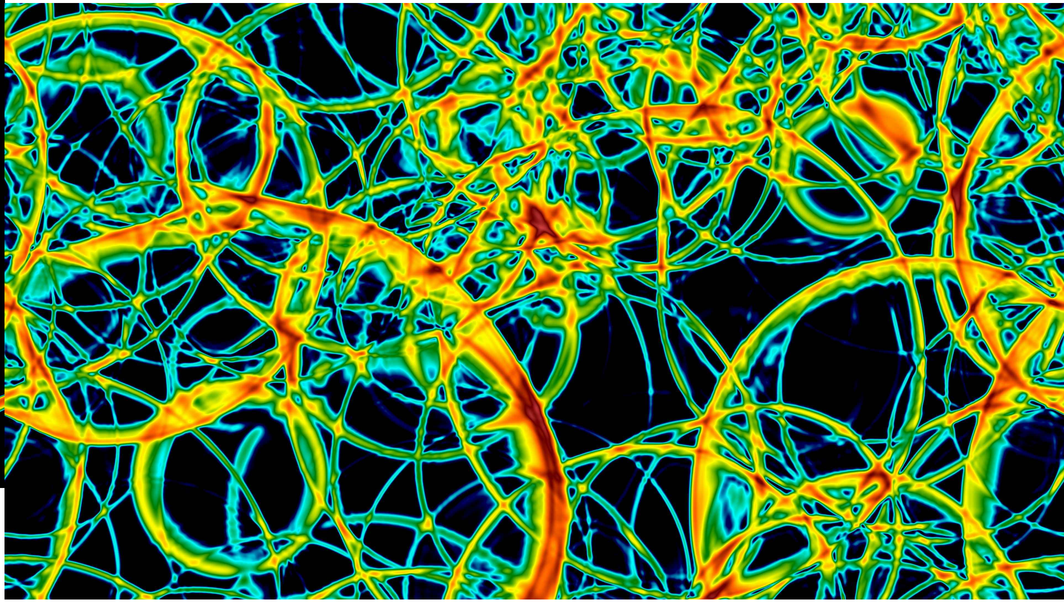
Kinetic energy of the plasma

Slices from 3D lattice simulations [],

Before collisions:



After collisions:



Kinetic energy distributed in shells around the bubble wall wall

[Images David Weir, Phil.Trans.Roy.Soc.Lond.A 376 (2018) 2114, 20170126]

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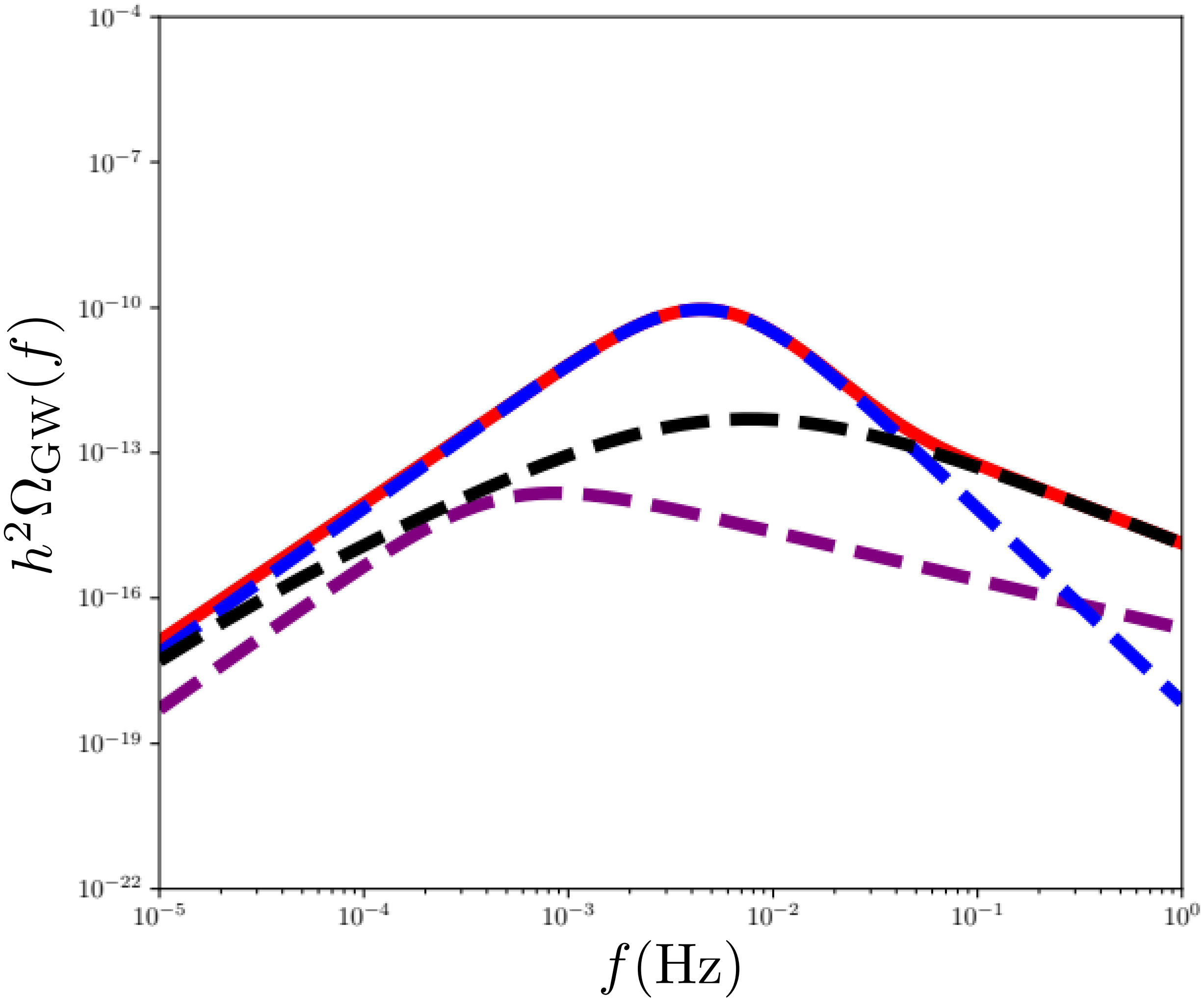

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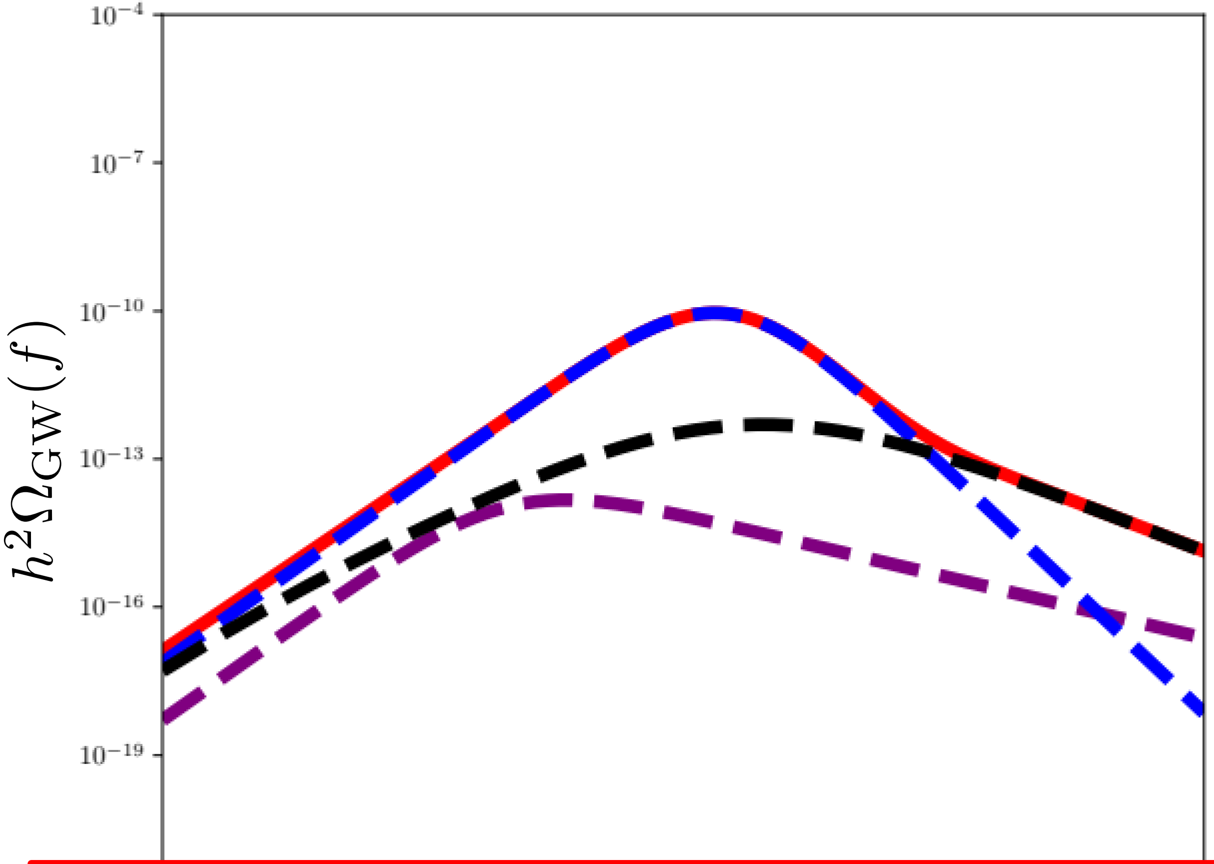

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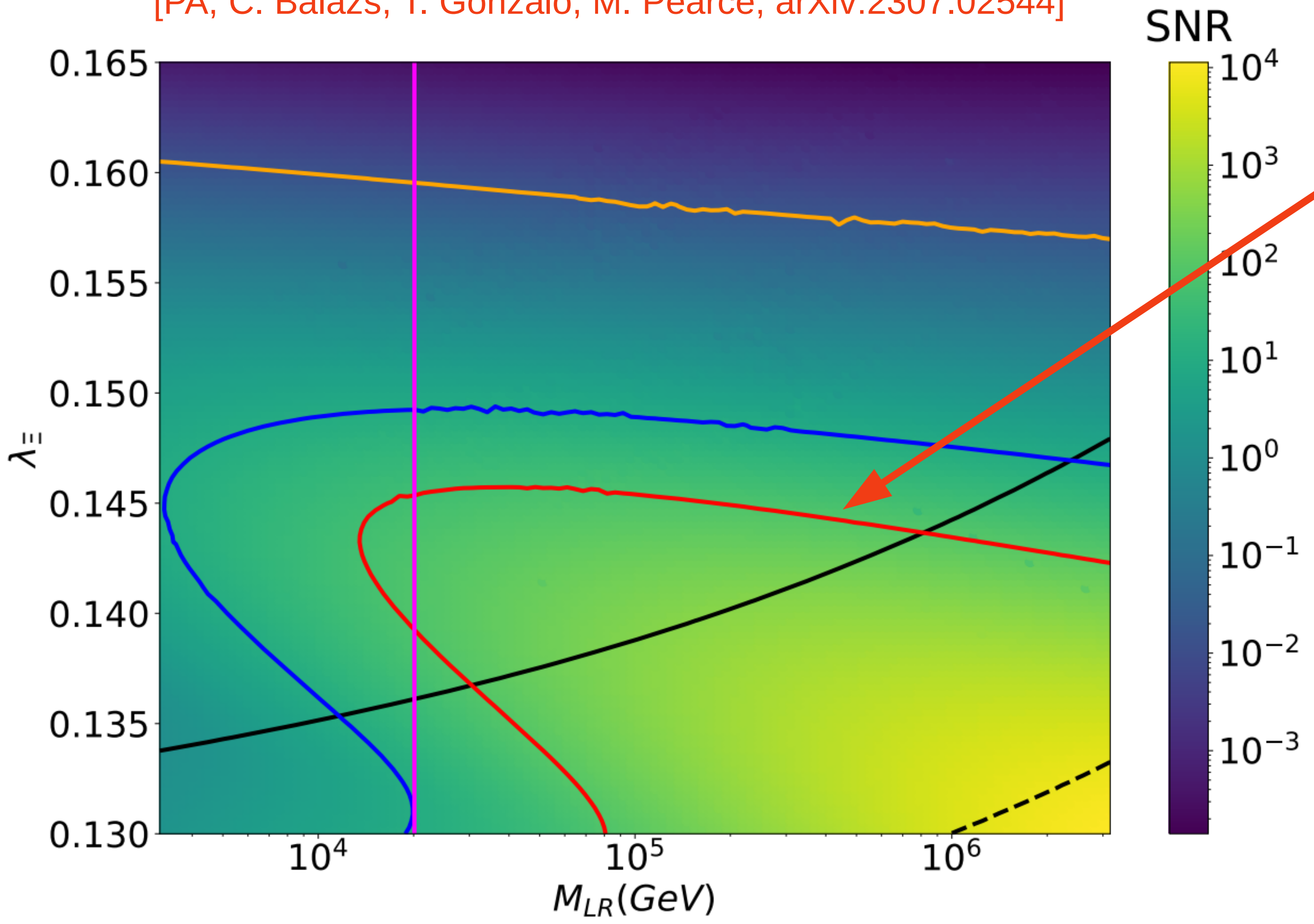
Understanding this quantitatively requires hyrdodynamical simulations and/or clever modeling of how it happens

We are entering an era
where
precise GWs predictions matter

Precise GWs predictions matter

LIGO data already constrains well motivated Pati-Salam GUT models

[PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544]



Constraint from LIGO data on a realistic model

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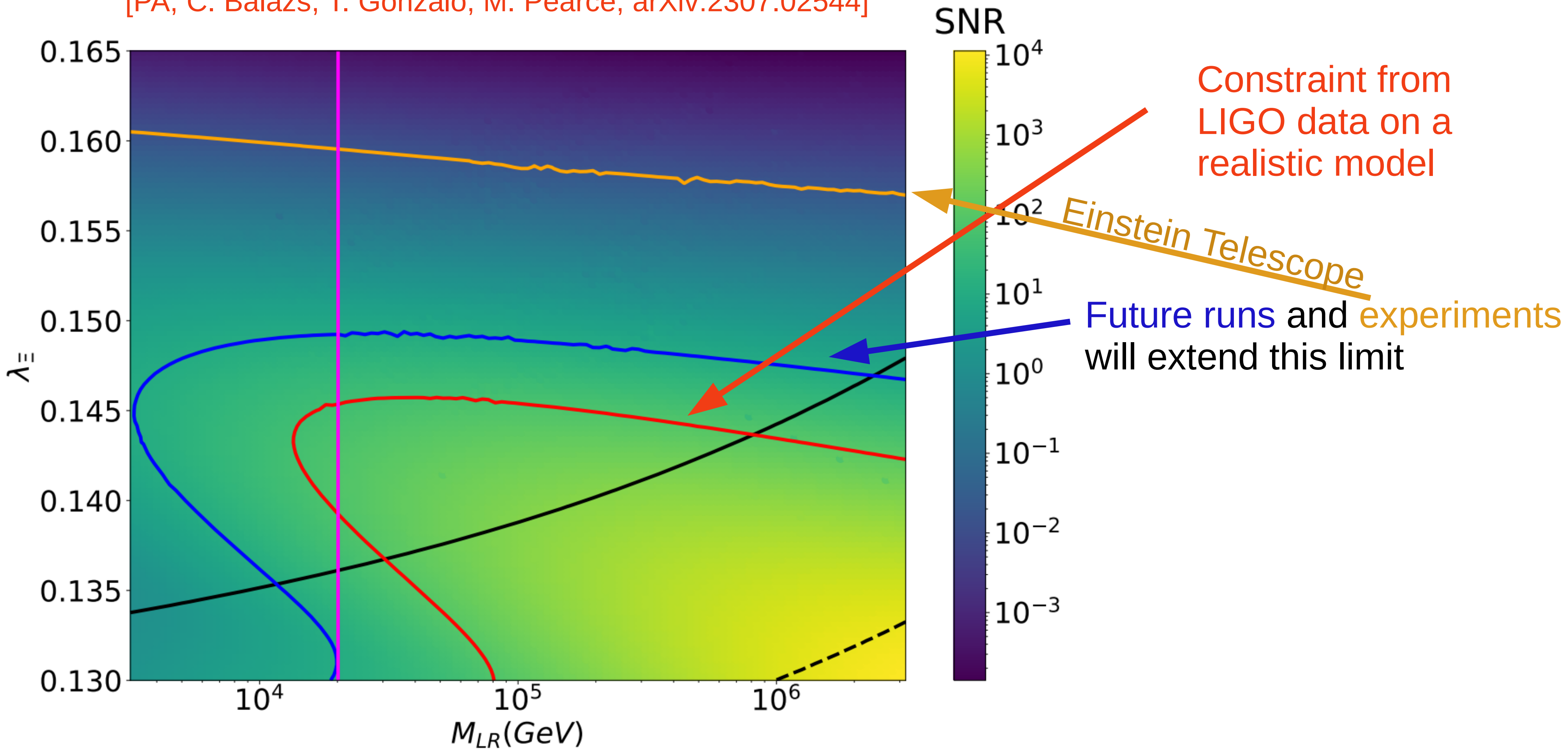

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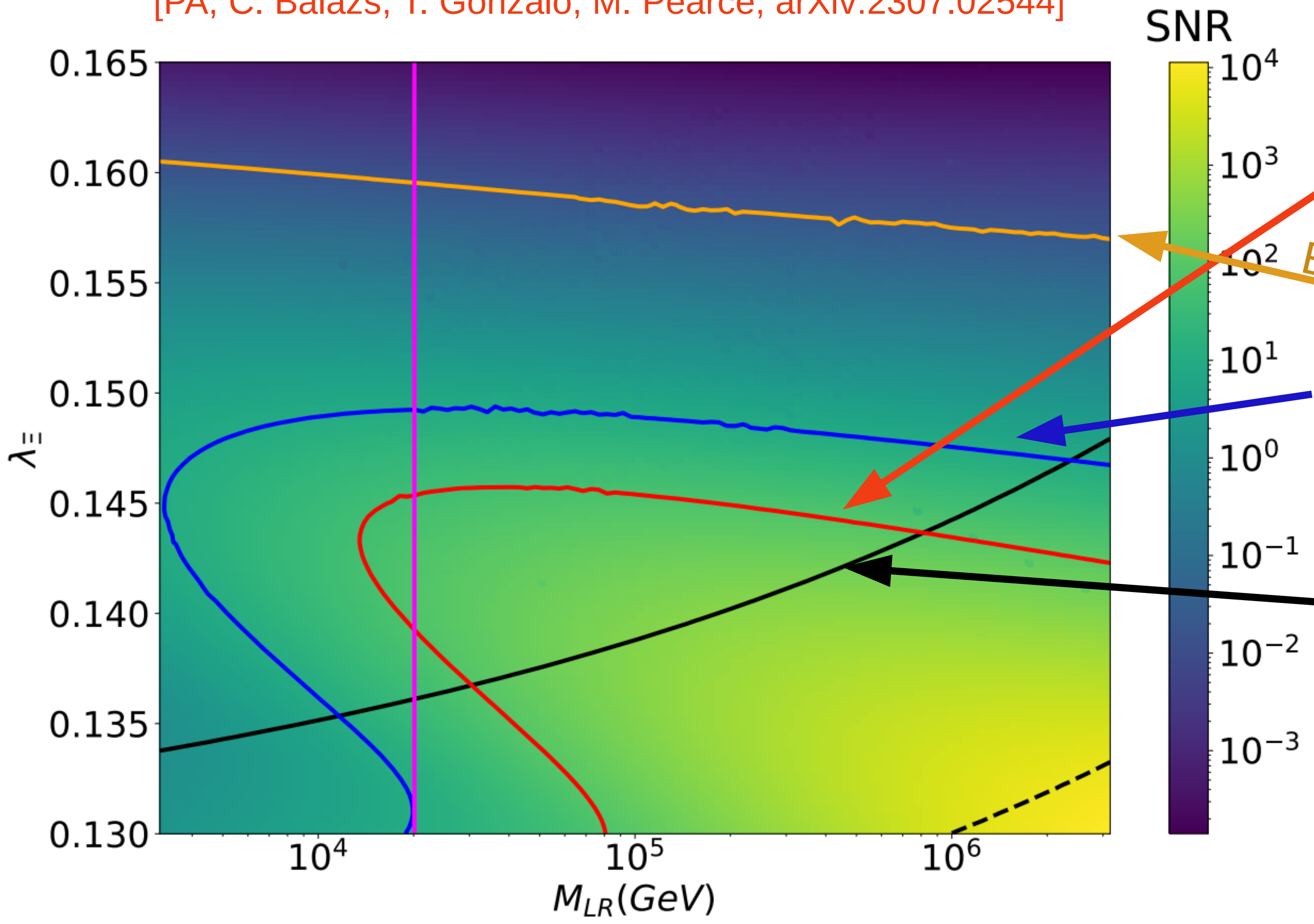

Constraint from LIGO data on a realistic model

Einstein Telescope

Future runs and experiments will extend this limit

Constraint from completion competes with LIGO constraint

Big news this summer:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

29 Jun 2023

Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

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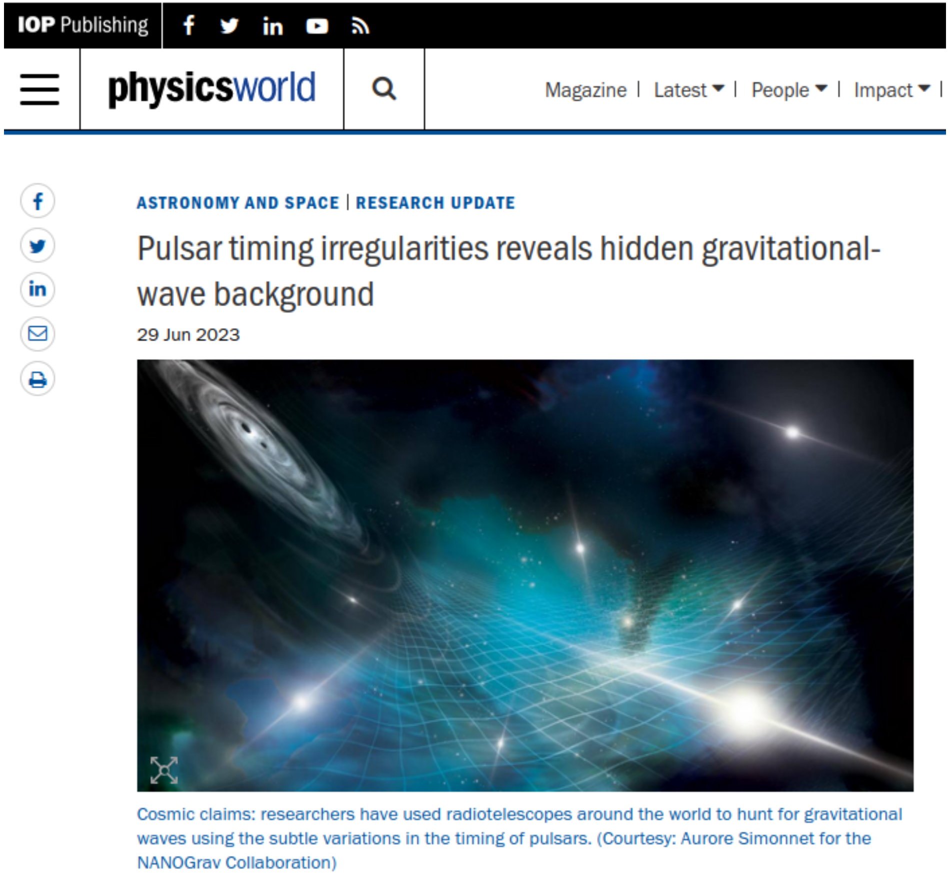
Conservative interpretation:
a population of
supermassive black holes binaries



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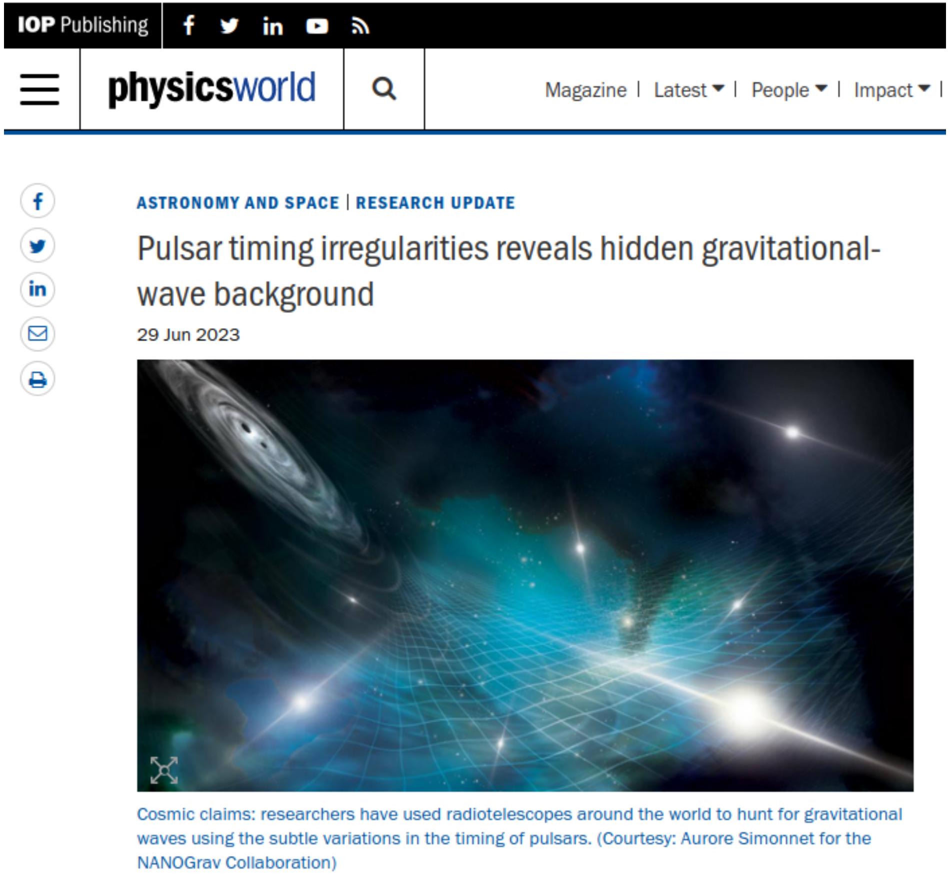
Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

Big news this summer:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

Conservative interpretation:
a population of
supermassive black holes binaries

But more exotic interpretations are possible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ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

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DOUBLE WARNING

For specific models these predictions require great care!

We looked at one model

prominantly cited by NANOGRAV

as able to explain nHz signals from PTAs...

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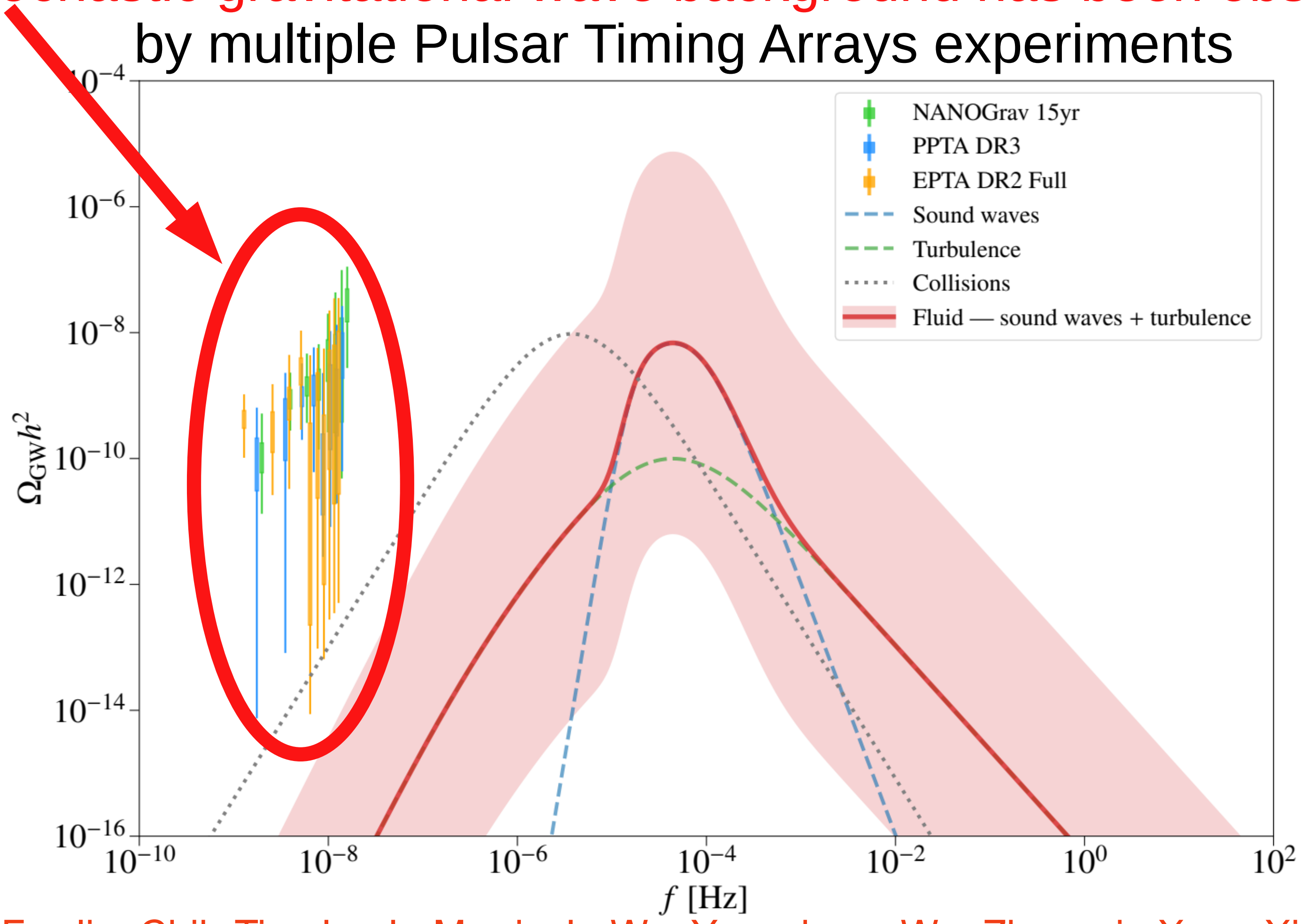
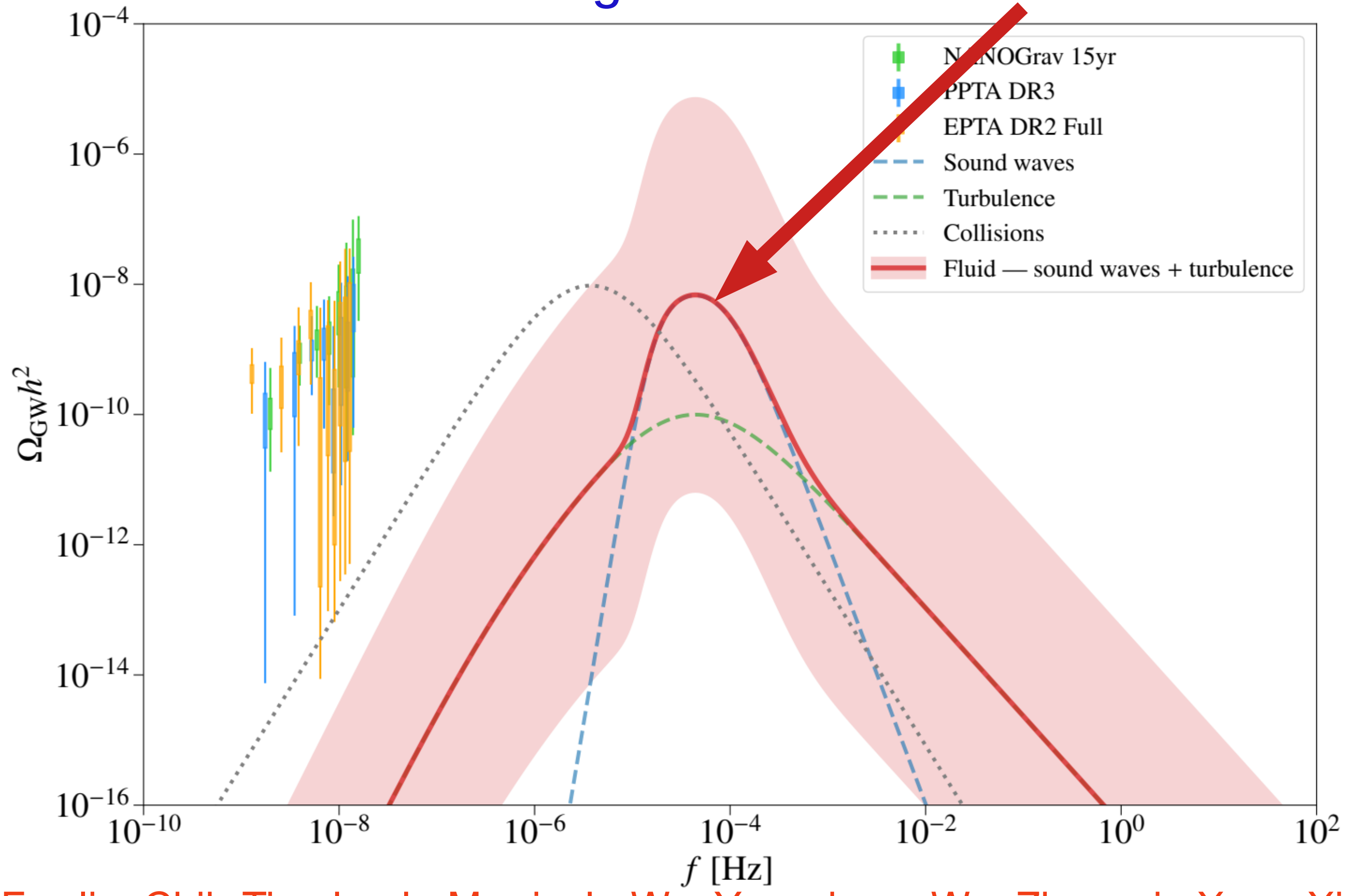


[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239]

But for the protypical model of supercooled PTs cited by NANOgrav as a possible explanation:

GWs can't fit the signal with careful calculation



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239]

Big news last month:
A stochastic gravitational wave background has been observed
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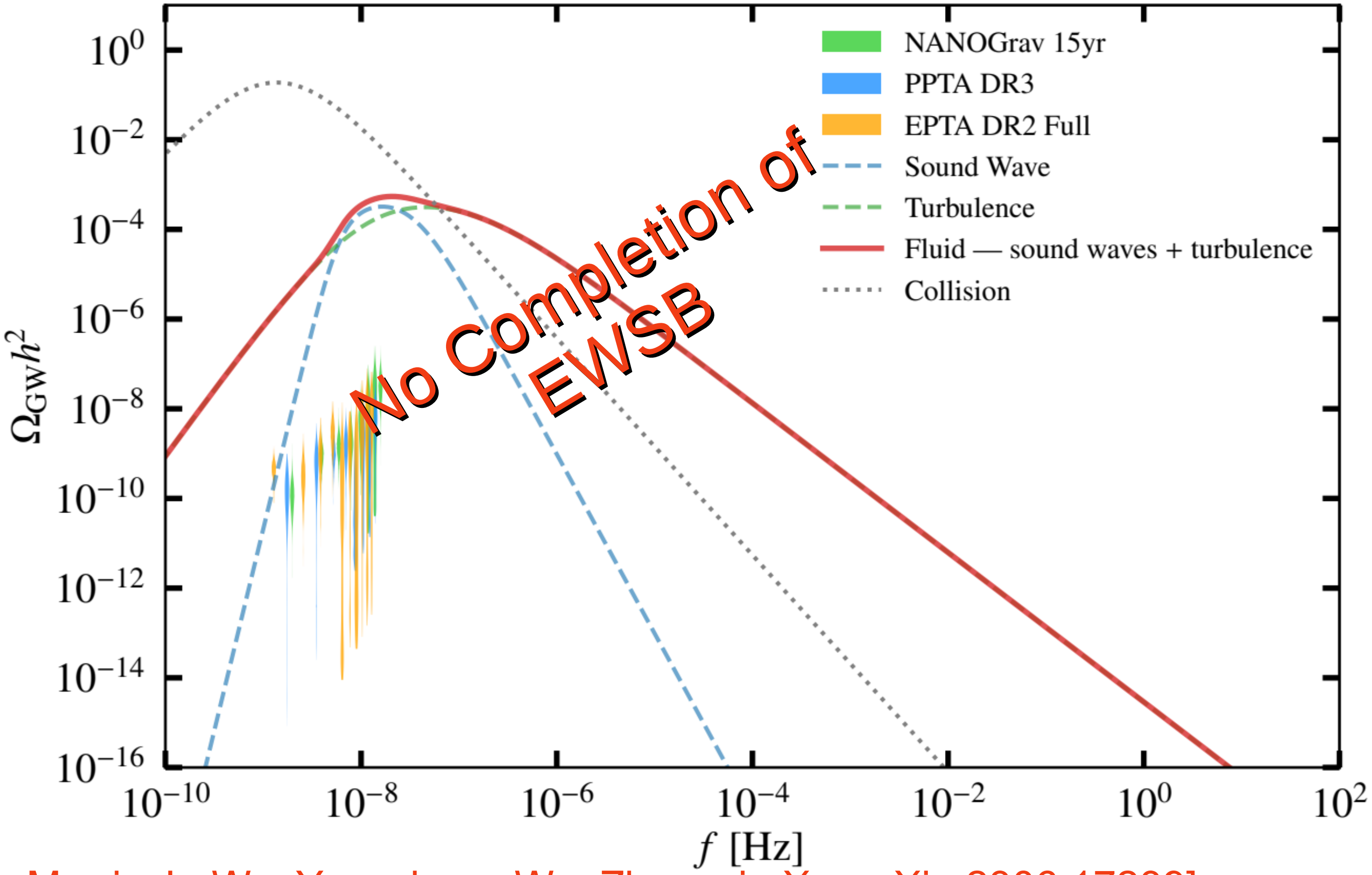
Larger signals are ruled out in this model because the PT does not complete

This is the first of the subtle effects I will discuss today!



[PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239]

From
particle physics theory
to GWs

From particle physics theory to GWs

There is a long chain of steps needed to make GW predictions

Effective Potential

Transition rates

Thermal parameters that influence GW spectrum

$$\hat{V}_{\text{eff}} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

PhaseTracing

Bubble dynamics, nucleation & growth

GW spectrum

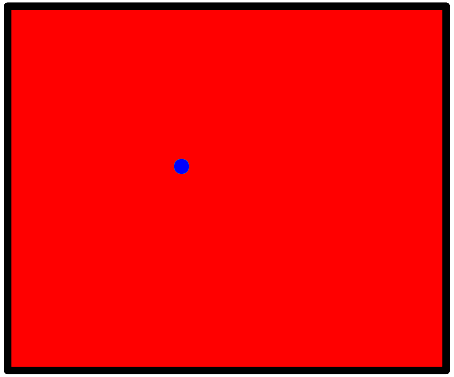
At every step there are challenges :

- open questions & active investigation
- Tensions between rigour and feasibility,
- Subtle issues leading to common misunderstandings / mistakes

Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume



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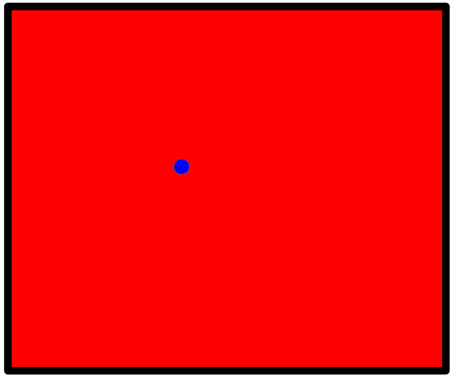
Nucleation: one bubble per Hubble volume

Often exstimated with simple heuristics

$S(t)/T = 140$ "bounce action" in $\Gamma(t) = Ae^{-S(t)}$



Hubble volume

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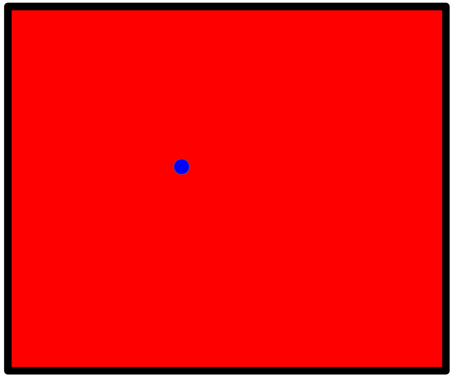

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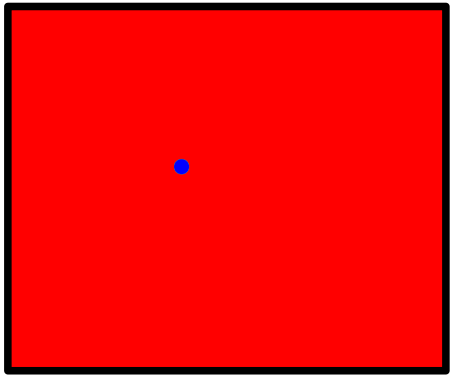

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Or solve $N(T_n) = 1$ $N(T) \approx \int_T^{T_c} dT' \frac{\Gamma(T')}{T' H^4(T')}$

If the barrier disolves quickly with temperature

$\longrightarrow$ Exponential nucleation rate $\longrightarrow$ Bubbles rapidly fill space

"Fast transition" or "low supercooling"

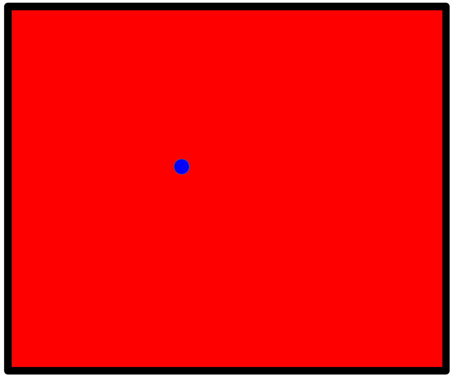
Does the Phase transiton complete?

Many studies only check nucleation

Nucleation: one bubble per Hubble volume

Not sufficient for scenarios with a lot of supercooling,

If the barrier persists to low temperatures,
→ nucleation rate can reach a maximum



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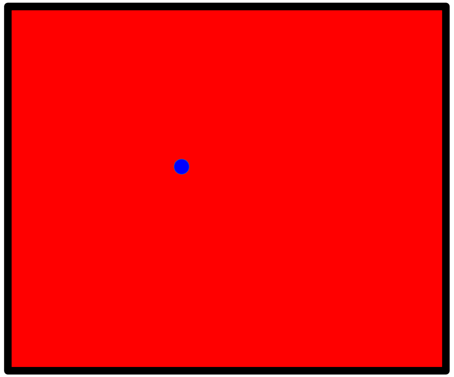

Hubble volume

For such slow transitions we must the false vacuum fraction $P_f \to 0$

$$P_f(T) = \exp\left[-\frac{4\pi}{3}\int_T^{T_c} \frac{dT'}{T'^4}\,\frac{\Gamma(T')}{H(T')}\left(\int_T^{T'} dT''\,\frac{v_w(T'')}{H(T'')}\right)^3\right]$$

Stochastic so actually check:
$P_f < \epsilon$

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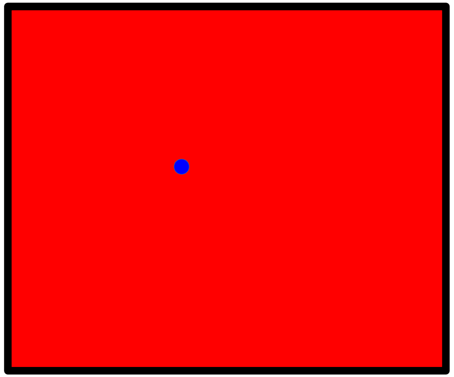

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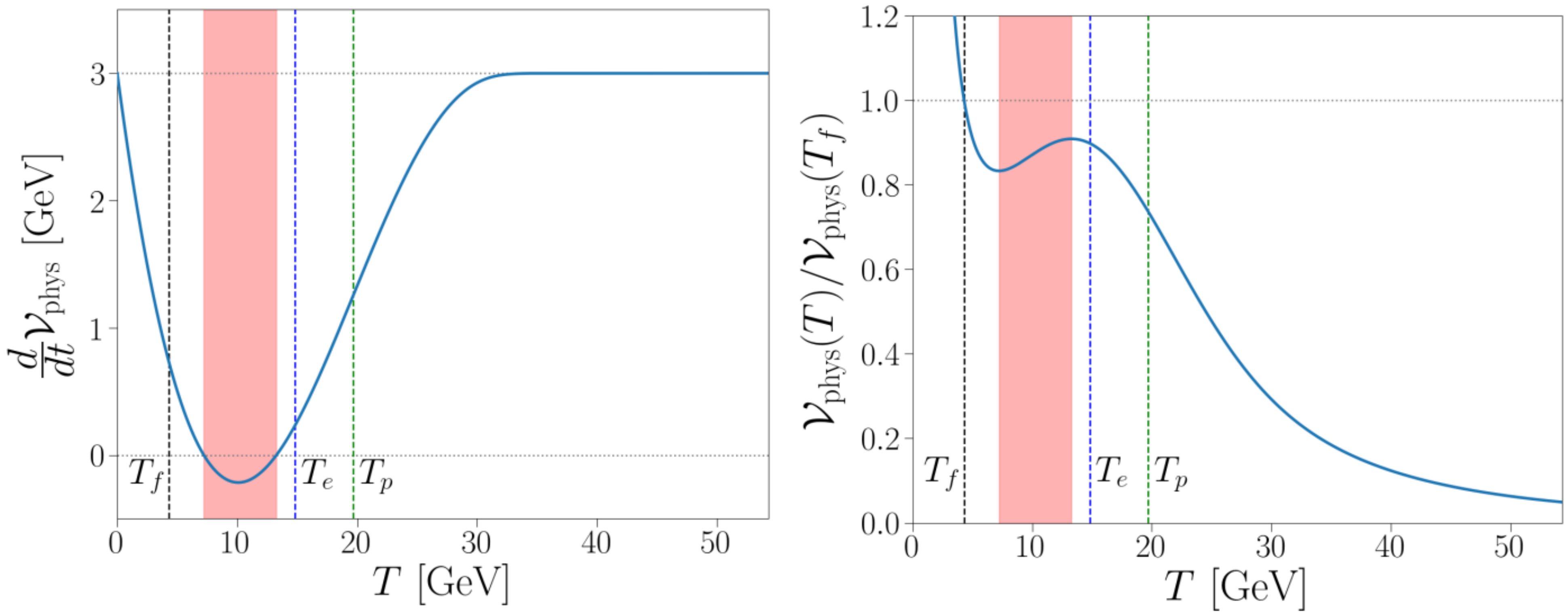
Stochastic so actually check:
$P_f < \epsilon$

Warning: even this may not be enough because space is expanding

Addional check for Percolation / completion



[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

To ensure it really completes, also require: $\frac{\mathrm{d}\mathcal{V}_f^{\rm phys}}{\mathrm{d}T} < 0$

Non-trivial because whole volume is expanding

Gravitational waves and thermal parameters

Recall $$h^2\Omega_{\text{GW-tot}} = h^2\Omega_{\text{coll}} + h^2\Omega_{\text{sw}} + h^2\Omega_{\text{turb}}$$

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_{\text{tot}}}\frac{d\rho_{\text{GW}}}{d\ln f}$$

energy density

$$\Omega_{\text{GW}}(f) \propto R_{\Omega}\, K^n L^m$$

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redshift factor

Redshift factor to account for redshifting from the transition time to today

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Kinetic energy fraction

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Kinetic energy fraction is the energy that can be available to source GWs

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Length scale
related to duration

redshift factor

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Redshift factor to account for redshifting from the transition time to today

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Length scale that is sensitive to the lifetime of the source

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Implicit dependence on the transition temperature and v_w

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Length scale that is sensitive to the lifetime of the source

Implicit dependence on the transition temperature and v_w

Powers depend on source & modelling, coeffs from simulations/calculations

False vacuum fraction $\longrightarrow$ several important milestone temperatures

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Completion temperature: $T_f\!: \; P_f(T_f) = 0.01$

False vacuum fraction $\longrightarrow$ several important milestone temperatures

Completion temperature: $T_f\colon\ P_f(T_f) = 0.01$

Percolation temperature: $T_p\colon\ P_f(T_p) = 0.71$

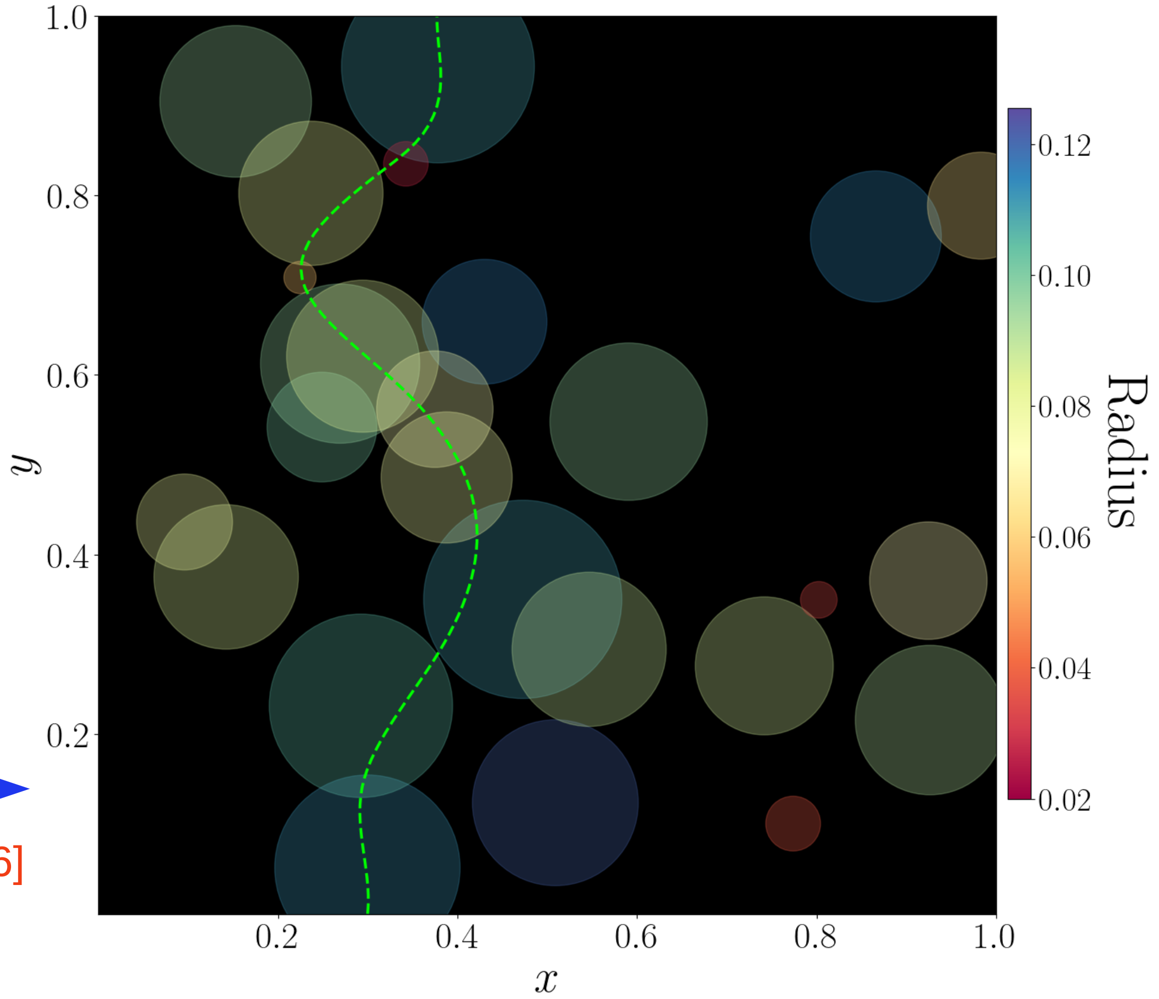
Percolation tempearture

T_p: $P_f(T_p) = 0.71$

- Percolation is when there is a connected path between bubbles across the space
- Strongly linked to bubble collisions
- Good choice for a temperature at which to evaluate the GWs spectrum

Example from simple simulation →

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]



False vacuum fraction $\longrightarrow$ several important milestone temperatures

Completion temperature: $T_f: \; P_f(T_f) = 0.01$

Percolation temperature: $T_p: \; P_f(T_p) = 0.71$

e-folding temperature: $T_e: \; P_f(T_e) = 1/e$

False vacuum fraction $\longrightarrow$ several important milestone temperatures

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e-folding temperature: T_e: $P_f(T_e) = 1/e$

The nucleation temperature is instead given by $N(T_n) = 1$

$$N(T) = \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$$

The nucleation temperature is frequently used for evaluating GW signals

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The nucleation temperature is frequently used for evaluating GW signals

but it may not exist…

and for slow transitions is decouples from the other the other temperatures

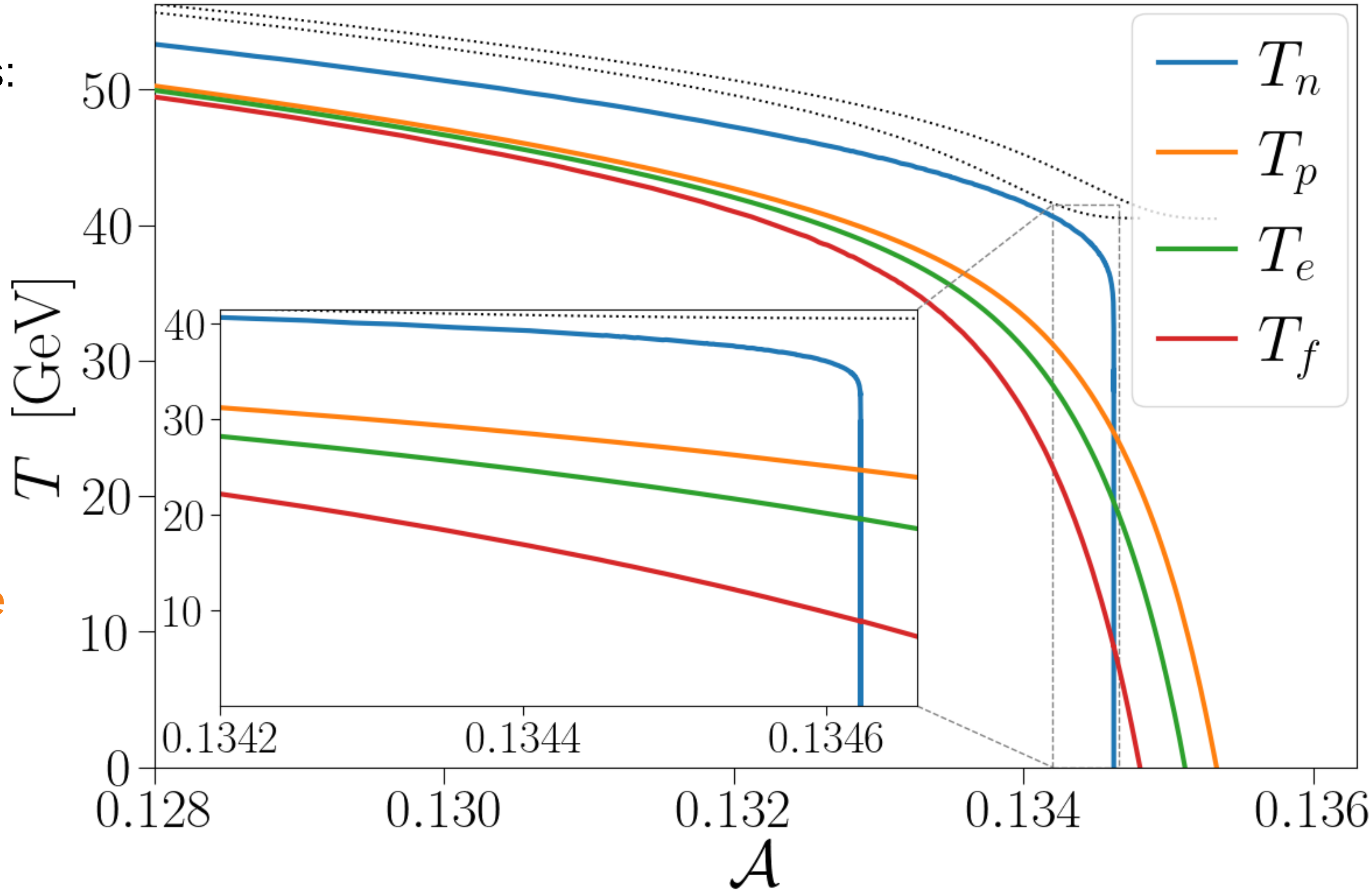
Milestone temperatures

[PA, C. Balázs, L. Morris, JCAP 03 (2023), 006]

Nucleation temperature is:

- Not related to bubble collisions
- Not related to other temperatures
- May not even exist

Percolation temperature is a better choice for GWs



The temperature choice really matters for gravitational wave signatures

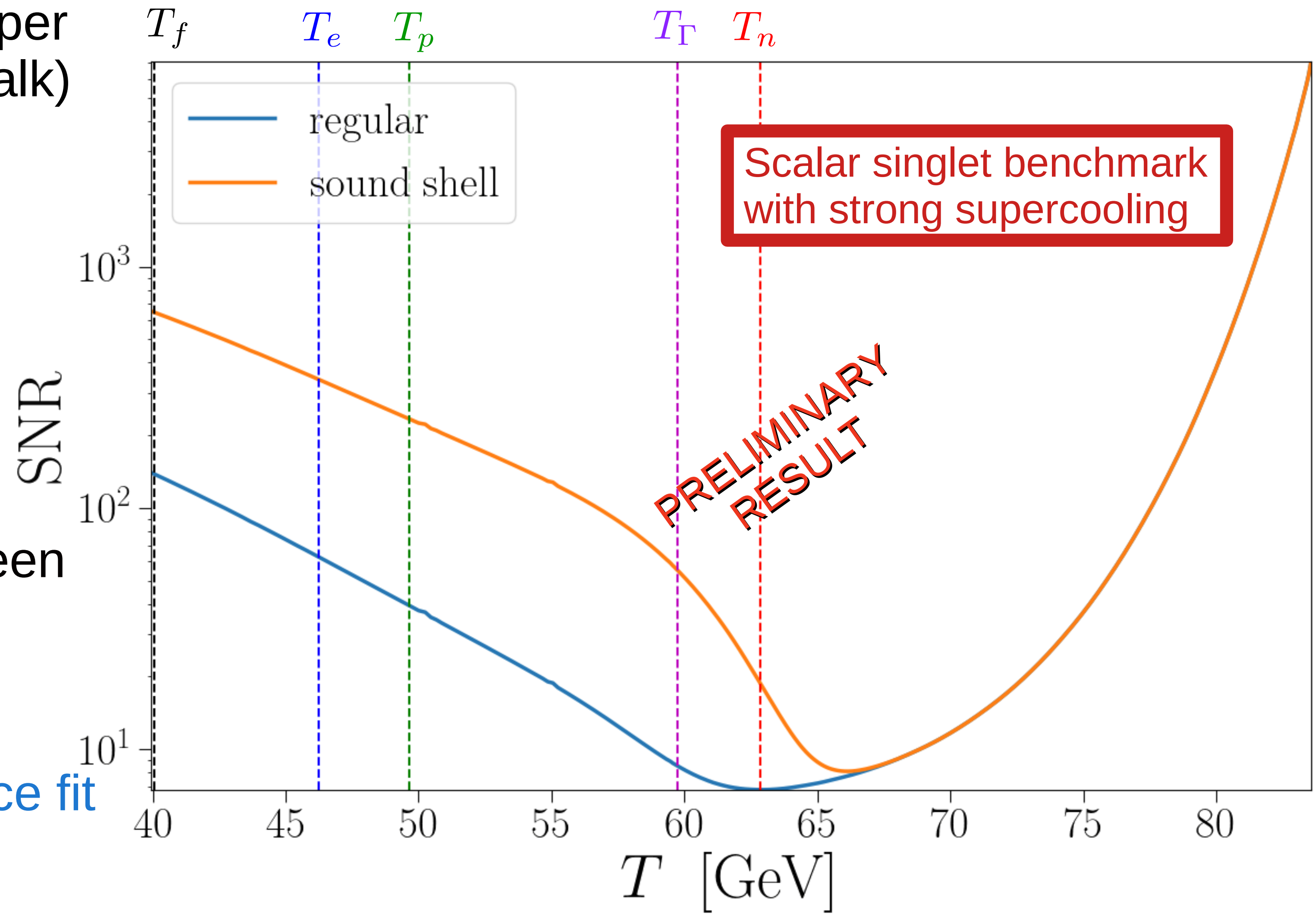
Temperature dependence

Point from same paper
(plot made for this talk)

Slow transition,
nucleation
far earlier than
percolation

Detectability
(SNR for LISA)
very different between
percolation vs
nucleation!

Sound shell and lattice fit
also very different



Gravitational waves and thermal parameters

Lattice fit to single broken power law for sound wave source :

[M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, PRD 96 (2017) 103520]

$$h^2\Omega_{\rm sw}^{\rm lat}(f) = 5.3\times 10^{-2}\, R_\Omega K^2 \left(\frac{H L_*}{c_{s,f}}\right) \Upsilon(\tau_{\rm sw}) S_{\rm sw}(f),$$

Shape

Speed of sound in false vacuum

Accounts for finite lifetime of source

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Sound shell model:

[Hindmarsh PRL 120 (2018) 071301, (+Hijazi) JCAP 12 (2019) 062, + (C. Gowling, D.C. Hooper and J. Torrado), JCAP 04 (2023) 061]

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Sound shell model is new but very promising

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Shape

Sound shell model is new but very promising

Turbulence also contributes, but not well modeled → Another uncertainty!

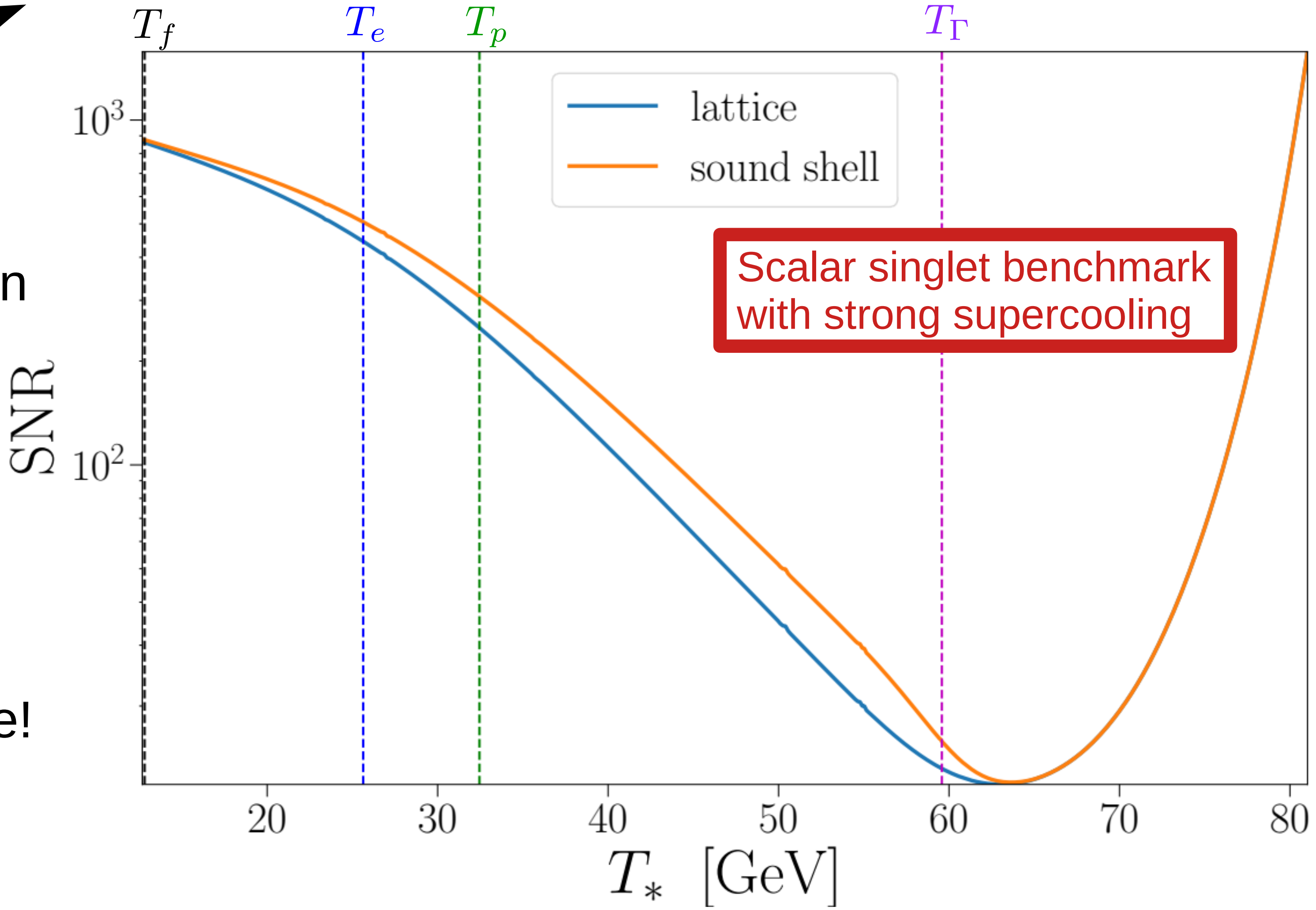
Temperature dependence

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

From here
(but plot simplified)

Another slow transition but percolates and completes *before* nucleation

LISA SNR varies more than an order of magnitude!



Temperature dependence

Many studies evaluate GW spectrim at the nucleation temperature

But the nucleation temperature is not really connceted to bubble collisions

Percolation is directly defined in terms of contact between bubbes

Nucleation is a bad choice, Percolation much better, but…

Temperature dependence

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Percolation criteria $P_f(T_p) = 0.71$ does not account for expanding space time

→ Temperature dependence represents a large uncertainty

Length scales / duration

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound waves:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T') P_f(T')}{T'^4 H(T')}$$

bubble number density

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bubble number density

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots ,$$

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1st order $\longrightarrow$ explonential nucleation rate $\Gamma(t) = \Gamma(t_*)\exp(\beta(t-t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = HT_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

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Widely used replacement

$$R_{\text{sep}} = (8\pi)^{\frac{1}{3}} \frac{v_w}{\beta}$$

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound waves:

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bubble number density

Best treatement

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

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Rough approximation

Times scales for sources gravitational waves affect the GWs signal

Depends on the particle physics model

Can be related to a length scale, mean bubble separation used in hydrodynamical simulations of sound:

$$R_{\rm sep}(T) = (n_B(T))^{-\frac{1}{3}} \qquad n_b(T) = T^3 \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')}$$

bubble number density

Best treatment

Often estimated by taylor expanding the bounce action $\Gamma(t) = Ae^{-S(t)}$

$$S(t) \approx S(t_*) + \left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*}(t-t_*) + \frac{1}{2}\left.\frac{\mathrm{d}^2S}{\mathrm{d}t^2}\right|_{t=t_*}(t-t_*)^2 + \cdots\,,$$

1st order $\longrightarrow$ explonential nucleation rate $\Gamma(t) = \Gamma(t_*)\exp(\beta(t-t_*)),$

$$\beta = -\left.\frac{\mathrm{d}S}{\mathrm{d}t}\right|_{t=t_*} = HT_* \left.\frac{\mathrm{d}S}{\mathrm{d}T}\right|_{T=T_*}$$

If Γ reaches a maximum
$\Rightarrow \beta < 0$ after or tiny close to maximum!

Times scales for sources gravitational waves affect the GWs signal

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2nd order $\longrightarrow$ Gaussian nucleation rate $\Gamma(t) = \Gamma(t_*)\exp\left(-\frac{\beta_{\rm V}^2}{2}(t-t_*)^2\right),$

$$\beta_{\rm V} = \sqrt{\frac{\mathrm{d}^2S}{\mathrm{d}t^2}}\Bigg|_{t=t_\Gamma}$$

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Can be used to replace mean bubble separation

$$R_{\rm sep} = \left(\sqrt{2\pi}\frac{\Gamma(T_\Gamma)}{\beta_{\rm V}}\right)^{-\frac{1}{3}}$$

T_Γ is where nucl rate (Γ) is maximised

Times scales for sources gravitational waves affect the GWs signal

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Rough approximation

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bubble number density

One more thing:

Alternative length scale - mean bubble radius

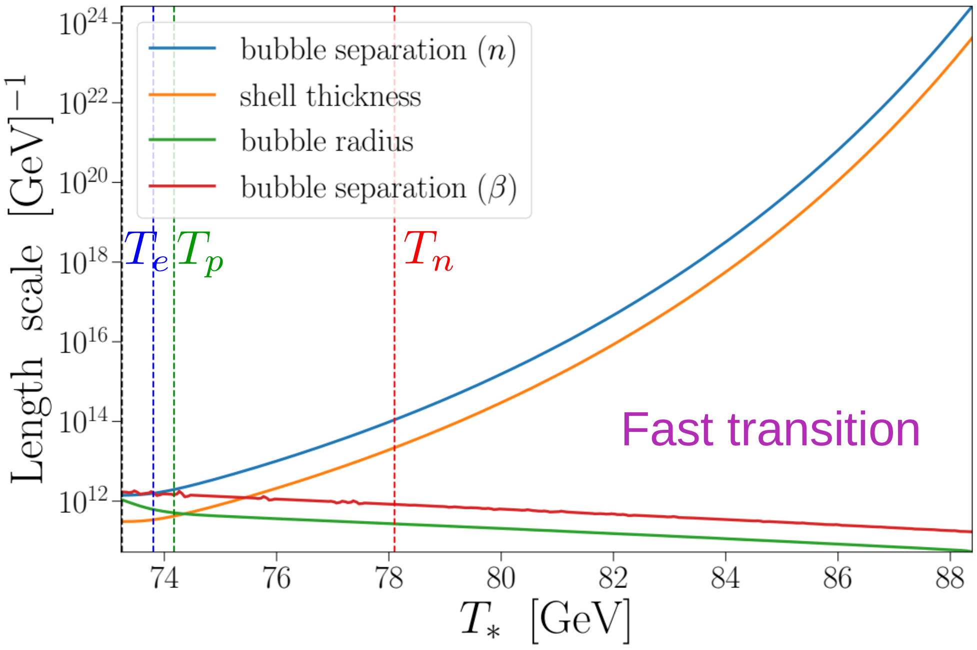
$$\bar{R}(T) = \frac{T^2}{n_B(T)} \int_T^{T_c} dT' \frac{\Gamma(T')P_f(T')}{T'^4 H(T')} \int_T^{T'} dT'' \frac{v_w(T'')}{H(T'')}.$$

This has been proposed in the literature but not used in simulations

For fast transitions

mean bubble separation varies a lot with T

Should not be used until $T \approx T_p$



[PA, L. Morris, Z. Xu, arXiv:2309.05474]

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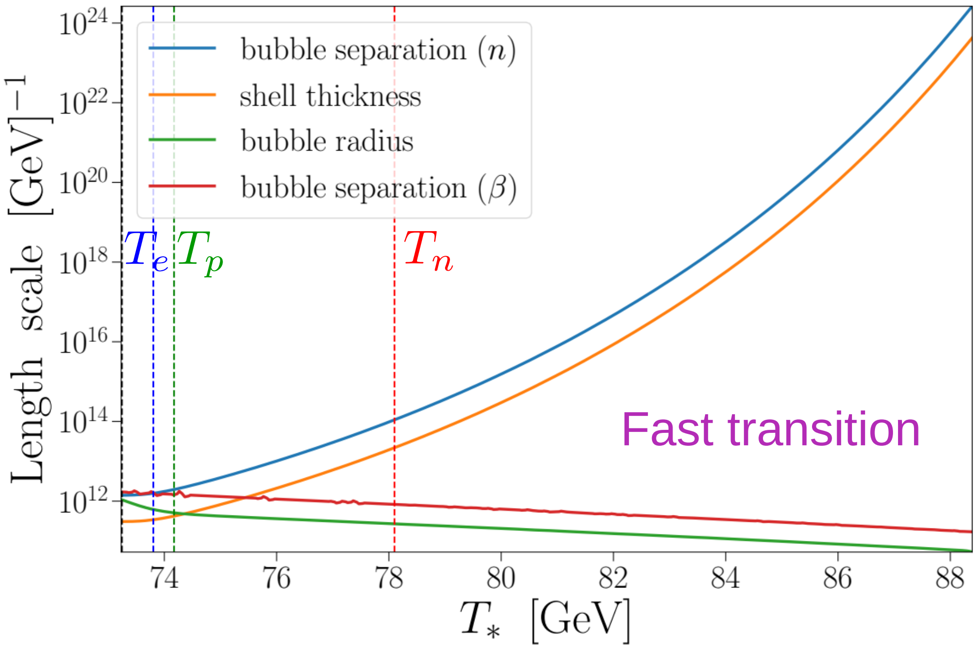
Estimating GWs with $\beta(T_p)$, factor 2 too low
Worse with $\beta(T_n)$ (very common)



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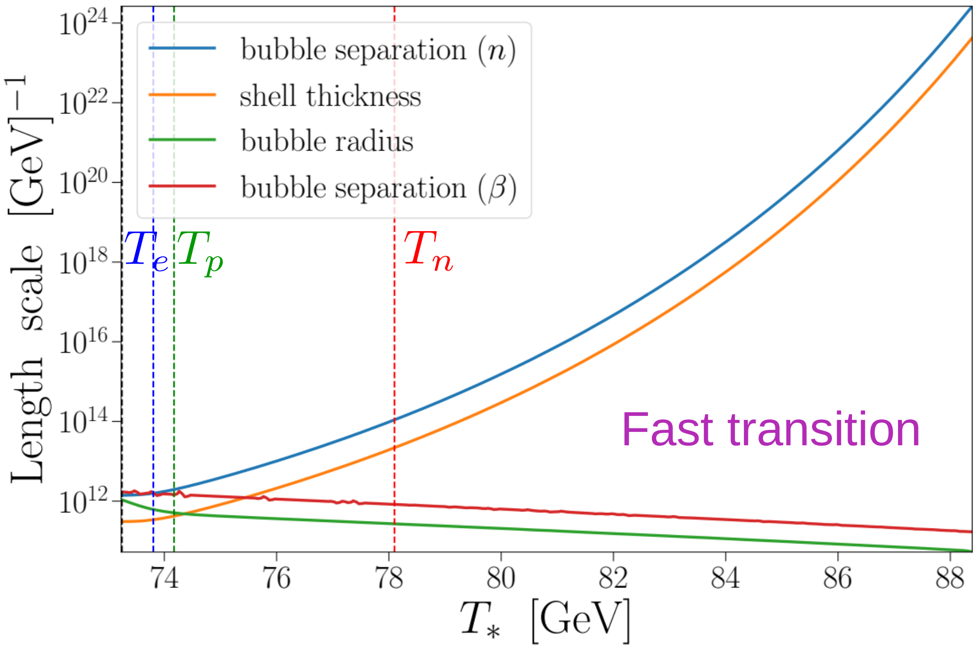

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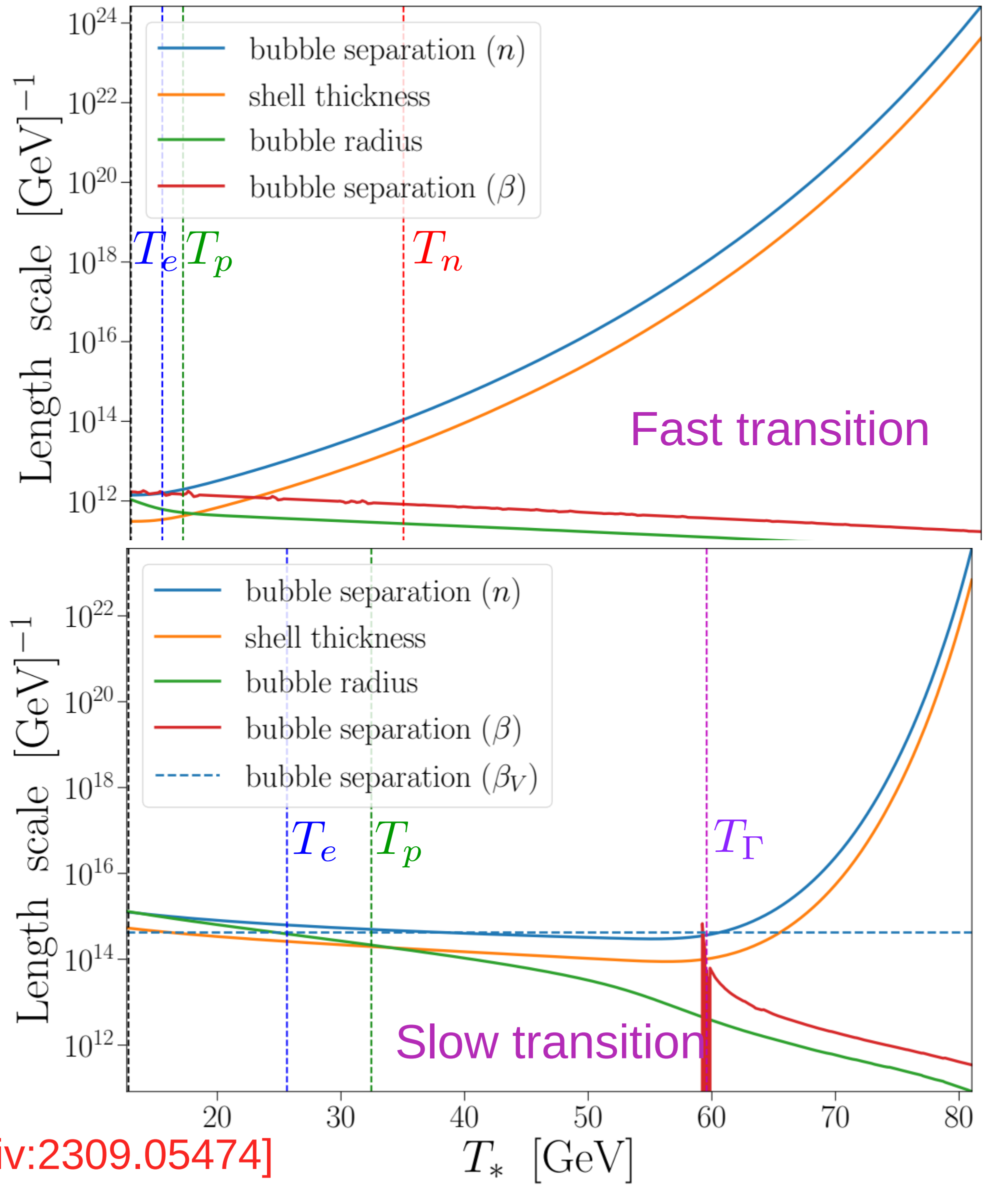

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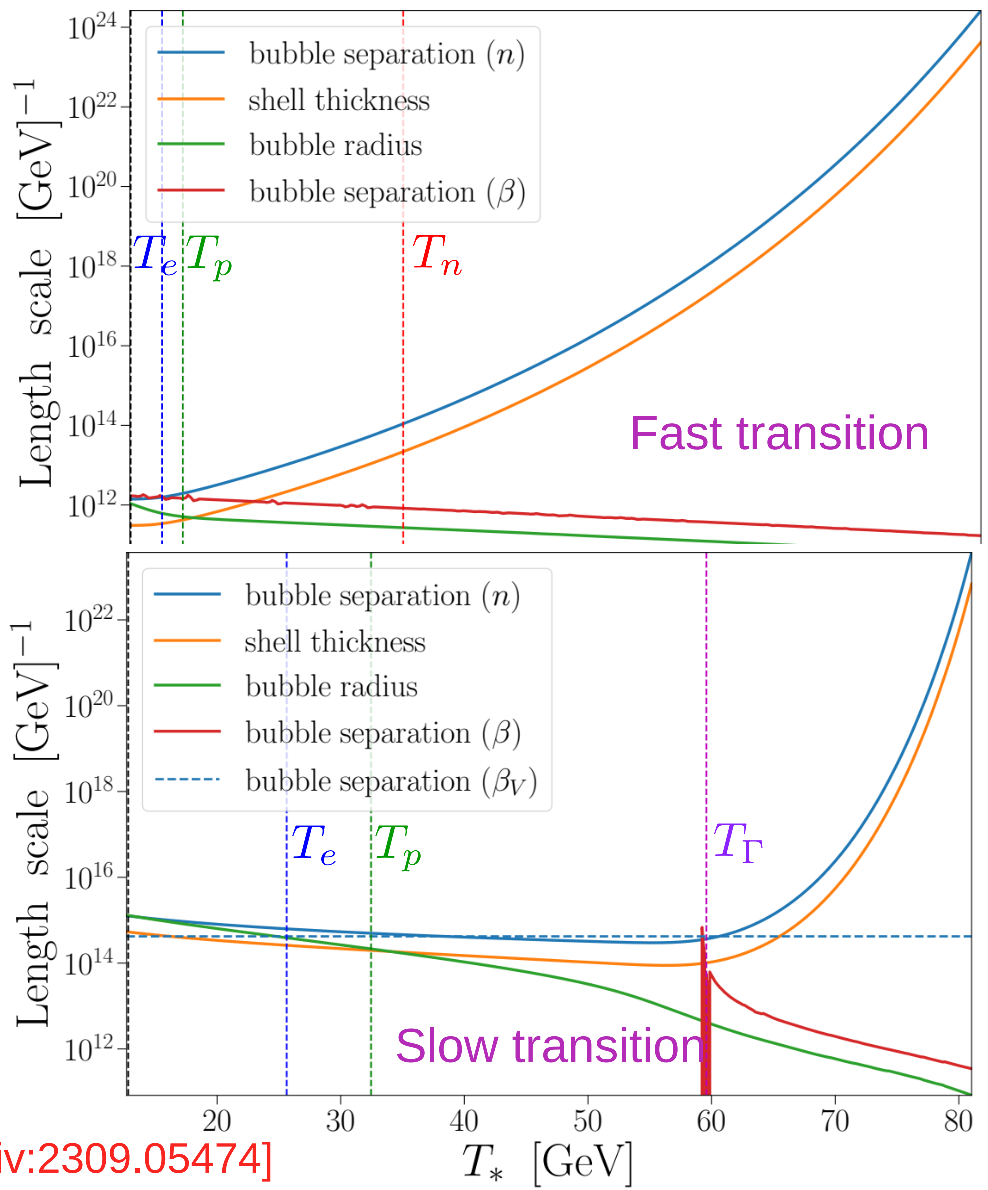

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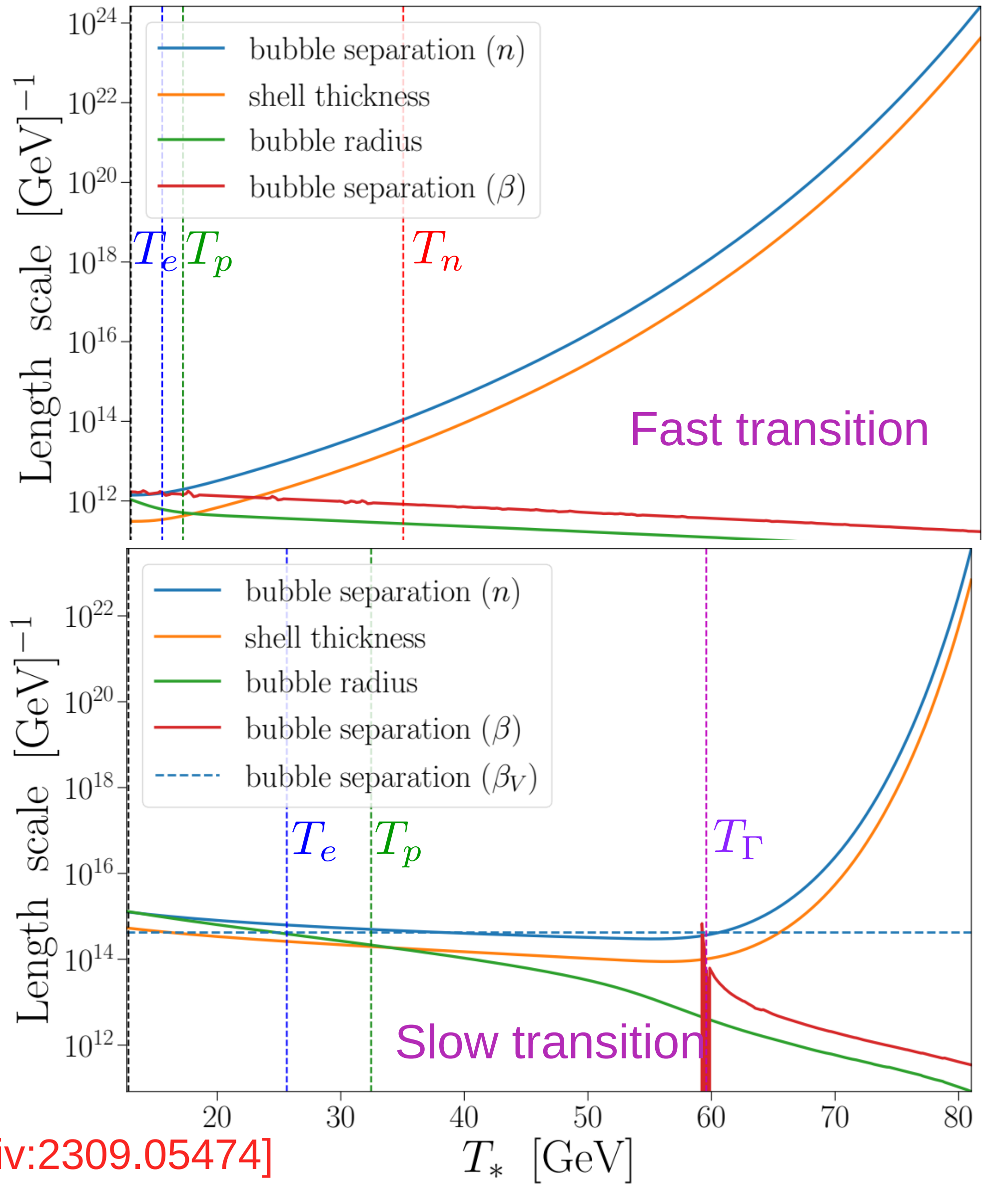
Mean bubble radius varies more as bubbles have longer to grow.

Using $\beta(T)$ makes no sense below T_Γ
orders of magnitude errors above

β_V ⟶ factor 1.5 drop in GW amplitide



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Kinetic Energy Fraction

Kinetic energy fraction $K = \frac{\rho_{\rm kin}(T_*)}{\rho_{\rm tot}(T_*)}$,

~ energy available
for a particular source of GWS

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Outside of hydrodynamical simulations approximations are used:

Average kinetic energy,
(single isolated bubble)

$$\rho_{\rm kin} = \frac{3}{\xi_w^3} \int_0^\infty d\xi \, \xi^2 w(\xi) v^2(\xi) \gamma^2(v(\xi))$$

[J.R.Espinosa, T.Konstandin, J.M.No, G.Servant, JCAP 06 (2010) 028]

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[J.R.Espinosa, T.Konstandin, J.M.No, G.Servant, JCAP 06 (2010) 028]

Bag Equation of state: $V(\phi_{\text{false}}) = \epsilon - aT^4 \longrightarrow$ ϵ = energy liberated from the vacuum

$V(\phi_{\text{true}}) = -aT^4$ $\quad$ $\rho_R = 3aT^4$ Radiation energy density

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"Latent heat"

$$\rho = V - T\frac{\partial V}{\partial T}$$

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$$p = -V$$

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$$\alpha_\rho = \frac{\Delta\rho}{\rho_R} \qquad \alpha_\theta = \frac{\Delta(V - \frac{1}{4}T\frac{\partial V}{\partial T})}{\rho_R} \qquad \alpha_p = \frac{\Delta p}{\rho_R}$$

"Latent heat"

$$\rho = V - T\frac{\partial V}{\partial T}$$

Trace anomaly

$$\theta = (\rho - 3p)/4$$

pressure

$$p = -V$$

Actually

The trace anomaly corresponds to the energy released that could source GWs

$$T_{00} = \underbrace{w\gamma^2 v^2}_{\rho_K} + \underbrace{\sum_i \left(\dot{\phi}_i^2 + (\nabla\phi)^2\right)}_{\rho_{\text{scalar}}} + \underbrace{\frac{3}{4}w}_{\rho_Q \text{ (heat)}} + \theta.$$

Trace anomaly

$$\theta = \frac{1}{4}(\rho - 3p) = V - \frac{1}{4}T\frac{\partial V}{\partial T}$$

[M. Hindmarsh, M. Hijazi , JCAP 12 (2019) 062]

This energy is then distributed amongst the fluid, the scalar field and heat

overestimate

$$\alpha_\rho = \frac{\Delta\rho}{\rho_R} > \alpha_\theta = \frac{\Delta(V - \frac{1}{4}T\frac{\partial V}{\partial T})}{\rho_R} > \alpha_p = \frac{\Delta p}{\rho_R}$$

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$\mathcal{O}(10)$

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Important for fast transitions

[PA, L. Morris, Z. Xu , arXiv:2309.05474]

pressure

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$\mathcal{O}(1/10)$

Efficiency coefficient

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$$\kappa := \frac{\rho_{\text{kin}}}{\theta_f - \theta_t} \qquad \rho_{\text{tot}} = \theta_f + \frac{3}{4} w_f, \quad K = \frac{\kappa(\theta_f - \theta_t)}{\theta_f + \frac{3}{4} w_f} = \frac{\kappa \alpha_\theta}{1 + \alpha_\theta + \delta} \qquad \delta = \frac{\theta_t}{\frac{3}{4} w_f}$$

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Pseudo-trace anomaly

$$\bar\theta(T) = \frac{1}{4}\left(\rho(T) - \frac{p(T)}{c_s^2(T)}\right)$$

Speed of sound ($c_s = \frac{1}{\sqrt{3}}$ in bag model)

[F. Giese, T. Konstandin and J. van de Vis, JCAP 07 (2020) 057,(+K. Schmitz), JCAP 01 (2021) 072]

Transition Solver

The good news is many of these issues can be avoided with careful numerical implementations

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→ Future releases (v2) will automate effective potential, link to DRalgo for best feasible handing of effective potential as well!

Conclusions

- Very exciting recent results indicate we have entered an era where GW experiments have sensitivy to SGBG from BSM physics
- Now things are real and we really need to understand uncertainties and make reliable predictions of GW spectra from BSM physics scenarios:
  - For slow transitions, checking the phase transition completes is essential.
  - The temperature dependence of predictions can be significant! The nucleation temperature is a bad choice, the percolation temperature seems reasonable.
  - The β approximation for length scale can lead to significant error even in fast transitions.
  - Latent heat (and pressure) approximations for the Kinetic energy fraction give significant errors for fast transitions
- Its very important that the theory community takes this seriously and BSM predictions are done as well as possible.
- TransitionSolver is here to help!

The END

Thanks for listening!

This talk is based on:

- PA, C. Balázs, A. Fowlie, L. Morris, L. Wu, arxiv:2305.02357, (Invited review for Progress in Particle and Nuclear Physics), (Accepted) 155 pages
- PA, C. Balázs, L. Morris, JCAP 03 (2023), 006, 57 pages
- PA, L. Morris, Z. Xu , arXiv:2309.05474,
- PA, A. Fowlie, Chih-Ting Lu, L. Morris, L. Wu, Yongcheng Wu, Zhongxiu Xu, arXiv:2306.17239,
- PA, C. Balázs, T. Gonzalo, M. Pearce, arXiv:2307.02544,
- PA, C. Balázs, A. Fowlie, L. Morris, G. White, Y. Zhang, JHEP 01 (2023) 050, 45 pages

Back up

/

Seminar versions

Comparison of predictions for a weakly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Variation	$h^2\Omega_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times 10^{-13})$	$h^2\Omega_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times 10^{-14})$	$f_{\mathrm{sw}}^{\mathrm{lat}}$ $(\times 10^{-5})$	$f_{\mathrm{sw}}^{\mathrm{ss}}$ $(\times 10^{-4})$	$h^2\Omega_{\mathrm{turb}}$ $(\times 10^{-16})$	f_{turb} $(\times 10^{-5})$	$\mathrm{SNR}_{\mathrm{lat}}$	$\mathrm{SNR}_{\mathrm{ss}}$	α $(\times 10^{-2})$	κ	K $(\times 10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
$T_* = T_n$	676.5	1046	2.189	1.098	8968	4.849	1.310	5.142	4.297	0.1857	7.597
$R_{\mathrm{sep}}(\beta)$	2.019	3.191	177.5	82.45	2.078	393.1	7.510	0.8449			
$K(\alpha(\theta))$	3.372	5.329			3.676		9.469	1.908	5.362	0.2011	10.23
$K(\alpha(p))$	1.428	2.256			1.649		4.010	0.8081	3.698	0.1682	5.997
$K(\alpha(\rho))$	14.45	22.84			14.61		40.59	8.172	10.35	0.2736	25.68
ϵ_2					11.03		10.11	2.064			
ϵ_3					290.2		11.21	3.406			
ϵ_4					301.7		11.26	3.462			

Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in K: trace anomaly approximation is quite good in this case

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ $(\times 10^{-13})$	$h^2\Omega_{\rm sw}^{\rm ss}$ $(\times 10^{-14})$	$f_{\rm sw}^{\rm lat}$ $(\times 10^{-5})$	$f_{\rm sw}^{\rm ss}$ $(\times 10^{-4})$	$h^2\Omega_{\rm turb}$ $(\times 10^{-16})$	$f_{\rm turb}$ $(\times 10^{-5})$	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α $(\times 10^{-2})$	κ	K $(\times 10^{-3})$
None	3.590	5.673	129.6	60.20	3.898	287.0	10.08	2.031	5.450	0.2074	10.64
$T_* = T_e$	2.552	4.042	159.9	73.75	2.662	354.2	8.763	1.204	5.575	0.2096	10.99
$T_* = T_f$	2.146	3.410	181.7	82.91	2.187	402.5	8.110	0.8892	5.771	0.2129	11.54
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Comparison of predictions for a weakly supercooled point in SSM

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

Differences in sound wave amplitude (sound shell): latent heat (and pressure) variants give substanial differences

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ ($\times 10^{-13}$)	$h^2\Omega_{\rm sw}^{\rm ss}$ ($\times 10^{-14}$)	$f_{\rm sw}^{\rm lat}$ ($\times 10^{-5}$)	$f_{\rm sw}^{\rm ss}$ ($\times 10^{-4}$)	$h^2\Omega_{\rm turb}$ ($\times 10^{-16}$)	$f_{\rm turb}$ ($\times 10^{-5}$)	$\mathrm{SNR}_{\rm lat}$	$\mathrm{SNR}_{\rm ss}$	α ($\times 10^{-2}$)	κ	K ($\times 10^{-3}$)
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Comparison of predictions for a weakly supercooled point in SSM

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Comparison of predictions for a strongly supercooled point

[PA, L. Morris, Z. Xu, arXiv:2309.05474]

However the variation in K estimates is much smaller for strongly supercooled scenarios

Variation	$h^2\Omega_{\rm sw}^{\rm lat}$ $(\times 10^{-7})$	$h^2\Omega_{\rm sw}^{\rm ss}$ $(\times 10^{-8})$	$f_{\rm sw}^{\rm lat}$ $(\times 10^{-6})$	$f_{\rm sw}^{\rm ss}$ $(\times 10^{-6})$	$h^2\Omega_{\rm turb}$ $(\times 10^{-10})$	$f_{\rm turb}$ $(\times 10^{-6})$	$\mathrm{SNR_{lat}}$	$\mathrm{SNR_{ss}}$	α	κ	K
None	1.861	3.748	9.345	23.48	6.348	20.70	249.6	307.7	1.651	0.7175	0.4536
$T_* = T_e$	4.318	8.872	7.908	19.12	14.74	17.52	443.7	498.2	4.257	0.8422	0.6950
$T_* = T_f$	17.04	35.42	4.111	9.722	81.84	9.106	864.5	876.4	71.06	0.9831	0.9803
$R_{\rm sep}(\beta_V)$	1.193	2.402	12.80	32.17	3.394	28.36	222.6	356.9			
$K(\alpha(\theta))$	1.819	3.663			6.227		244.9	301.5	1.605	0.7269	0.4478
$K(\alpha(p))$	1.768	3.560			6.083		239.2	294.2	1.564	0.7269	0.4409
$K(\alpha(\rho))$	1.967	3.962			6.646		261.4	323.0	1.728	0.7383	0.4677
ϵ_2					17.95		700.0	742.2			
ϵ_3					0		18.36	130.9			
ϵ_4					288.4		11210	11230			

From particle physics theory to GWs

$$\hat{V}_{\text{eff}} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Effective Potential: can be computed perturbatively with finite temperature quantum field theory

However there are problems appling this for phase transitons at finite temp

- Unphysical Gauge dependence

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Particle physics theories have gauge freedom $\phi \to \phi - \frac{\partial \psi}{\partial t}$

Like gauge freedom of electromagnetism $\mathbf{A} \to \mathbf{A} + \boldsymbol{\nabla}\psi$

Physics doesn't change

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Physics doesn't change

But the effective potential *does* depend on the gauge – not an obervable

Presents challenges in the phase tracing and transition rate calculations

From particle physics theory to GWs

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Effective Potential: can be computed perturbatively with finite temperature quantum field theory

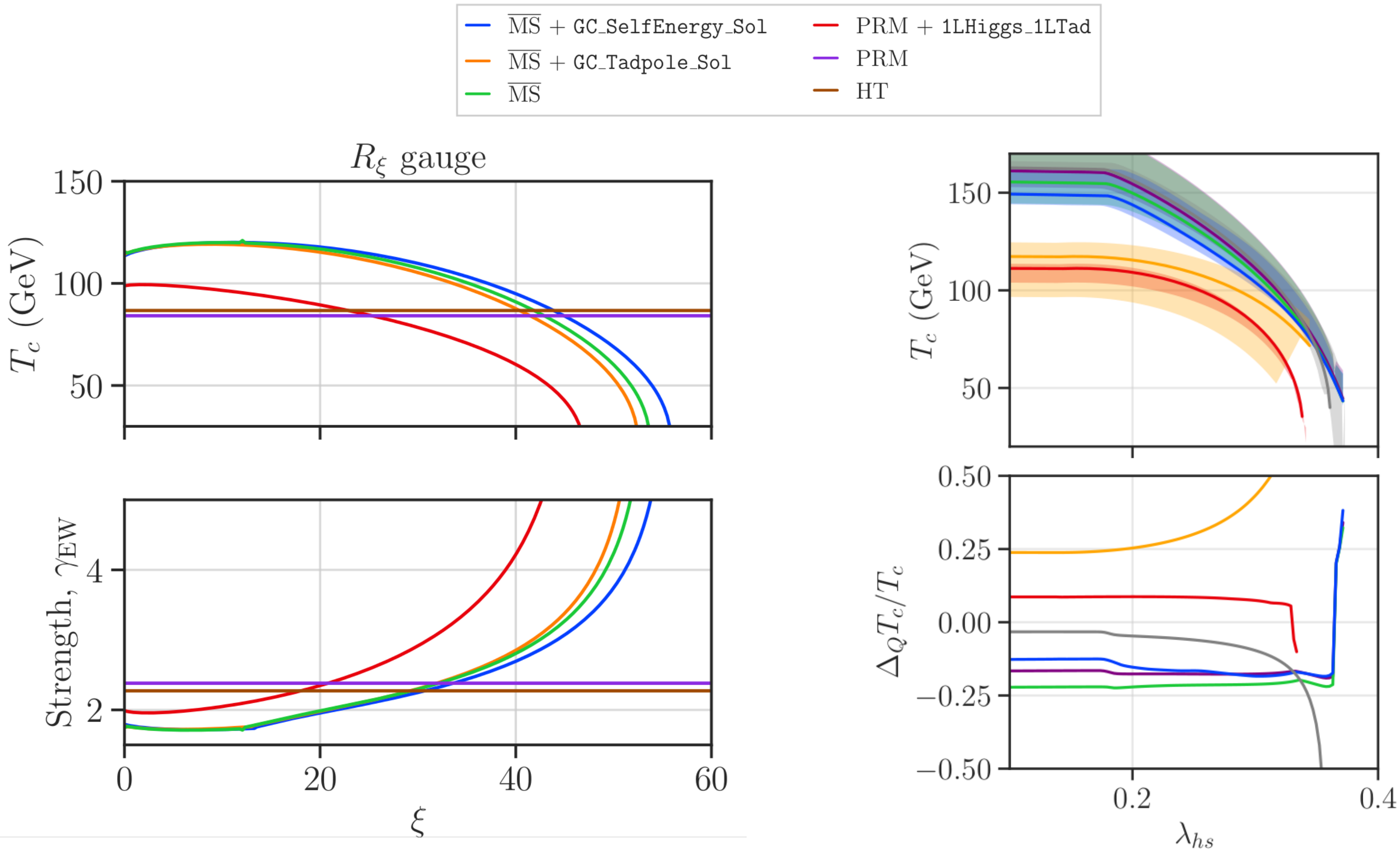
However there are problems appling this for phase transitons at finite temp

- Unphysical Gauge dependence
- Infrared divergences / problems with perturbativity for large T^2/m^2
- Many different scales in the problem
- thus large dependence on the renormalisation scale

Effective Potential

[PA, C. Balazs, A. Fowlie, L. Morris, G. White and Y.~Zhang, JHEP 01 (2023) 050]

Significant variance from gauge and renormalisation scale



Effective Potential

These issues have substantial impact on uncertainties in GW predictions

[Djuna Croon, Oliver Gould, Philipp Schicho, Tuomas V. I. Tenkanen, Graham White, JHEP 04 (2021) 055]

$\Delta\Omega_{\mathrm{GW}}/\Omega_{\mathrm{GW}}$	4d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$
Gauge dependence	$\mathcal{O}(10^1)$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$
Higher loop orders	unknown
Nucleation corrections	unknown

Effective Potential

These issues have substantial impact on uncertainties in GW predictions

[Djuna Croon, Oliver Gould, Philipp Schicho, Tuomas V. I. Tenkanen, Graham White, JHEP 04 (2021) 055]

$\Delta\Omega_{\rm GW}/\Omega_{\rm GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$

High temperture effects can be resummed by effective field theory techniques

But still room for non-perurbative effects

Effective Potential

These issues have substantial impact on uncertainties in GW predictions

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Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

High temperture effects can be resummed by effective field theory techniques

But non-perurbative effects may cause problems

From particle physics theory to GWs

$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$

Effective Potential:

Most rigorous approach is to do this non-perturbatively on lattice

This is how we know SM EW and QCD transtions are smooth cross-overs

[K. Kajantie, M. Laine, J. Peisa, K. Rummukainen, M. Shaposhnikov, PRL 77 (1996) 2887-2890,
Y. Aoki, G. Endrodi*, Z. Fodor*, S. D. Katz*, and K. K. Szabo, Nature, 443:675–678, 2006]
[*Eötvös affiliation]

Downside: Very time consuming to do this on the lattice

Not feasible for many transitions / models with huge parameter spaces

$\longrightarrow$ Tension between rigour and feasability

From particle physics theory to GWs

$$\hat{V}_{\text{eff}} \rightarrow \phi_i^{min}(T) \rightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \rightarrow P_f(T), N(T) \rightarrow \alpha, \beta, v_w \rightarrow h^2\Omega(f)$$

Effective Potential:

- Standard: 4D Perturbative approach with “Daisy resummation” — Easy to implement, Feasible for scans
- Better: 3D EFT Perturbative calculation — Hard to implement*, Feasible for scans
- Gold standard: non-perturbative lattice — Hard to implement, Not feasible for scans

* Very recently DRalgo code was developed to make this easier!
[Andreas Ekstedt, Philipp Schicho, Tuomas V. I. Tenkanen, Comp.Phys.Comm. 288 (2023) 108725]

State of the art: match to 3DEFT models with lattice results where possible,
use 3DEFT where not available (or create new lattice results...)

See e.g. [PRD 100 (2019) 11, 115024, Phys.Rev.Lett. 126 (2021) 17, 171802]

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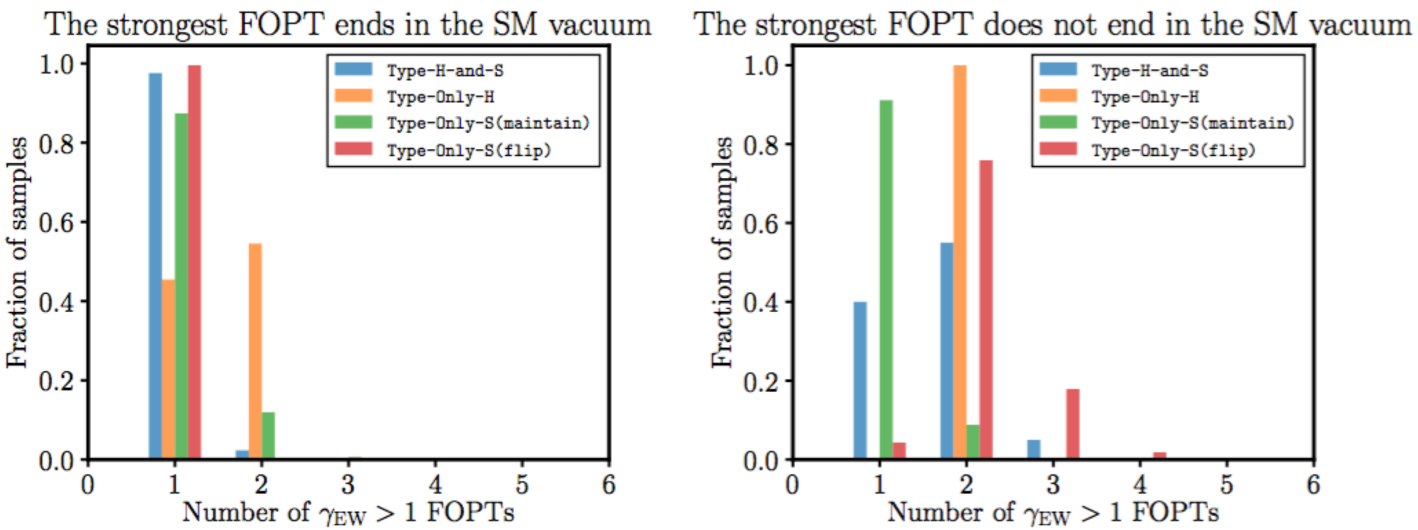
PhaseTracing

This is not straightforward:

multiple FOPTs and possible paths common in realsitic models



[PA, Csaba Balazs, Andrew Fowlie, Giancarlo Pozzo, Graham White, Yang Zhang, JHEP 11 (2019) 151]

From particle physics theory to GWs

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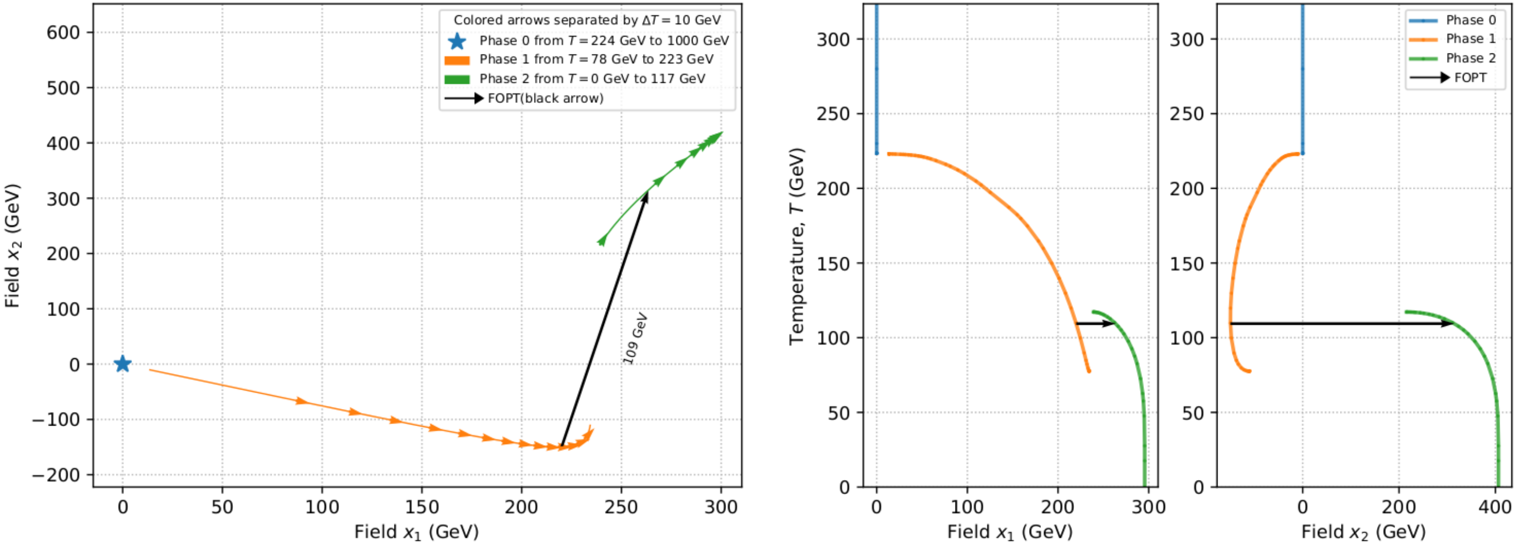

Careful algorithims needed to handle this, e.g.

PhaseTracer

[PhaseTracer, PA, Csaba Balazs, Andrew Fowlie, Yang Zhang, Eur.Phys.J.C 80 (2020) 6, 567]

From particle physics theory to GWs

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Transition rates   Semi-classical approx $\Gamma \approx A e^{-B}$

Action at saddle point

Fluctuations around saddle point

B solved by finding a “bounce” instanton solution numerically

Tricky numerical problem, many public bounce solvers

CosmoTransitions [C. L. Wainwright, CPC 183 (2012) 2006–2013,],

AnyBubble [A. Masoumi, K. D. Olum and B. Shlaer, JCAP 1701 (2017) 051],

BubbleProfiler [PA, Balazs, Bardsley, Fowlie, Harries & White CPC 244 (2019) 448-468]

SimpleBounce [Ryosuke Sato, CPC 258 (2021) 107566]

All bounce solvers to date have some signifcant drawbacks

(numerical stability, reliability, noise/precision, speed, number of fields)

From particle physics theory to GWs

$$\hat{V}_{\rm eff} \longrightarrow \phi_i^{min}(T) \longrightarrow \Gamma(\phi_i^{min} \to \phi_j^{min}) \longrightarrow P_f(T), N(T) \longrightarrow \alpha, \beta, v_w \longrightarrow h^2\Omega(f)$$

Action at saddle point

Transition rates   Semi-classical approx $\Gamma \approx A e^{-B}$

Fluctuations around saddle point

A usually assumed less important,
Often estimated on dimensional grounds

$$A \approx T^4$$

$$A \approx T^4 \left(B/(2\pi T)^{3/2}\right)$$

Problem: what if A has exponential dependence?

$\longrightarrow$ Calculate it directly $\longrightarrow$ BubbleDet

[Ekstedt, Gould, and Hirvonen, arXiv:2308.15652]

From particle physics theory to GWs

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Gauge dependence for critical temperatures can be obtained with hbar expansion

[H.H. Patel and M.J.Ramsey-Musolf, JHEP 07 (2011) 029]

Downside: can‘t be combined with Daisy resummation, fewer precision corrections

From particle physics theory to GWs

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Gauge independent calc for critical temperatures can be obtained with hbar expansion

[H.H. Patel and M.J.Ramsey-Musolf, JHEP 07 (2011) 029]

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Gauge independent calc for critical temperatures can be obtained with hbar expansion

[Lofgren, Ramsey-Musolf, Schicho,Tenkanen, PRL, 130 (2023) 25, 251801,
(+Hirvonen) JHEP 07 (2022) 135]

Downside: can't go beyond NLO, not yet implemented in any public software

From particle physics theory to GWs

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Bubble dynamics, nucleation & growth

Does the phase transition complete?

Interactions with plasma at the bubble wall

This depends on the bubble wall velocity v_w

Need to calculate friction $p_{\rm tot} = p_{\rm driving} - p_{\rm friction}$

From vacuum energy difference

Friction grows as bubble wall velocity increases

Completion and (indirectly) the GW spectrum depend on the bubble wall velocity v_w

Interactions with plasma at the bubble wall

Need to calculate friction $p_{\rm tot} = p_{\rm driving} - p_{\rm friction}$

From vacuum energy difference

Friction grows with v_w $\longrightarrow$ Expect a terminal v_w when $p_{\rm driving} = p_{\rm friction}$

Calculating friction - very active topic

Unresolved Issues

Common approaches:

- Vary v_w as an input parameter / uncertainty
- Fix to e.g. $v_w \approx c$
- Use special choice (Champman-Jouguet velocity)

In the Landau gauge you actually get:

$$V_T = \frac{T^4}{2\pi^2}\left[\sum_i n_\phi J_B\left(\frac{m^2_{\phi_i}}{T^2}\right) + \sum_j n_V J_B\left(\frac{m^2_{V_j}}{T^2}\right) - \sum_l n_f J_F\left(\frac{m^2_{f_l}}{T^2}\right)\right]$$

Field dependent masses of the scalar bosons, vector bosons and fermions

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These are functions for bosons and fermions of the form:

$$J_B(y^2) = \int_0^\infty dk\ k^2 \log\left[1 - e^{-\sqrt{k^2+y^2}}\right] \quad J_F(y^2) = \int_0^\infty dk\ k^2 \log\left[1 + e^{-\sqrt{k^2+y^2}}\right]$$

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To get insight do a high temperature expansion $y^2 \ll 1$ i.e. $T \gg m^2$

$$J_B^{HT}(y^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}y^2 - \frac{\pi}{6}y^3 + \ldots \qquad J_F^{HT}(y^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}y^2 \ldots$$

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$$V_T \approx \frac{T^2}{24}\left[\sum_i n_\phi m^2_{\phi_i} + \sum_j n_V m^2_{V_j} - \sum_l n_f m^2_{f_l}\right] - \frac{T}{12\pi}\left[\sum_i n_\phi m^3_{\phi_i} + \sum_j n_V m^3_{V_j}\right]$$

In the Landau gauge you actually get:

$$V_T = \frac{T^4}{2\pi^2}\left[\sum_i n_\phi J_B\left(\frac{m^2_{\phi_i}}{T^2}\right) + \sum_j n_V J_B\left(\frac{m^2_{V_j}}{T^2}\right) - \sum_l n_f J_F\left(\frac{m^2_{f_l}}{T^2}\right)\right]$$

These are functions for bosons and fermions of the form:

$$J_B(y^2) = \int_0^\infty dk\ k^2 \log\left[1 - e^{-\sqrt{k^2+y^2}}\right] \quad J_F(y^2) = \int_0^\infty dk\ k^2 \log\left[1 + e^{-\sqrt{k^2+y^2}}\right]$$

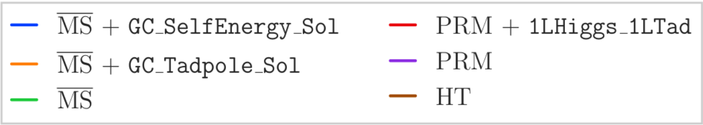
To get insight do a high temperature expansion $y^2 \ll 1$ i.e. $T \gg m^2$

$$J_B^{HT}(y^2) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}y^2 - \frac{\pi}{6}y^3 + \ldots \qquad J_F^{HT}(y^2) \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24}y^2 \ldots$$

$$V_T \approx \frac{T^2}{24}\left[\sum_i n_\phi m^2_{\phi_i} + \sum_j n_V m^2_{V_j} - \sum_l n_f m^2_{f_l}\right] - \frac{T}{12\pi}\left[\sum_i n_\phi m^3_{\phi_i} + \sum_j n_V m^3_{V_j}\right]$$

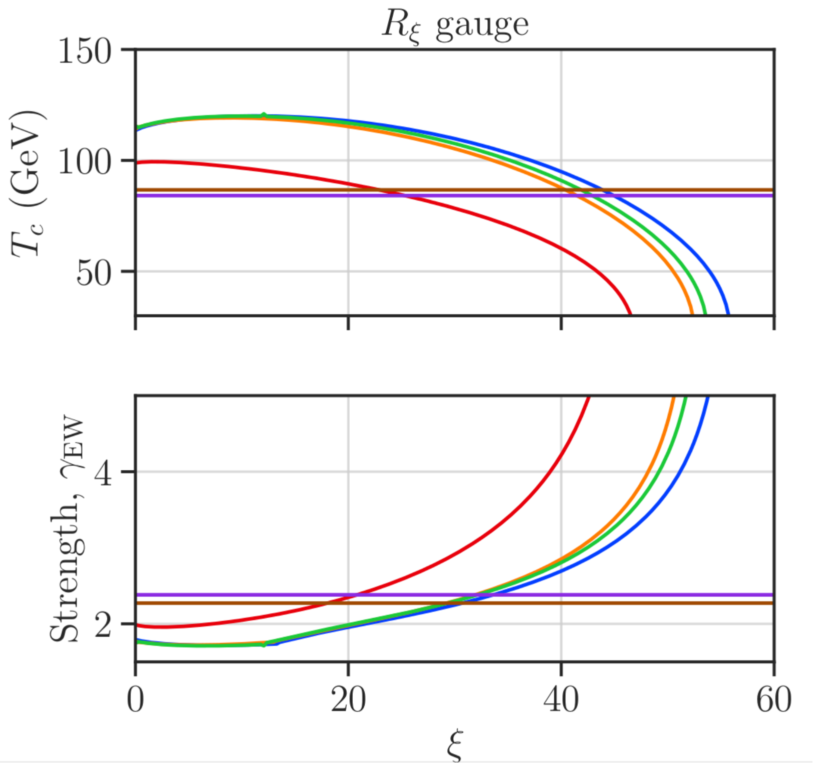
$\sim T^2\phi^2$ Quadratic term! $\sim T\phi^3$ Cubic term!

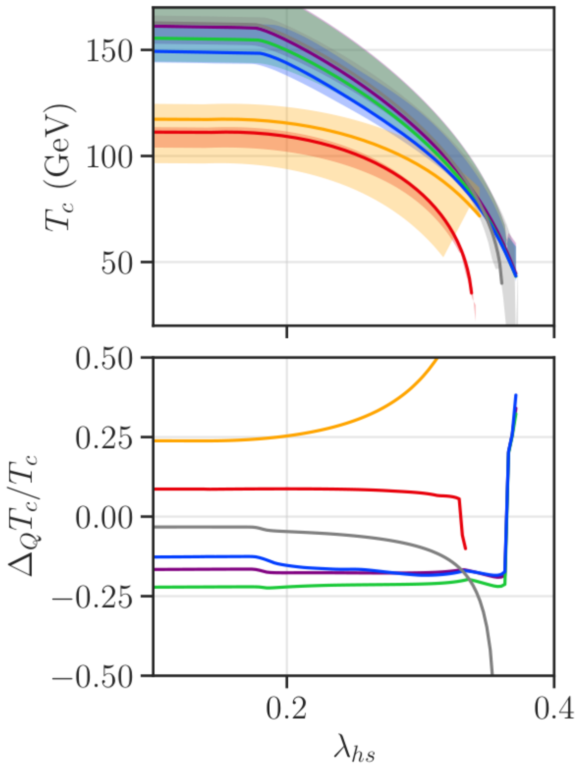
Effective Potential



Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale





Effective Potential

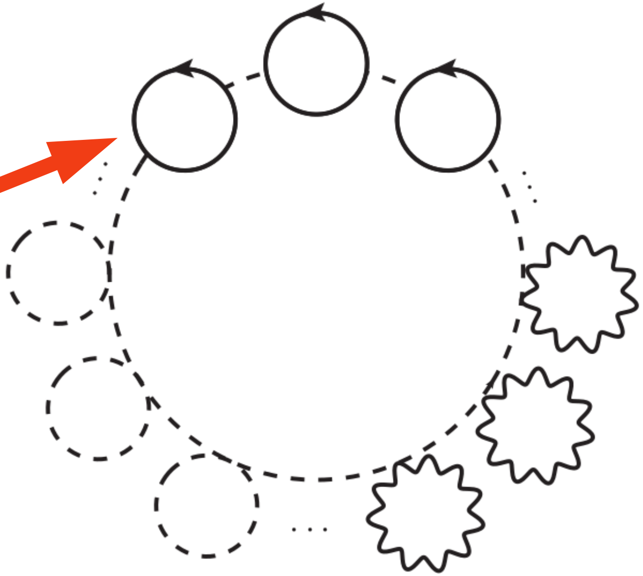
Perturbative estimates of the effective potential can be tricky

Significant variance from gauge and renormalisation scale

Resummation needed to to deal with high temperatures spoiling perturbativity

Daisy diagram with N-loops

Individual petals are inserted one-loop corrections



Resum daisy diagrams for leading order $\frac{T^2}{m^2}$

Effective Potential

- Better resummation by constructing a 3DEFT often called Dimensional Reductions (see e.g. D.Croon, O.Gould, P.Schicho, T.Tenkanen and G.White JHEP 04 (2021) 055)

- This can be done via automation of DRalgo for best feasible handing of effective potential as well!

- Gold standard is really to do things non-peturbatively on the lattice

- Not really feasible for scans in BSM models with many parameters

- But can be done for most exciting cases.

Bag model approximation breaks down when some masses $M \sim T$

We expect exactly this in EW transitions

Need to generalise from constant ϵ

Efficiency coefficient

Kinetic energy fraction $\longrightarrow$ $K = \frac{\kappa \alpha_i}{1+\alpha_i}$

Characterises energy released By PT

$\longrightarrow$ Extract ϵ, a from potential + get κ from JCAP 06 (2010) 028 for given v_w

Common generalisations

$$\alpha_\rho = \frac{\Delta \rho}{\rho_R}$$

"Latent heat"

$$\rho = V - T\frac{\partial V}{\partial T}$$

$$\alpha_\theta = \frac{\Delta(V - \frac{1}{4}T\frac{\partial V}{\partial T})}{\rho_R}$$

Trace anomaly

$$\theta = (\rho - 3p)/4$$

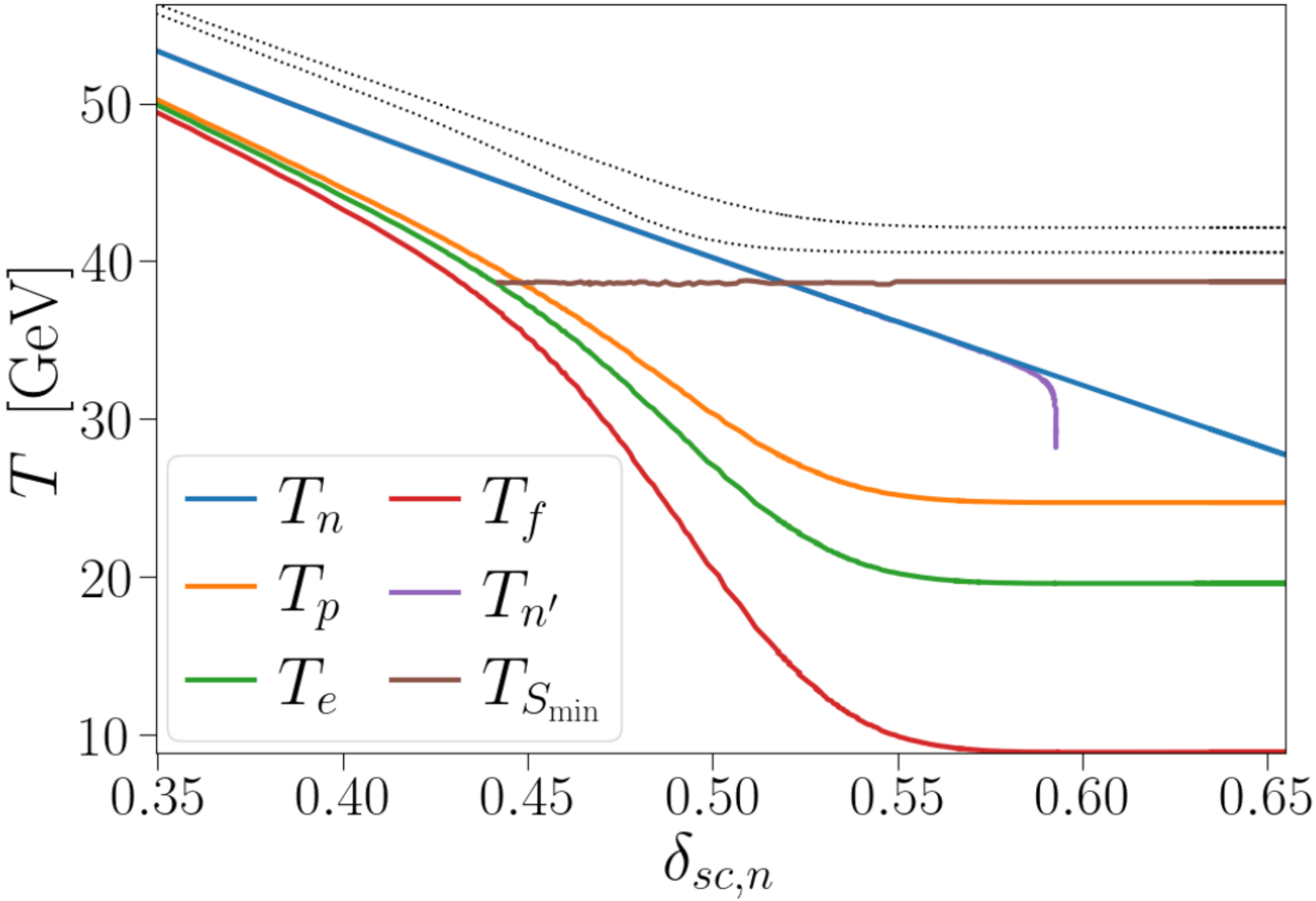
$$= V - \frac{1}{4}T\frac{\partial V}{\partial T}$$

$$\alpha_p = \frac{\Delta p}{\rho_R}$$

pressure

$$p = -V$$

Temperature dependence



Big news last month:
A stochastic gravitational wave background has been observed
by multiple Pulsar Timing Arrays experiments

Conservative interpretation would be supermassive black holes

Nonetheless SGWB is now a real thing to be used as data!

Now we really need to think about how precise our calculations are!

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ASTRONOMY AND SPACE | RESEARCH UPDATE

Pulsar timing irregularities reveals hidden gravitational-wave background

29 Jun 2023

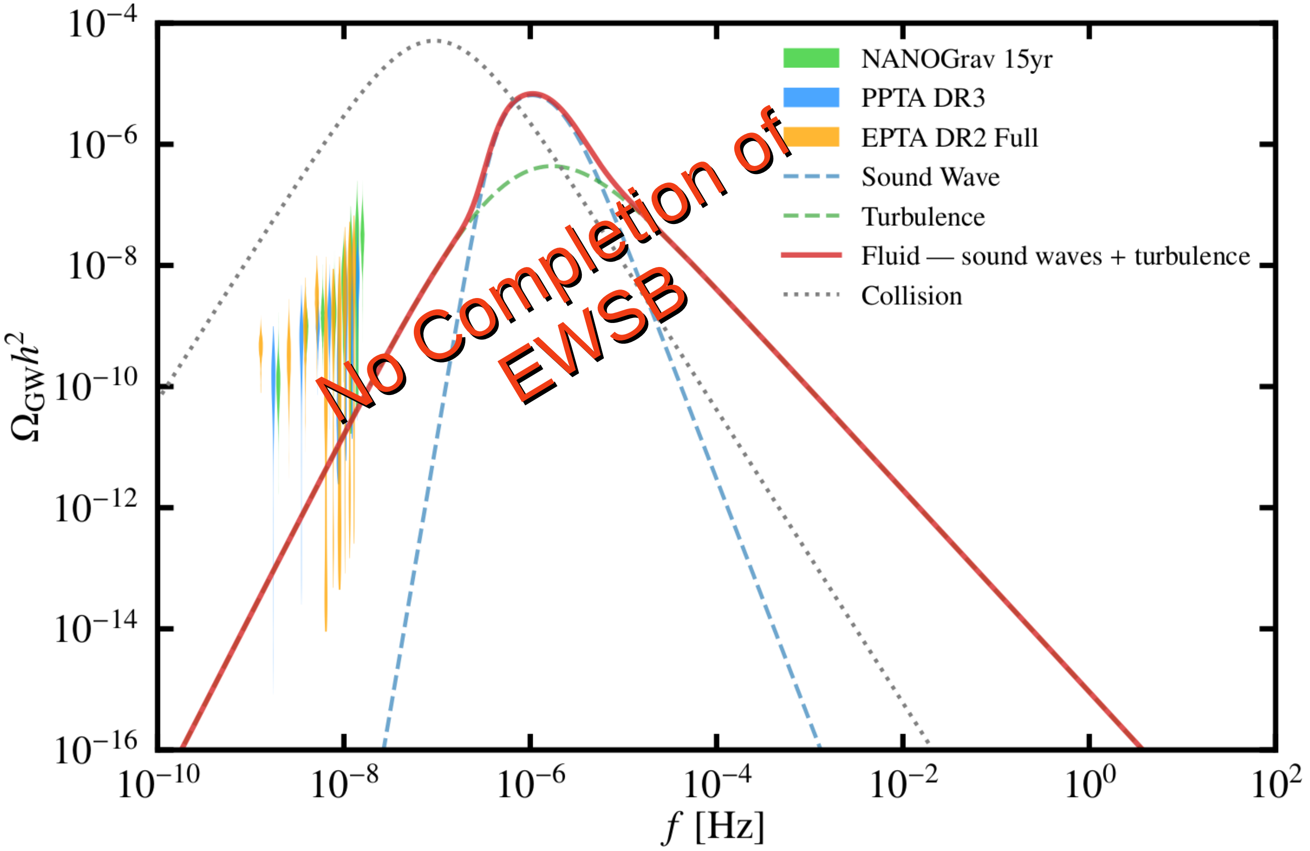
Cosmic claims: researchers have used radiotelescopes around the world to hunt for gravitational waves using the subtle variations in the timing of pulsars. (Courtesy: Aurore Simonnet for the NANOGrav Collaboration)

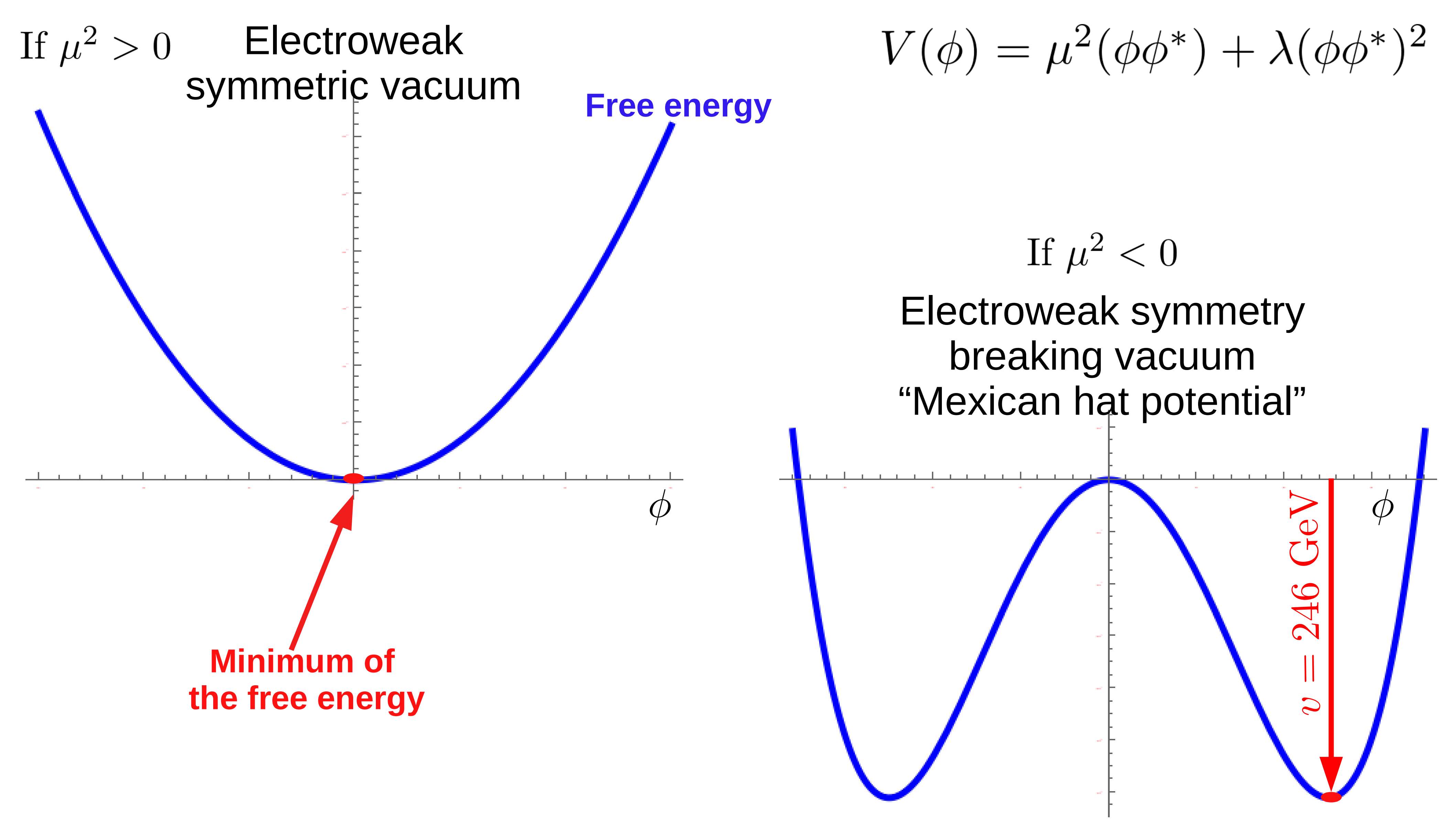
Big news last month:

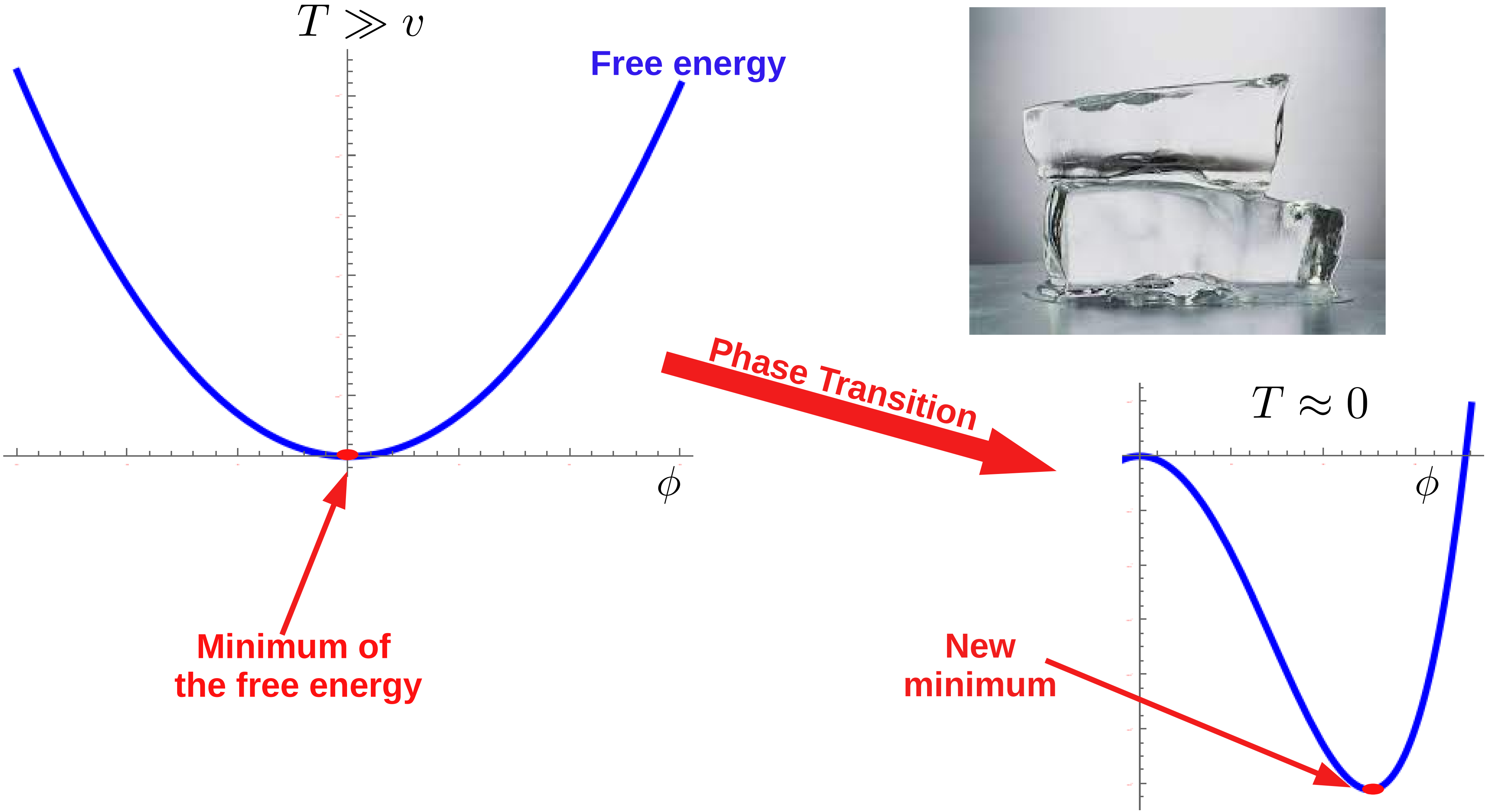
A stochastic gravitational wave background has been observed

by multiple Pulsar Timing Arrays experiments

Larger signals are ruled out in this model precisely because of one of the subtle effects I will discuss today!




If $\mu^2 > 0$
Electroweak symmetric vacuum
Free energy
ϕ
Minimum of the free energy
$V(\phi) = \mu^2(\phi\phi^*) + \lambda(\phi\phi^*)^2$
If $\mu^2 < 0$
Electroweak symmetry breaking vacuum
“Mexican hat potential”
$v = 246\ \mathrm{GeV}$
ϕ


$T \gg v$
Free energy
ϕ
Minimum of the free energy
Phase Transition
$T \approx 0$
ϕ
New minimum

GWs from First Order Phase Transitions

There are many subtleties and challenges in calculating GW spectra from cosmological PTs

For example this makes it easy to mistakenly predict a given model explains the data, get the wrong projections for future experiments or miss correlated features/constraints

I will discuss some subtle issues from JCAP 03 (2023), 006 and our review arxiv:2305.02357

- Nucleation is not enough – check PT completes
- Temperature dependence is very important, using most relevant temperatures really matters
- Hidden assumptions and approximations in thermal parameters and fits to calculations or simulations of gravitatonal wave spectra
- Resummation and gauge invariance in the effective potential treatment

  There are many other details I can’t cover, see original papers for details

Check the phase transiton completes

Especially for EWBG studies a common procedure was just finding FOPTS and checking it had $\gamma = v/T_c \gtrsim 1$

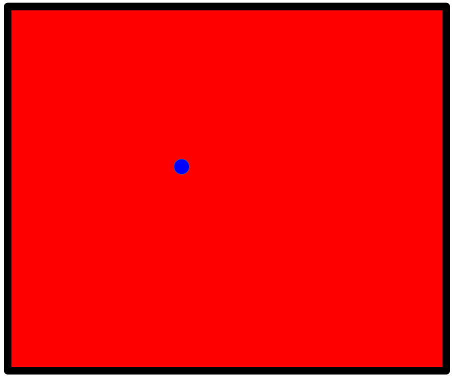
More careful studies checked that bubbles nucleate (one per Hubble volume)

$$N(T) = \int_T^{T_c} dT' \, \frac{\Gamma(T')}{T' H^4(T')}$$

$$N(T_n) = 1$$

Nucleation rate is computed from the bounce action, obtained from a bounce solver (e.g. BubbleProfiler, CosmoTransitions)

Hubble volume



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Much better to calculate the false vacuum fraction

$$P_f(T) = \exp\left[-\frac{4\pi}{3} v_w^3 \int_T^{T_c} \frac{\Gamma(T')dT'}{T'^4 H(T')} \left(\int_T^{T'} \frac{dT''}{H(T'')}\right)^3\right]$$

Check this can be reduced to a sufficiently small value, e.g. $P_f(T_f) < 0.01$

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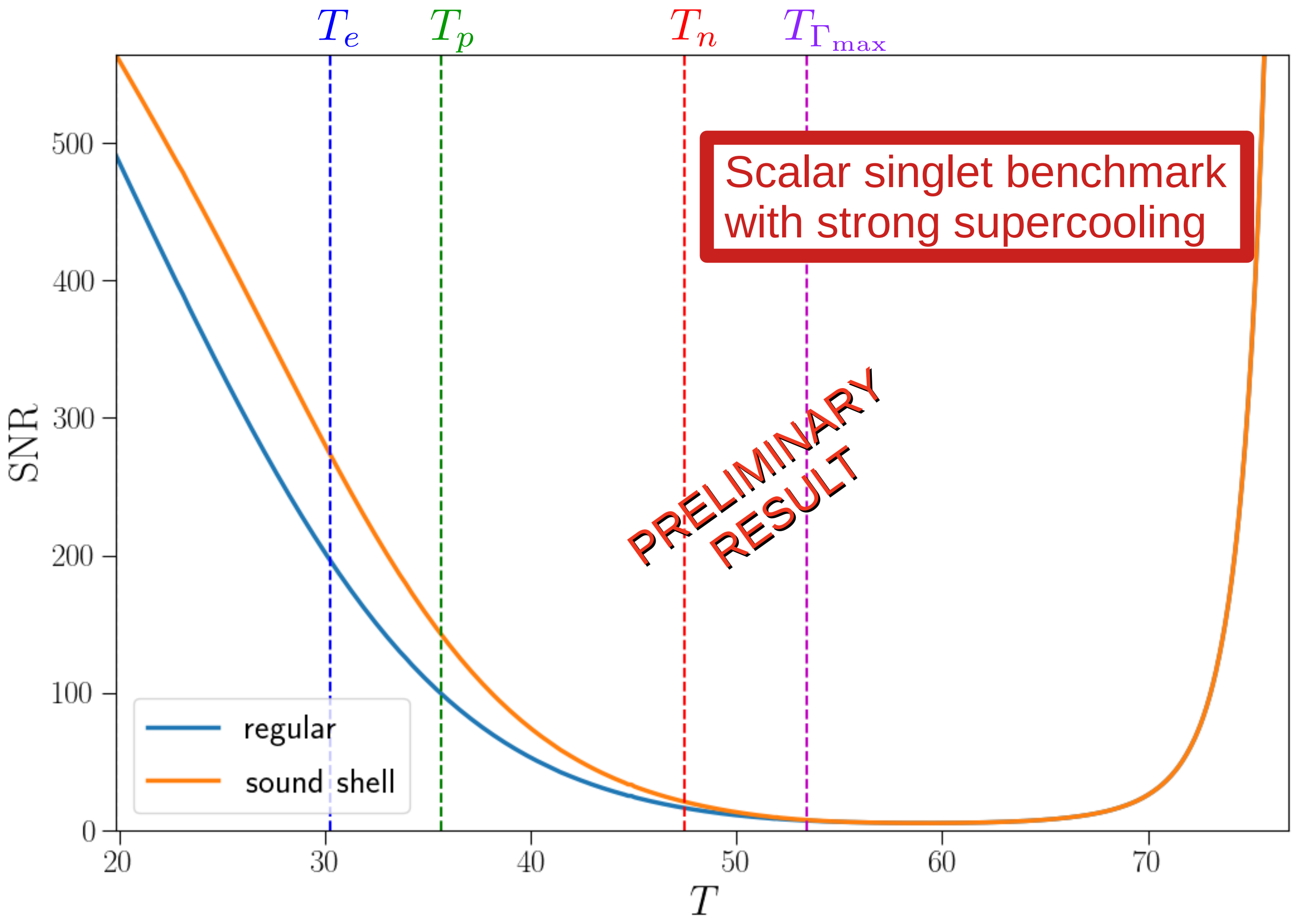
Check this can be reduced to a sufficiently small value, e.g. $P_f(T_f) < 0.01$

Warning: even this may not be enough to guarantee completion since space between the bubbles is also growing.

Temperature dependence



Very significant difference between SNR at percolation vs nucleation!

[PA, Lachlan Morris, Zhongxiu Xu, Prelininary findings]