Weakly coupled asymptotic safety up to four loops

Tom Steudtner

Technische Universität Dortmund

in collaboration with
Daniel Litim, Nahzaan Riyaz, Emmanuel Stamou

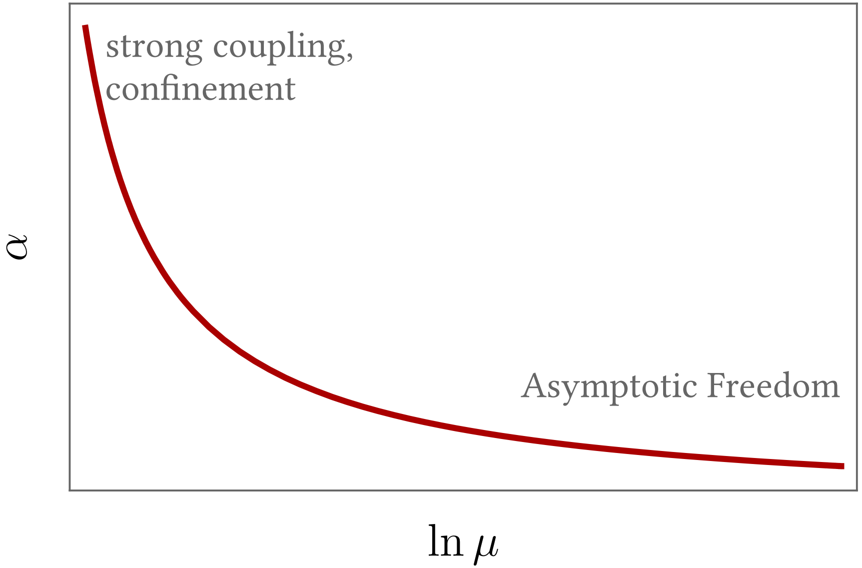
[2307.08747], [ongoing work]

NCBJ Seminar, October 24$^{\text{th}}$, 2023

Outline

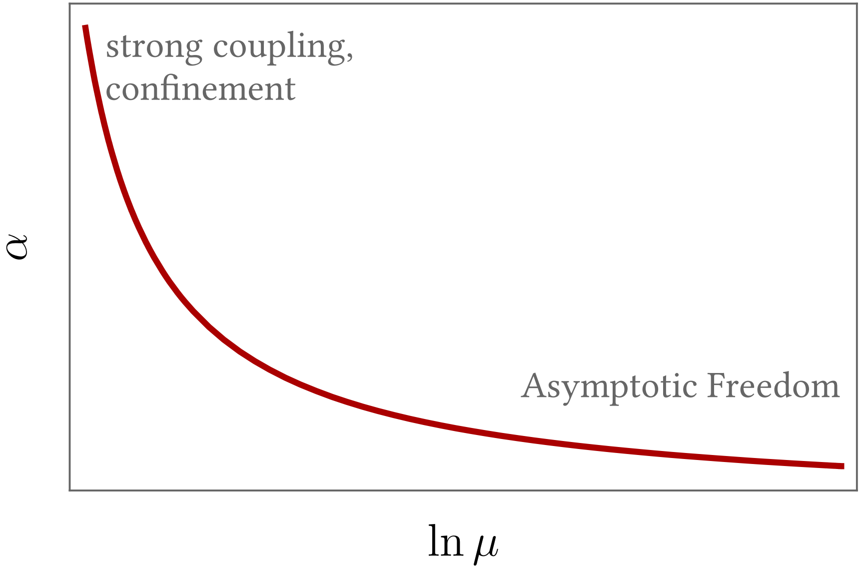
Motivation

» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]



Motivation

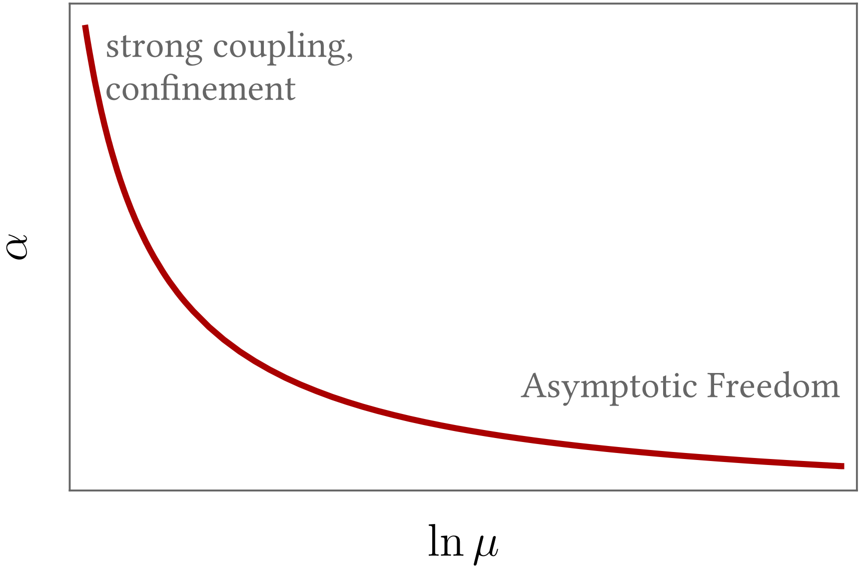
» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]

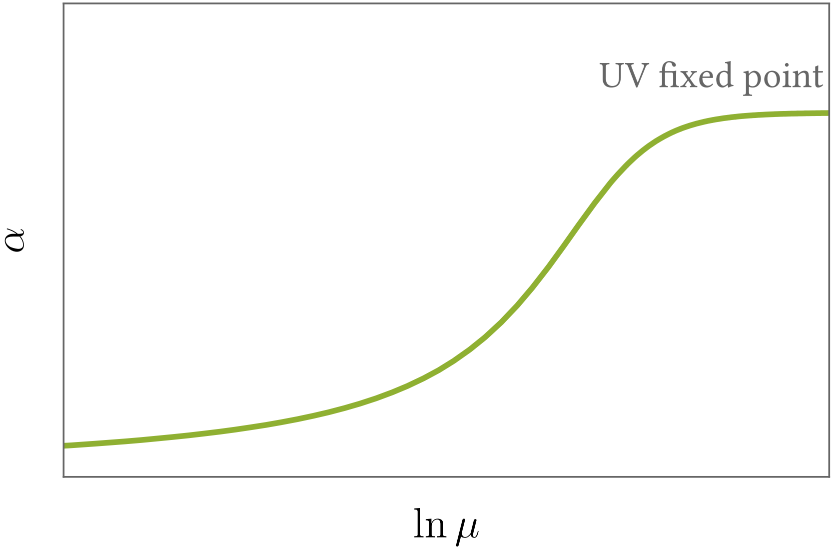


⟶ "UV complete": well defined at high energies
⟶ theory remains predictive
⟶ UV regime is perturbative!

Motivation

» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]



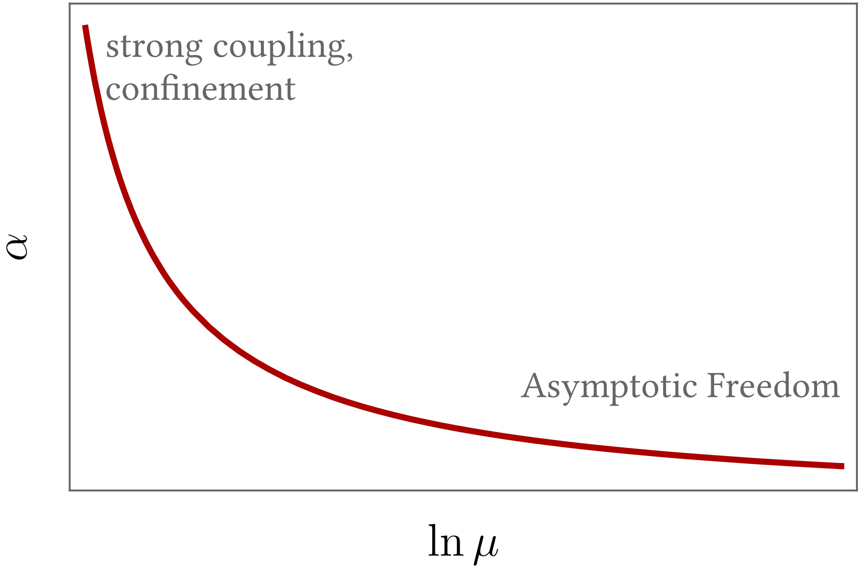


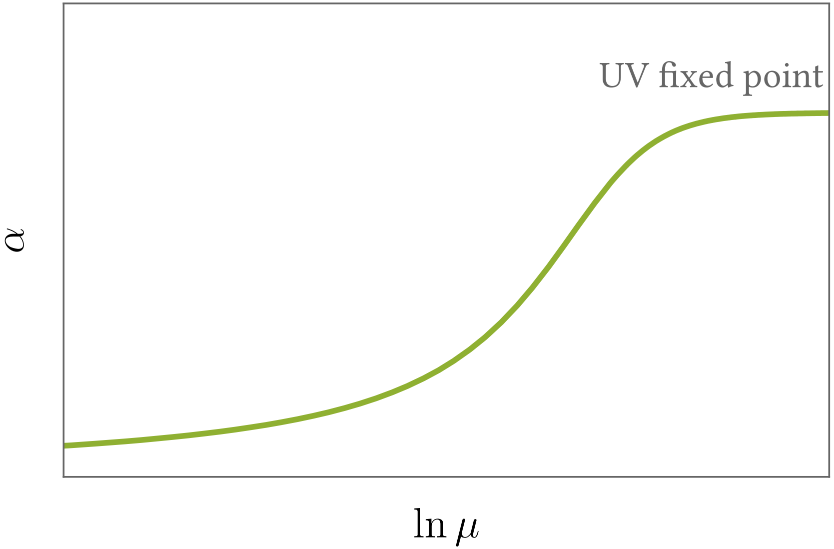
→ "UV complete": well defined at high energies
→ theory remains predictive
→ UV regime is perturbative!

» Generalisation: Asymptotic Safety [Weinberg (1980)]

Motivation

» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]





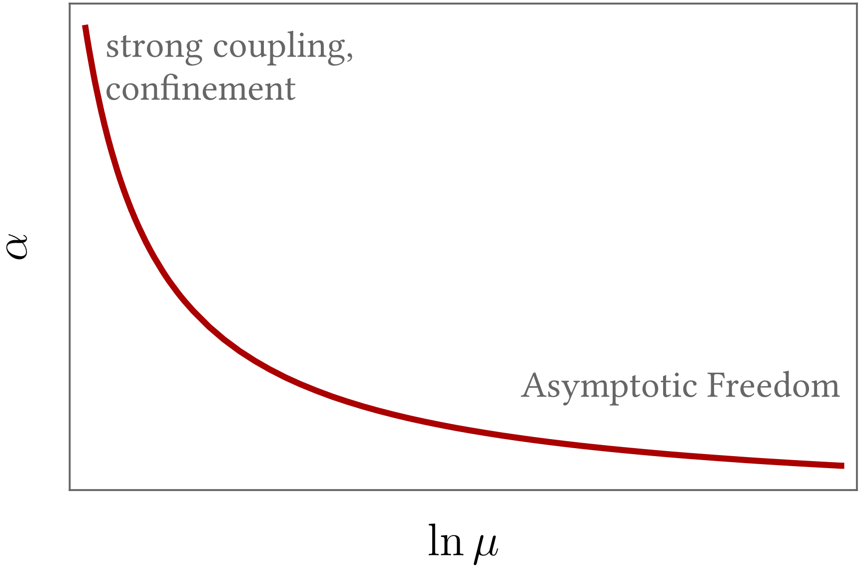
⟶ "UV complete": well defined at high energies
⟶ theory remains predictive
⟶ UV regime is perturbative!

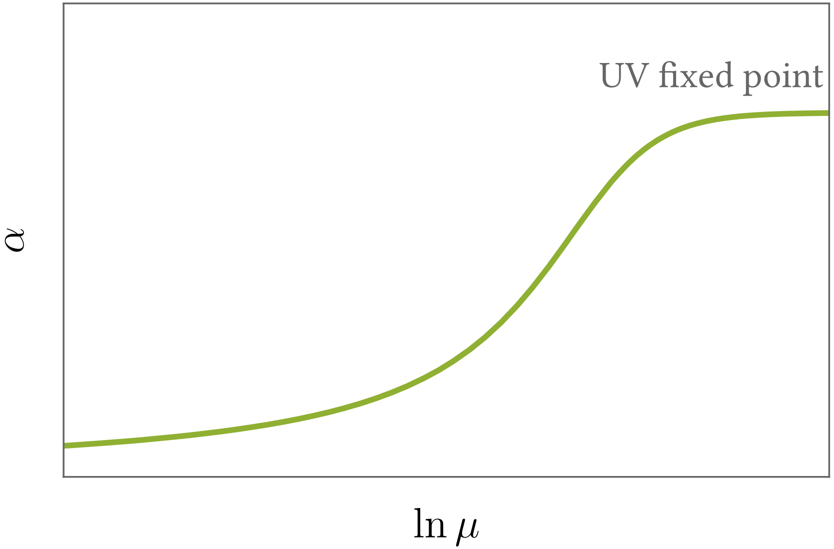
» Generalisation: Asymptotic Safety [Weinberg (1980)]

⟶ "exotic"
⟶ known reliable examples away from d=4
⟶ Perturbation theory? Strongly coupled?

Motivation

» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]





⟶ "UV complete": well defined at high energies
⟶ theory remains predictive
⟶ UV regime is perturbative!

» Generalisation: Asymptotic Safety [Weinberg (1980)]

⟶ "exotic"
⟶ known reliable examples away from d=4
⟶ Perturbation theory? Strongly coupled?

Wish list: 4d, weakly coupled, renormalizable

Perturbative Safety beyond Freedom

- scalars, fermions, gauge bosons
- charged matter beyond AF
- non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Perturbative Safety beyond Freedom

» scalars, fermions, gauge bosons

» charged matter beyond AF

» non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Still difficult to find an actual QFT! Perturbative reliability?

Perturbative Safety beyond Freedom

» scalars, fermions, gauge bosons

» charged matter beyond AF

» non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Still difficult to find an actual QFT! Perturbative reliability?

⟶ known examples are variants of the Litim-Sannino model
(or equivalent theories)

[Litim, Sannino 2014, 2015]

Perturbative Safety beyond Freedom

» scalars, fermions, gauge bosons

» charged matter beyond AF

» non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Still difficult to find an actual QFT! Perturbative reliability?

⟶ known examples are variants of the **Litim-Sannino model**
(or equivalent theories)

[Litim, Sannino 2014, 2015]

⟶ at (Veneziano) large N_f,N_c there is **strict perturbative control**
UV FP guaranteed to exist to all orders in perturbation theory!

Perturbative Safety beyond Freedom

» scalars, fermions, gauge bosons
» charged matter beyond AF
» non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Still difficult to find an actual QFT! Perturbative reliability?

⟶ known examples are variants of the **Litim-Sannino model** (or equivalent theories) [Litim, Sannino 2014, 2015]

⟶ at (Veneziano) large N_f,N_c there is **strict perturbative control**
UV FP guaranteed to exist to all orders in perturbation theory!

⟶ 3 families of QFTs, all equivalent in Veneziano limit (triality) [Bond, Litim, TS 2019]

$SU(N_c)$	$SO(N_c)$ or $Sp(N_c)$
Dirac, (LiSa)	Majorana fermions

Perturbative Safety beyond Freedom

» scalars, fermions, gauge bosons

» charged matter beyond AF

» non-abelian gauge, Yukawa interactions, scalar self-interactions [Bond, Litim 2016 & 2018]

Still difficult to find an actual QFT! Perturbative reliability?

⟶ known examples are variants of the **Litim-Sannino model** (or equivalent theories) [Litim, Sannino 2014, 2015]

⟶ at (Veneziano) large N_f,N_c there is **strict perturbative control**
UV FP guaranteed to exist to all orders in perturbation theory!

⟶ 3 families of QFTs, all equivalent in Veneziano limit (triality) [Bond, Litim, TS 2019]

$SU(N_c)$	$SO(N_c)$ or $Sp(N_c)$
Dirac, (LiSa)	Majorana fermions

⟶ no other simple theories under strict perturbative control [TS 2020 (PhD thesis)]

II. Litim-Sannino Model

Litim–Sannino–Model (LiSa)

Field		$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
'Quarks'	ψ_L	N_c	N_f	1
	ψ_R	N_c	1	N_f
'complex Meson'	ϕ	1	N_f	$\overline{N_f}$

Litim–Sannino–Model (LiSa)

Field		$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
'Quarks'	ψ_L	N_c	N_f	1
	ψ_R	N_c	1	N_f
'complex Meson'	ϕ	1	N_f	$\overline{N_f}$

$$\mathcal{L} = -\frac{1}{4} F^{A\mu\nu} F^{A}{}_{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}}$$
$$+ \operatorname{Tr}\left[\overline{\psi} i \not{D} \psi\right] - y \operatorname{Tr}\left[\overline{\psi}\left(\phi\, \mathcal{P}_R + \phi^\dagger\, \mathcal{P}_L\right)\psi\right]$$
$$+ \operatorname{Tr}\left[\partial^\mu \phi^\dagger \partial_\mu \phi\right] - m^2 \operatorname{Tr}\left[\phi^\dagger \phi\right] - u \operatorname{Tr}\left[\phi^\dagger \phi \phi^\dagger \phi\right] - v \operatorname{Tr}\left[\phi^\dagger \phi\right] \operatorname{Tr}\left[\phi^\dagger \phi\right]$$

single trace

double trace

gauge sector (QCD)

↓ ψ

Yukawa (chiral!)

↑ ϕ

scalar sector

Litim–Sannino–Model (LiSa)

Field		$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
'Quarks'	ψ_L	N_c	N_f	1
	ψ_R	N_c	1	N_f
'complex Meson'	ϕ	1	N_f	$\overline{N_f}$

$$\mathcal{L} = -\frac{1}{4} F^{A\mu\nu} F^{A}{}_{\mu\nu} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}}$$
$$+ \mathrm{Tr}\left[\overline{\psi} i \not{D} \psi\right] - y\, \mathrm{Tr}\left[\overline{\psi}\left(\phi\, \mathcal{P}_R + \phi^\dagger \mathcal{P}_L\right)\psi\right]$$
$$+ \mathrm{Tr}\left[\partial^\mu \phi^\dagger \partial_\mu \phi\right] - m^2\, \mathrm{Tr}\left[\phi^\dagger \phi\right] - u\, \mathrm{Tr}\left[\phi^\dagger \phi \phi^\dagger \phi\right] - v\, \mathrm{Tr}\left[\phi^\dagger \phi\right]\, \mathrm{Tr}\left[\phi^\dagger \phi\right]$$

single trace

double trace

gauge sector (QCD)

↓ ψ

Yukawa (chiral!)

↑ ϕ

scalar sector

→ interacting fixed points under perturbative control

Exact perturbative control

- Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$
- introduce ‘t Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

Exact perturbative control

» Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$

» introduce 't Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

» small and tunable expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\qquad -\frac{11}{2} < \epsilon < \infty$

Exact perturbative control

» Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$

» introduce ‘t Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

» small and tunable expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\quad -\frac{11}{2} < \epsilon < \infty$

» 1-Loop part of gauge beta function: $\beta_g = \alpha_g^2\, \left[\frac{4}{3}\epsilon + \mathcal{O}(\alpha^1)\right]$

» Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$

» introduce ‘t Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

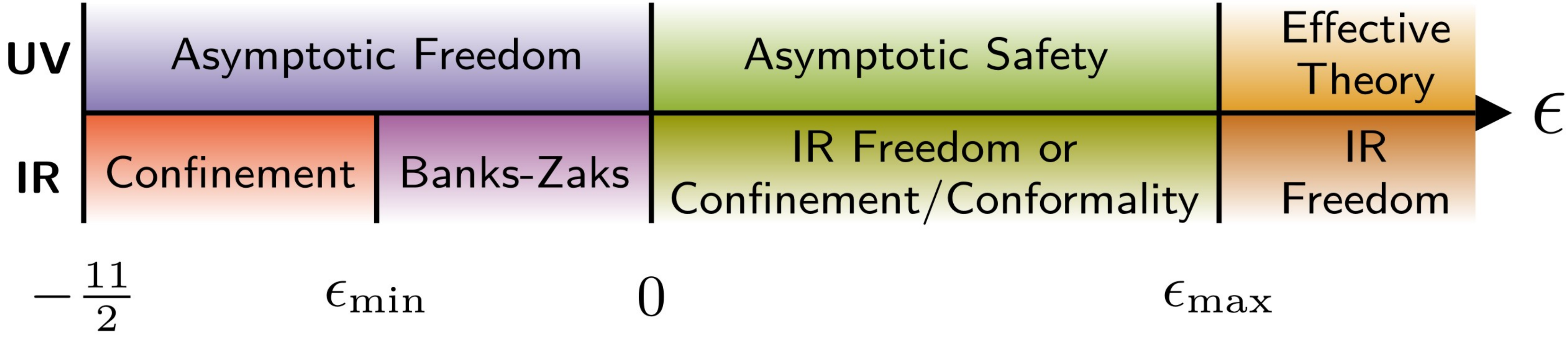
» small and tunable expansion parameter: $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\qquad -\frac{11}{2} < \epsilon < \infty$

» 1-Loop part of gauge beta function: $\beta_g = \alpha_g^2\, \left[\frac{4}{3}\epsilon + \mathcal{O}(\alpha^1)\right]$

» conformal expansion: $\alpha^* = \epsilon\, a_{\text{LO}} + \epsilon^2\, a_{\text{NLO}} + \epsilon^3\, a_{\text{NNLO}} + \ldots$

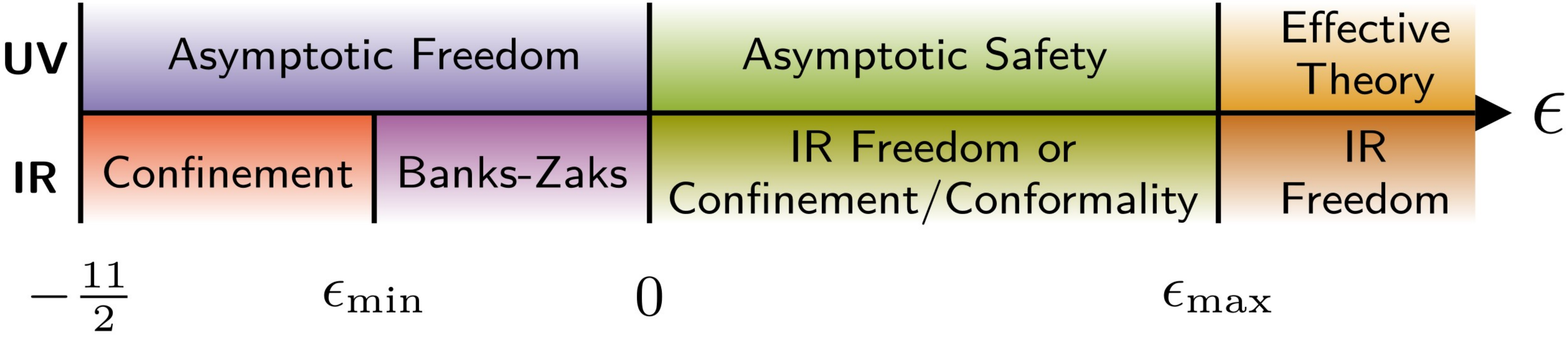
a_{LO}	a_{NLO}	a_{NNLO}
2-loop gauge 1-loop Yukawa 1-loop quartic	3-loop gauge 2-loop Yukawa 2-loop quartic	4-loop gauge 3-loop Yukawa 3-loop quartic
[Litim,Sannino, 2014]	[Bond, Medina, Litim, TS, 2017]	[Litim, Riyaz, Stamou, TS, *2023*]

Conformal Windows of LiSa



Conformal Windows of LiSa

Gaussian is UV FP

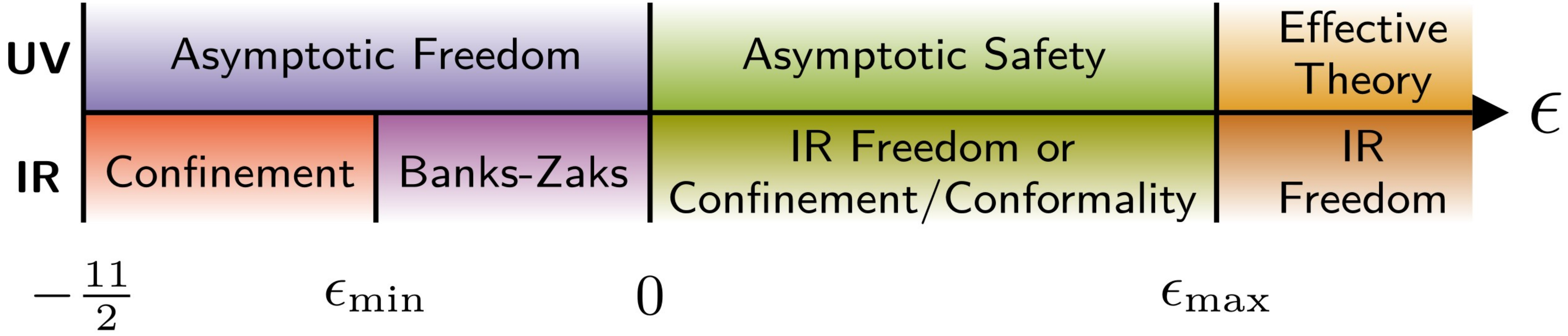


Conformal Windows of LiSa

Gaussian is UV FP

fully interacting UV FP

$\alpha^*_{g,y,u,v} \neq 0$

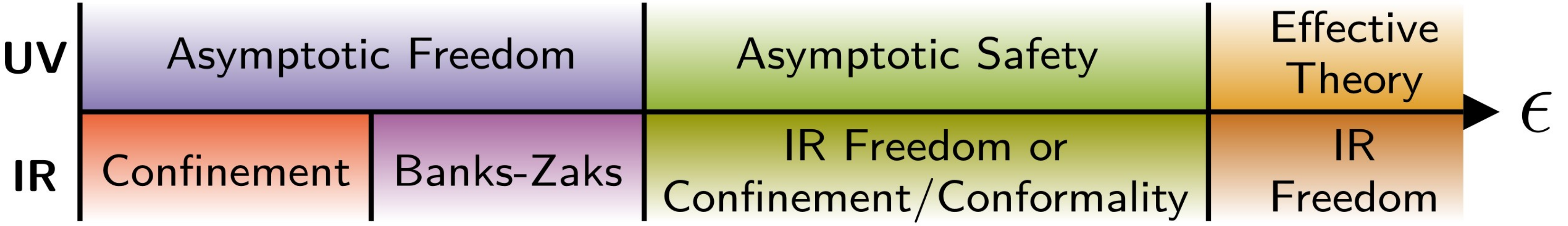


Conformal Windows of LiSa

Gaussian is UV FP

fully interacting UV FP

$\alpha^*_{g,y,u,v} \neq 0$



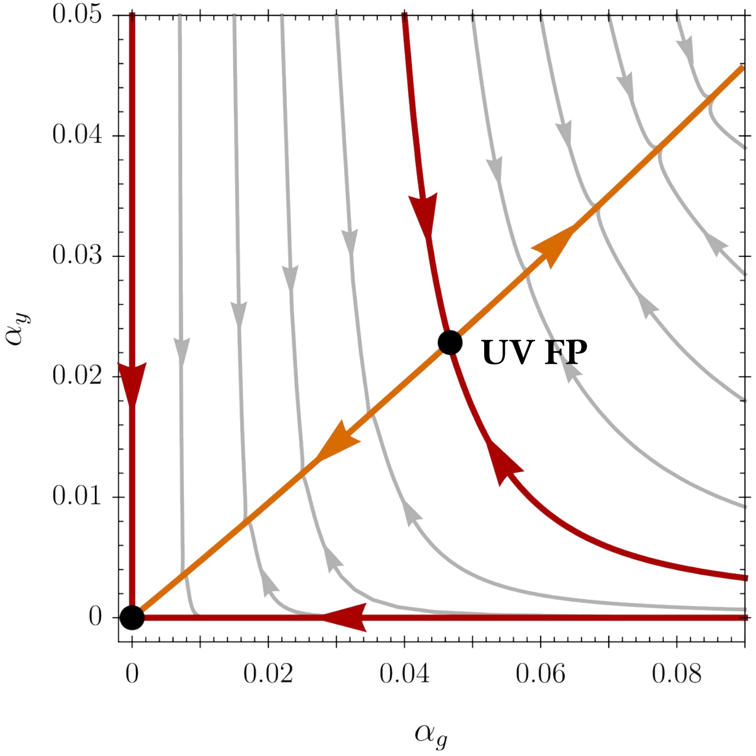
$-\frac{11}{2}$ ϵ_{min} 0 ϵ_{max}

$\longrightarrow$ disappears outside of UV conformal window $[0,\ \epsilon_{\text{max}}]$

$\longrightarrow$ determine ϵ_{max} $\longrightarrow$ $(N_f,\ N_c)_{\text{min}}$

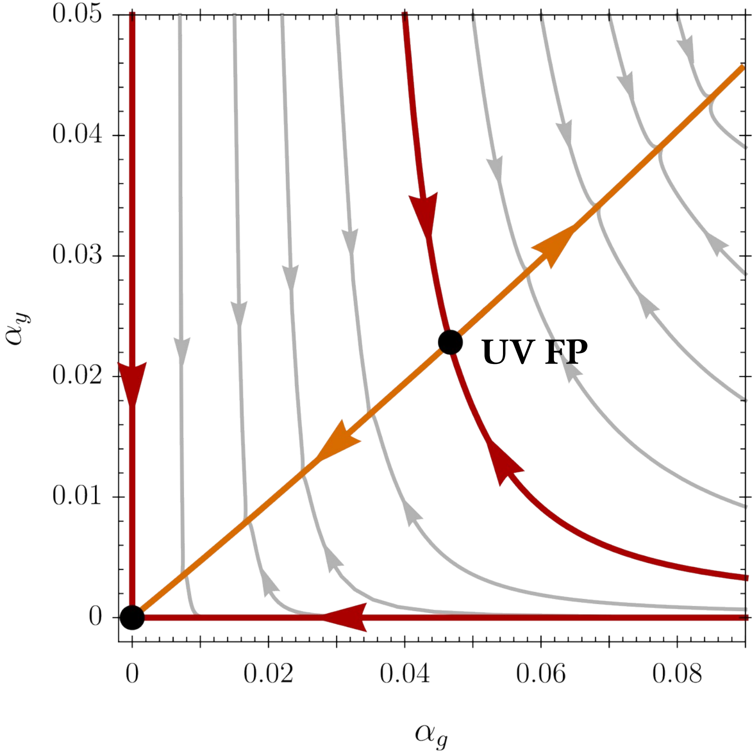
$\longrightarrow$ determine why

UV fixed point

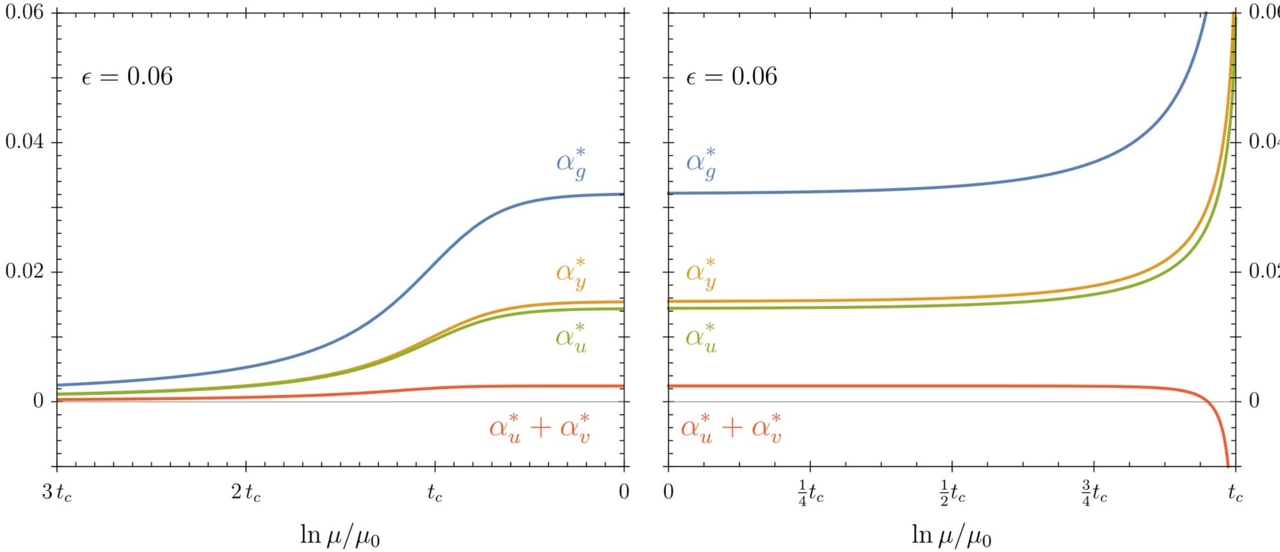


1 relevant and 3 irrelevant directions

UV fixed point



1 relevant and 3 irrelevant directions



III. Computation

Computation

» obtain β_g at 4 loops, $\beta_{y,u,v}$ at 3 loops and evaluate $\beta_{g,y,u,v} = 0$

Computation

- obtain β_g at 4 loops, $\beta_{y,u,v}$ at 3 loops and evaluate $\beta_{g,y,u,v} = 0$
- 4 loop gauge, 3 loop Yukawa RGEs are known for all ren. QFTs

  [Pool, Thomsen (2019)] [Bednyakov, Pikelner (2021)] [Davies, Herren, Thomsen (2021)]

Computation

» obtain β_g at 4 loops, $\beta_{y,u,v}$ at 3 loops and evaluate $\beta_{g,y,u,v} = 0$

» 4 loop gauge, 3 loop Yukawa RGEs are known for all ren. QFTs

[Pool, Thomsen (2019)] [Bednyakov, Pikelner (2021)] [Davies, Herren, Thomsen (2021)]

» Quartic 3-loop: pure scalar and Yukawa contributions are known for generic theories

[TS (2021)] [TS, Jack, Osborn (2023)]

Computation

» obtain β_g at 4 loops, $\beta_{y,u,v}$ at 3 loops and evaluate $\beta_{g,y,u,v} = 0$

» 4 loop gauge, 3 loop Yukawa RGEs are known for all ren. QFTs

[Pool, Thomsen (2019)] [Bednyakov, Pikelner (2021)] [Davies, Herren, Thomsen (2021)]

» Quartic 3-loop: pure scalar and Yukawa contributions are known for generic theories

[TS (2021)] [TS, Jack, Osborn (2023)]

» missing: gauge contributions to 3-loop Quartic beta functions

» determine (finite N) in two different ways

$\longrightarrow$ direct loop computation in LiSa

$\longrightarrow$ use template RGEs, extract from literature

Direct computation

» everything 3-loop (check against literature), 33.5k diagrams

» own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]

» RGEs from counterterms, need to distinguish UV and IR poles (massless fields, no ext. momenta)

Direct computation

» everything 3-loop (check against literature), 33.5k diagrams

» own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]

» RGEs from counterterms, need to distinguish UV and IR poles (massless fields, no ext. momenta)

» Infrared rearrangement with mass parameter [Misiak, Munz, Chetyrkin, '94 & '98]

$$\int_p \cdots \frac{1}{(p-q)^2} = \int_p \cdots \left(\frac{1}{\underbrace{p^2 - m_{\mathrm{IRA}}^2}_{\mathrm{DoD}=-2}} + \underbrace{\frac{2\,p\cdot q - q^2 - m_{\mathrm{IRA}}^2}{p^2 - m_{\mathrm{IRA}}^2} \frac{1}{(p-q)^2}}_{\mathrm{DoD}=-3} \right)$$

⟶ cut away finite terms systematically

⟶ IR divergencies are regulated due to m_{IRA}^2

Direct computation

» everything 3-loop (check against literature), 33.5k diagrams

» own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]

» RGEs from counterterms, need to distinguish UV and IR poles (massless fields, no ext. momenta)

» Infrared rearrangement with mass parameter [Misiak, Munz, Chetyrkin, '94 & '98]

$$\int_p \cdots \frac{1}{(p-q)^2} = \int_p \cdots \left(\underbrace{\frac{1}{p^2 - m_{\text{IRA}}^2}}_{\text{DoD}=-2} + \underbrace{\frac{2\,p\cdot q - q^2 - \cancel{m_{\text{IRA}}^2}}{p^2 - m_{\text{IRA}}^2} \frac{1}{(p-q)^2}}_{\text{DoD}=-3} \right)$$

⟶ cut away finite terms systematically

⟶ IR divergencies are regulated due to m_{IRA}^2

⟶ neglect numerator mass, include gluon and scalar mass counterterm

Direct computation

» everything 3-loop (check against literature), 33.5k diagrams

» own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]

» RGEs from counterterms, need to distinguish UV and IR poles (massless fields, no ext. momenta)

» Infrared rearrangement with mass parameter [Misiak, Munz, Chetyrkin, '94 & '98]

$$\int_p \cdots \frac{1}{(p-q)^2} = \int_p \cdots \left(\underbrace{\frac{1}{p^2 - m_{\mathrm{IRA}}^2}}_{\mathrm{DoD}=-2} + \underbrace{\frac{2\,p\cdot q - q^2 - \cancel{m_{\mathrm{IRA}}^2}}{p^2 - m_{\mathrm{IRA}}^2} \frac{1}{(p-q)^2}}_{\mathrm{DoD}=-3} \right)$$

⟶ cut away finite terms systematically

⟶ IR divergencies are regulated due to m_{IRA}^2

⟶ neglect numerator mass, include gluon and scalar mass counterterm

⟶ only massive vacuum integrals remain, common mass parameter

⟶ reduction formulas from LiteRed [Lee '12 & '13], 3 Master integrals @ 3L [Martin, Robertson, '16]

Direct computation

» everything 3-loop (check against literature), 33.5k diagrams

» own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]

» RGEs from counterterms, need to distinguish UV and IR poles (massless fields, no ext. momenta)

» Infrared rearrangement with mass parameter [Misiak, Munz, Chetyrkin, '94 & '98]

$$\int_p \cdots \frac{1}{(p-q)^2} = \int_p \cdots \left(\underbrace{\frac{1}{p^2 - m_{\mathrm{IRA}}^2}}_{\mathrm{DoD}=-2} + \underbrace{\frac{2\,p\cdot q - q^2 - \cancel{m_{\mathrm{IRA}}^2}}{p^2 - m_{\mathrm{IRA}}^2} \frac{1}{(p-q)^2}}_{\mathrm{DoD}=-3} \right)$$

⟶ cut away finite terms systematically

⟶ IR divergencies are regulated due to m_{IRA}^2

⟶ neglect numerator mass, include gluon and scalar mass counterterm

⟶ only massive vacuum integrals remain, common mass parameter

⟶ reduction formulas from LiteRed [Lee '12 & '13], 3 Master integrals @ 3L [Martin, Robertson, '16]

⟶ no γ_5 ambiguity [Chetyrkin, Zoller '12]

2^{nd} Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\, \phi^a \psi^i \psi^j - \frac{1}{4!} \lambda_{abcd}\, \phi^a \phi^b \phi^c \phi^d$$

2nd Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\,\phi^a\psi^i\psi^j - \frac{1}{4!}\lambda_{abcd}\,\phi^a\phi^b\phi^c\phi^d$$

» RGE ansatz can be written down for such tensors

$$\beta^\lambda_{abcd} = c_1\,\lambda_{abef}\lambda_{efcd} + c_2\,\mathrm{Tr}\left[y^a y^e\right]\lambda_{ebcd} + c_3\,\mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

⟶ all possible physics contained in tensor contractions, c_i are universal

2^{nd} Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\,\phi^a\psi^i\psi^j - \frac{1}{4!}\lambda_{abcd}\,\phi^a\phi^b\phi^c\phi^d$$

» RGE ansatz can be written down for such tensors

$$\beta^\lambda_{abcd} = c_1\,\lambda_{abef}\lambda_{efcd} + c_2\,\mathrm{Tr}\left[y^a y^e\right]\lambda_{ebcd} + c_3\,\mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

$\longrightarrow$ all possible physics contained in tensor contractions, c_i are universal

» fix coefficients from literature

2^{nd} Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\, \phi^a \psi^i \psi^j - \tfrac{1}{4!} \lambda_{abcd}\, \phi^a \phi^b \phi^c \phi^d$$

» RGE ansatz can be written down for such tensors

$$\beta^\lambda_{abcd} = c_1\, \lambda_{abef} \lambda_{efcd} + c_2\, \mathrm{Tr}\left[y^a y^e\right] \lambda_{ebcd} + c_3\, \mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

⟶ all possible physics contained in tensor contractions, c_i are universal

» fix coefficients from literature

» quartic RGE in gaugeless limit complete [TS 2021] [Jack, Osborn, TS, 2023]

» gauge contributions, only for charged fermions

2nd Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\,\phi^a\psi^i\psi^j - \frac{1}{4!}\lambda_{abcd}\,\phi^a\phi^b\phi^c\phi^d$$

» RGE ansatz can be written down for such tensors

$$\beta^{\lambda}_{abcd} = c_1\,\lambda_{abef}\lambda_{efcd} + c_2\,\mathrm{Tr}\left[y^a y^e\right]\lambda_{ebcd} + c_3\,\mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

⟶ all possible physics contained in tensor contractions, c_i are universal

» fix coefficients from literature

» quartic RGE in gaugeless limit complete [TS 2021] [Jack, Osborn, TS, 2023]

» gauge contributions, only for charged fermions

» reduction of TS due to gauge invariance

2^{nd} Method: Template RGEs

» template action with generalised fields, couplings:

$$\mathcal{L} \supset -y^a_{ij}\,\phi^a\psi^i\psi^j - \tfrac{1}{4!}\lambda_{abcd}\,\phi^a\phi^b\phi^c\phi^d$$

» RGE ansatz can be written down for such tensors

$$\beta^\lambda_{abcd} = c_1\,\lambda_{abef}\lambda_{efcd} + c_2\,\mathrm{Tr}\left[y^a y^e\right]\lambda_{ebcd} + c_3\,\mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

$\longrightarrow$ all possible physics contained in tensor contractions, c_i are universal

» fix coefficients from literature

» quartic RGE in gaugeless limit complete [TS 2021] [Jack, Osborn, TS, 2023]

» gauge contributions, only for charged fermions

» reduction of TS due to gauge invariance

» use SM 3L results [Chetyrkin, Zoller '12 –'13] [Bednyakov, Pikelner, Velizhanin '13] and QED-like gauge-Yukawa theory [Marquard, Boyack, Maciejko '18]

$\longrightarrow$ unable to fix all coefficients, but enough to compute LiSa RGEs!

IV. Conformal window

How to probe the UV conformal window

- beta functions $\beta_{g,y,u,v}$

  $\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$

How to probe the UV conformal window

» beta functions $\beta_{g,y,u,v}$

$\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$

$\longrightarrow$ critical exponents ϑ_i as eigenvalues of stability matrix $M_{xx'} = \frac{\partial \beta_x}{\partial \alpha_{x'}}\Big|_{\alpha=\alpha^*}$

$$(\alpha_x - \alpha_x^*) = c_{x,i}\left(\frac{\mu}{\mu_0}\right)^{\vartheta_i}$$

$$\vartheta_1 < 0 < \vartheta_{2,3,4}$$

How to probe the UV conformal window

» beta functions $\beta_{g,y,u,v}$

$\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$

$\longrightarrow$ critical exponents ϑ_i as eigenvalues of stability matrix $M_{xx'} = \left.\dfrac{\partial \beta_x}{\partial \alpha_{x'}}\right|_{\alpha=\alpha^*}$

$$(\alpha_x - \alpha_x^*) = c_{x,i} \left(\frac{\mu}{\mu_0}\right)^{\vartheta_i}$$

$$\vartheta_1 < 0 < \vartheta_{2,3,4}$$

» typical shape (all loop orders)

$$\mathcal{L} \supset -u\,\mathrm{Tr}\left[\phi^\dagger\phi\phi^\dagger\phi\right] - v\,\mathrm{Tr}\left[\phi^\dagger\phi\right]\,\mathrm{Tr}\left[\phi^\dagger\phi\right]$$

$$\beta_g = \alpha_g^2\left[\tfrac{4}{3}\epsilon + b_g(\alpha_{g,y,u},\epsilon)\right]$$

$$\beta_y = \alpha_y\, b_y(\alpha_{g,y,u},\epsilon)$$

$$\beta_u = b_u(\alpha_{g,y,u},\epsilon)$$

} "single trace"

How to probe the UV conformal window

- beta functions $\beta_{g,y,u,v}$

  $\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$

  $\longrightarrow$ critical exponents ϑ_i as eigenvalues of stability matrix $M_{xx'} = \left.\dfrac{\partial \beta_x}{\partial \alpha_{x'}}\right|_{\alpha=\alpha^*}$

$$(\alpha_x - \alpha^*_x) = c_{x,i} \left(\frac{\mu}{\mu_0}\right)^{\vartheta_i} \qquad \vartheta_1 < 0 < \vartheta_{2,3,4}$$

- typical shape (all loop orders)

$$\mathcal{L} \supset -u \, \mathrm{Tr}\left[\phi^\dagger \phi \phi^\dagger \phi\right] - v \, \mathrm{Tr}\left[\phi^\dagger \phi\right] \, \mathrm{Tr}\left[\phi^\dagger \phi\right]$$

$$\beta_g = \alpha_g^2 \left[\tfrac{4}{3}\epsilon + b_g(\alpha_{g,y,u}, \epsilon)\right]$$

$$\beta_y = \alpha_y \, b_y(\alpha_{g,y,u}, \epsilon)$$

$$\beta_u = b_u(\alpha_{g,y,u}, \epsilon)$$

"single trace"

$$\beta_v = f_0(\alpha_{g,y,u}, \epsilon) + f_1(\alpha_{g,y,u}, \epsilon)\, \alpha_v + f_2(\alpha_{g,y,u}, \epsilon)\, \alpha_v^2$$

$\longrightarrow$ quadratic shape, up to two solutions $\alpha_v^{*\pm}$ for each $\alpha^*_{g,y,u}$

"double trace"

Constraints on the UV conformal window

» strong coupling $\alpha_x^* \gtrsim 1$

Constraints on the UV conformal window

» strong coupling $\alpha_x^* \gtrsim 1$ → not the case, entire window weakly coupled

Constraints on the UV conformal window

- strong coupling $\alpha_x^* \gtrsim 1$ $\longrightarrow$ not the case, entire window weakly coupled
- vacuum instability: FP potential needs to be bounded from below

$$\alpha_u^* > 0 \qquad \alpha_u^* + \alpha_v^* > 0$$

Constraints on the UV conformal window

- strong coupling $\alpha_x^* \gtrsim 1$ $\longrightarrow$ not the case, entire window weakly coupled
- vacuum instability: FP potential needs to be bounded from below

$$\alpha_u^* > 0 \qquad \alpha_u^* + \alpha_v^* > 0$$

- FP merger: UV FP collides with another solution at $\epsilon_{\max}$, both become complex

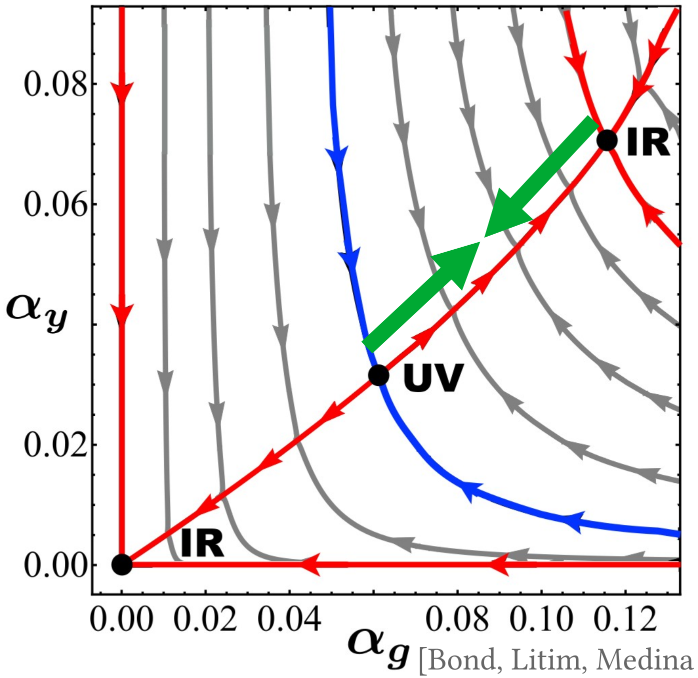
Constraints on the UV conformal window

» strong coupling $\alpha_x^* \gtrsim 1$ $\longrightarrow$ not the case, entire window weakly coupled

» vacuum instability: FP potential needs to be bounded from below

$$\alpha_u^* > 0 \qquad \alpha_u^* + \alpha_v^* > 0$$

» FP merger: UV FP collides with another solution at $\epsilon_{\max}$, both become complex



[Bond, Litim, Medina '20]

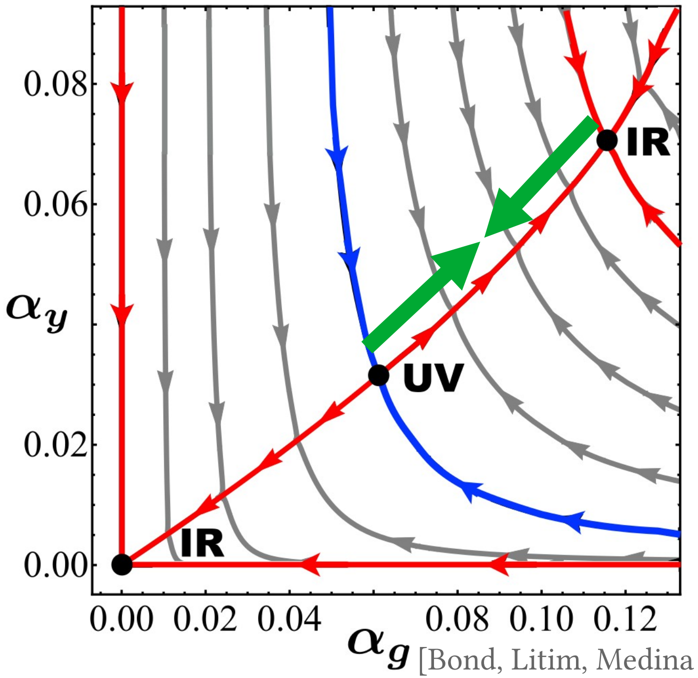
Constraints on the UV conformal window

» strong coupling $\alpha_x^* \gtrsim 1$ → not the case, entire window weakly coupled

» vacuum instability: FP potential needs to be bounded from below

$$\alpha_u^* > 0 \qquad \alpha_u^* + \alpha_v^* > 0$$

» FP merger: UV FP collides with another solution at $\epsilon_{\max}$, both become complex



[Bond, Litim, Medina '20]

→ single trace merger in $\alpha^*_{g,y,u}$ system

$$\vartheta_1(\epsilon_{\max}) = 0$$

→ double trace merger: two solutions α_v^* for same $\alpha^*_{g,y,u}$

$$\vartheta_3(\epsilon_{\max}) = 0$$

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

$\longrightarrow$ series is exact up to third term

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

⟶ series is exact up to third term

⟶ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \dots$$

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

→ series is exact up to third term

→ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \ldots$$

→ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \ldots$$

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

$\longrightarrow$ series is exact up to third term

$\longrightarrow$ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \ldots$$

$\longrightarrow$ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \ldots$$

$\longrightarrow$ no hint for double-trace merger,
unstable second solution

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

⟶ series is exact up to third term

⟶ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \ldots$$

⟶ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \ldots$$

⟶ no hint for double-trace merger,
unstable second solution

⟶ power series is short, bad convergence

⟶ can employ Padè resummation in ϵ

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

→ series is exact up to third term

→ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \ldots$$

→ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \ldots$$

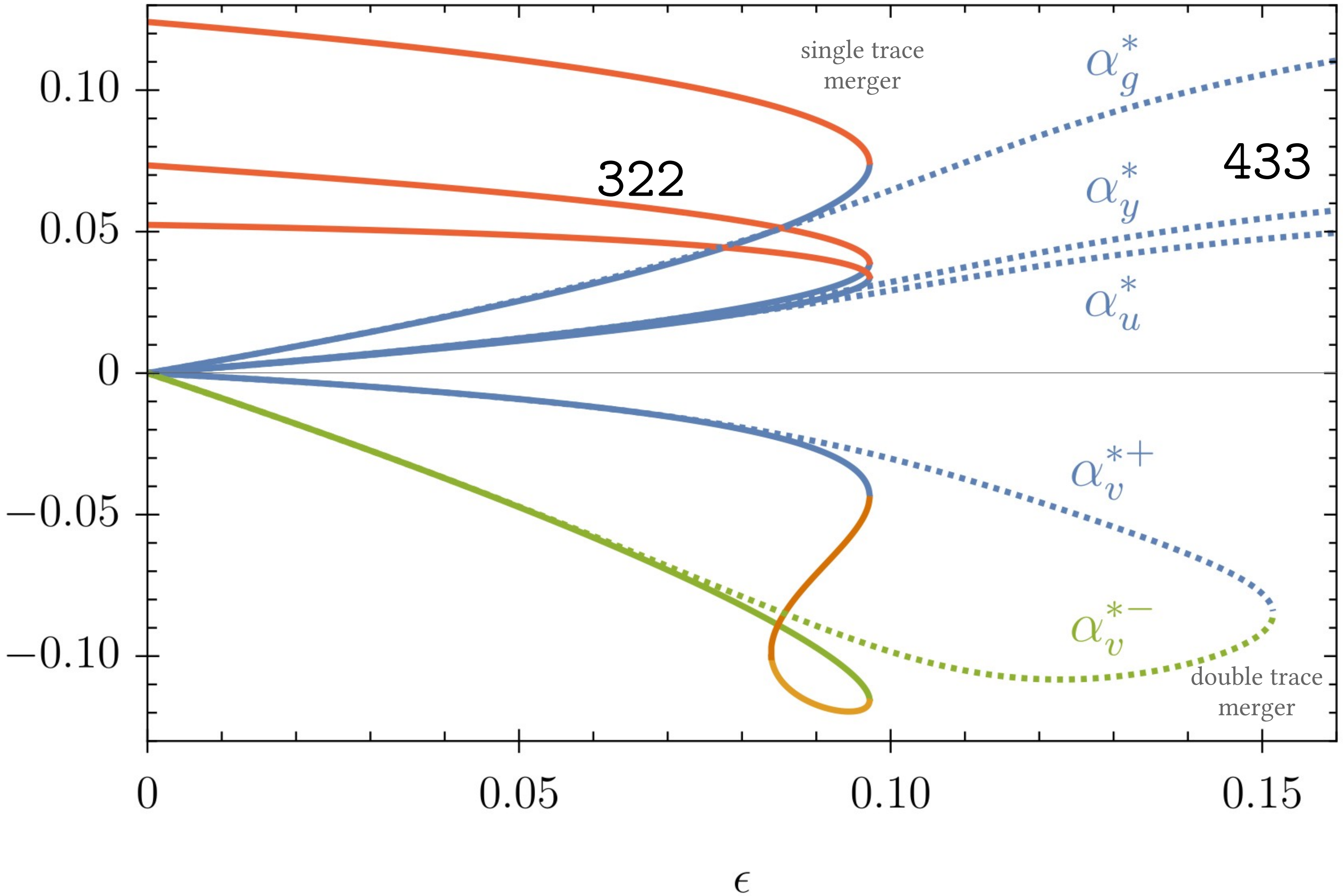
→ no hint for double-trace merger, unstable second solution

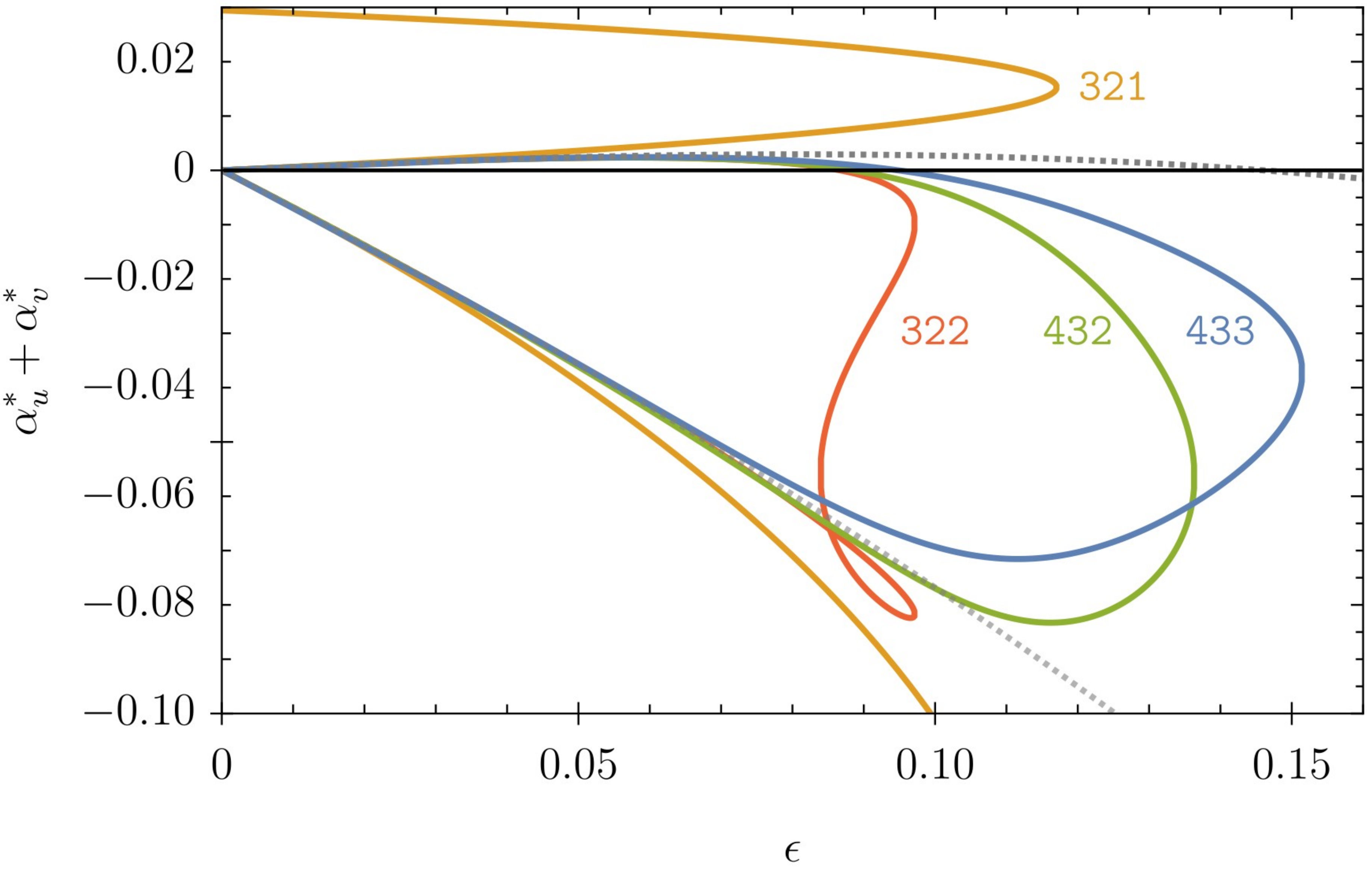
→ power series is short, bad convergence

→ can employ Padè resummation in ϵ

» $\beta_{g,y,u,v} = 0$

→ resums higher orders in ϵ


single trace
merger
322
433
α_g^*
α_y^*
α_u^*
α_v^{*+}
α_v^{*-}
double trace
merger
0.10
0.05
0
−0.05
−0.10
0
0.05
0.10
0.15
ϵ



321
322
432
433
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.10
0
0.05
0.10
0.15
$\alpha_u^* + \alpha_v^*$
ϵ

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

⟶ series is exact up to third term

⟶ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \ldots$$

⟶ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \ldots$$

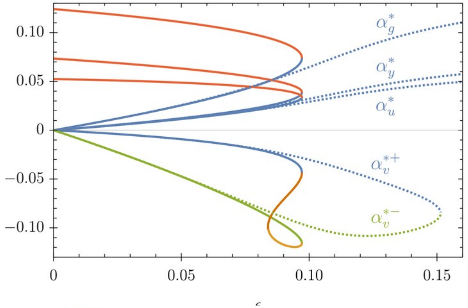
⟶ no hint for double-trace merger, unstable second solution

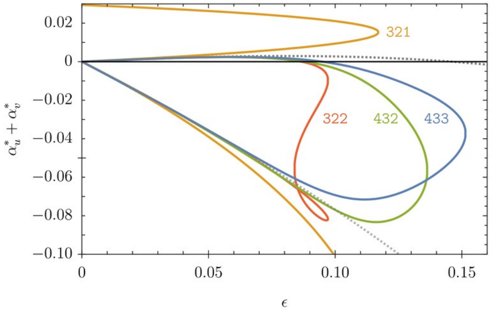
⟶ power series is short, bad convergence

⟶ can employ Padè resummation in ϵ

» $\beta_{g,y,u,v} = 0$

⟶ resums higher orders in ϵ





⟶ hints at vacuum instability

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

⟶ series is exact up to third term

⟶ hints for vacuum instability

$$\alpha_u^* + \alpha_v^* \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \dots$$

⟶ hints for single trace merger

$$\vartheta_1 \approx -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 6.947\,\epsilon^4 + \dots$$

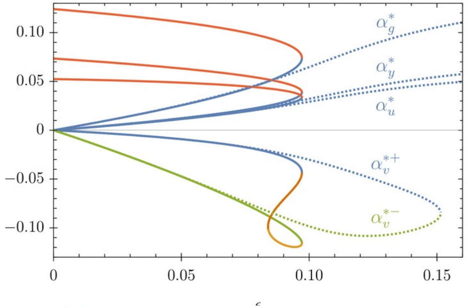
⟶ no hint for double-trace merger, unstable second solution

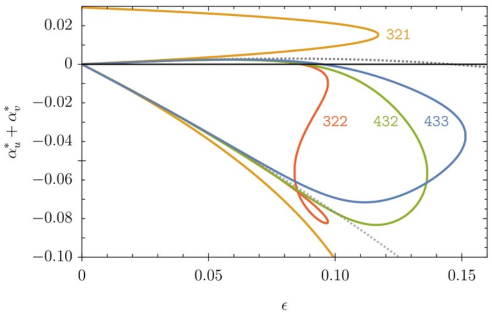
⟶ power series is short, bad convergence

⟶ can employ Padè resummation in ϵ

» $\beta_{g,y,u,v} = 0$

⟶ resums higher orders in ϵ



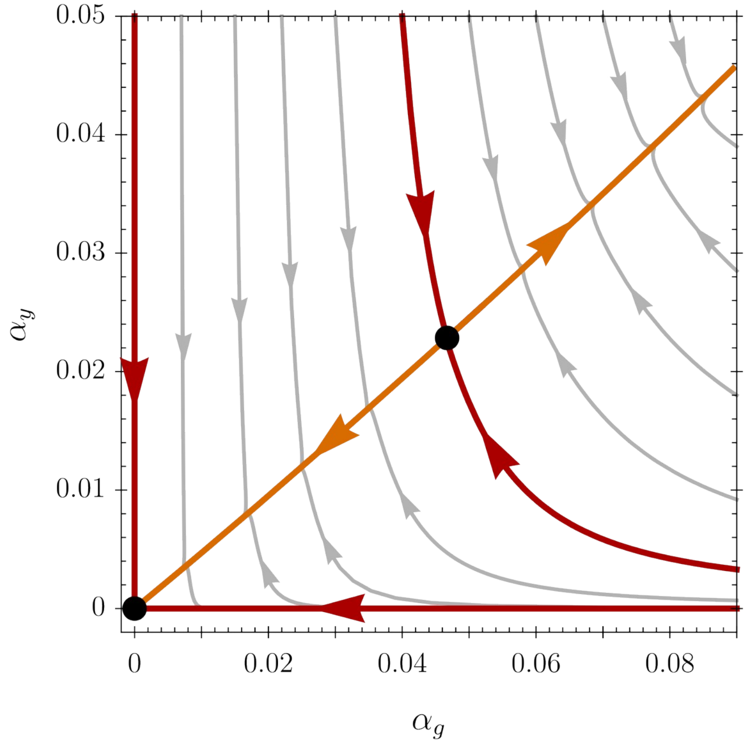


⟶ hints at vacuum instability

» $\beta_{g,\text{eff}} = 0$

⟶ RGE along relevant separatrix

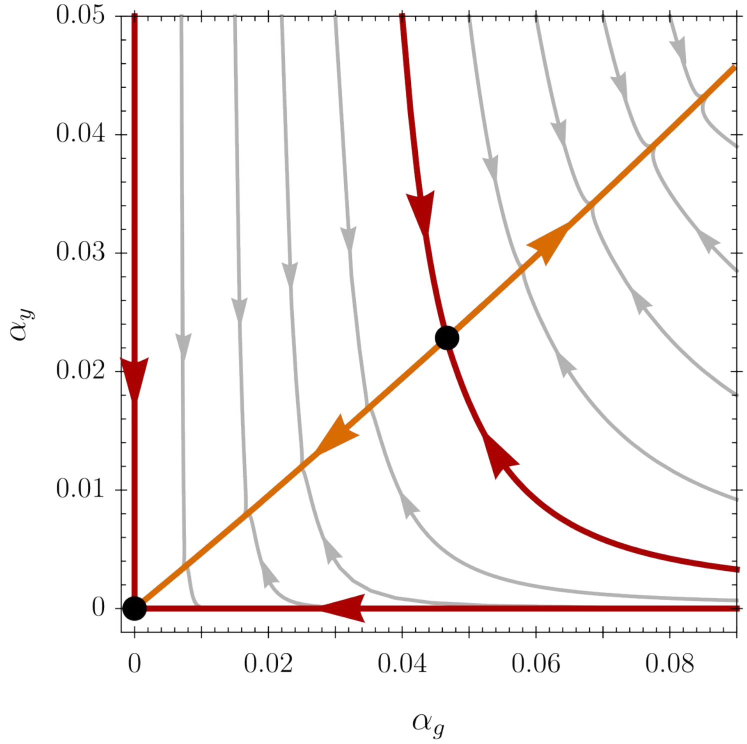
Separatrix Expansion



⟶ RGE along relevant separatrix

⟶ $\beta_{g,\mathrm{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

Separatrix Expansion



→ RGE along relevant separatrix

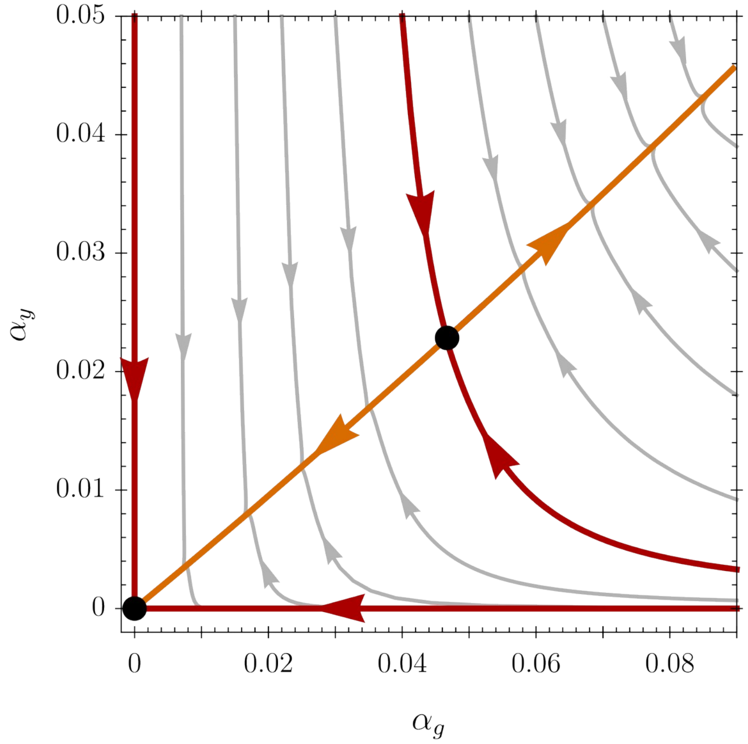
→ $\beta_{g,\text{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

→ Expansion around Gaussian (weak branch)

$$\beta_{g,\text{eff}} = \sum_{\ell=1}^{\infty} A_\ell(\epsilon)\, \alpha_g^{\ell+1}$$

→ complete ϵ dependence up to A_4

Separatrix Expansion



→ RGE along relevant separatrix

→ $\beta_{g,\text{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

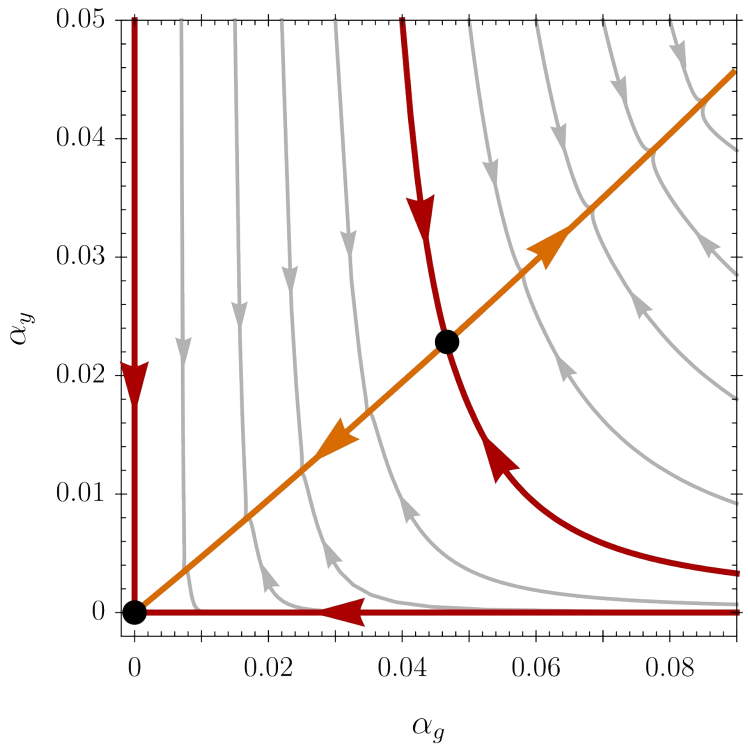
→ Expansion around Gaussian (weak branch)

$$\beta_{g,\text{eff}} = \sum_{\ell=1}^{\infty} A_\ell(\epsilon)\, \alpha_g^{\ell+1}$$

→ complete ϵ dependence up to A_4

→ more consistency than $\beta_{g,y,u,v} = 0$

Separatrix Expansion



→ RGE along relevant separatrix

→ $\beta_{g,\text{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

→ Expansion around Gaussian (weak branch)

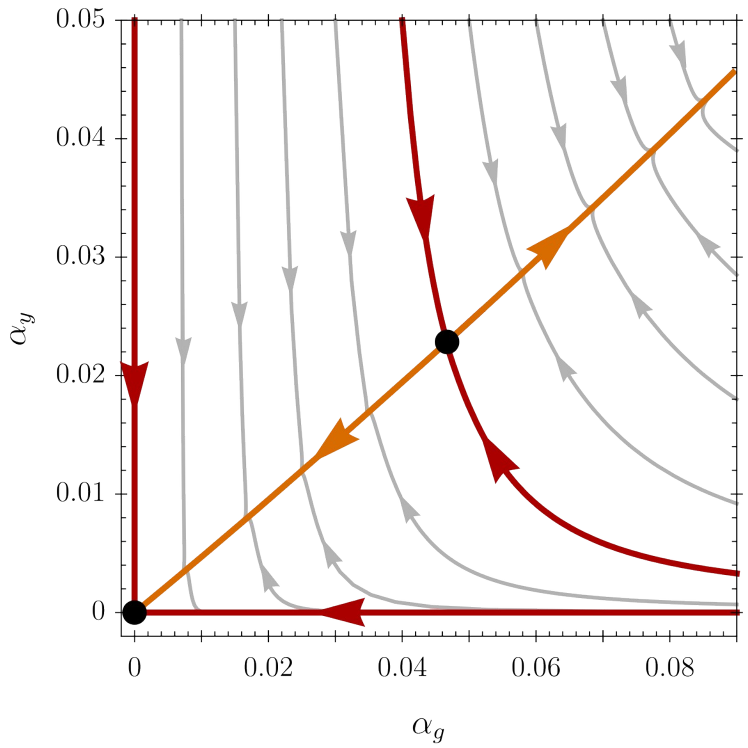
$$\beta_{g,\text{eff}} = \sum_{\ell=1}^{\infty} A_\ell(\epsilon)\, \alpha_g^{\ell+1}$$

→ complete ϵ dependence up to A_4

→ more consistency than $\beta_{g,y,u,v} = 0$

→ Padè and Padè-Borel resummation in α_g

Separatrix Expansion



→ RGE along relevant separatrix

→ $\beta_{g,\text{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

→ Expansion around Gaussian (weak branch)

$$\beta_{g,\text{eff}} = \sum_{\ell=1}^{\infty} A_\ell(\epsilon)\, \alpha_g^{\ell+1}$$

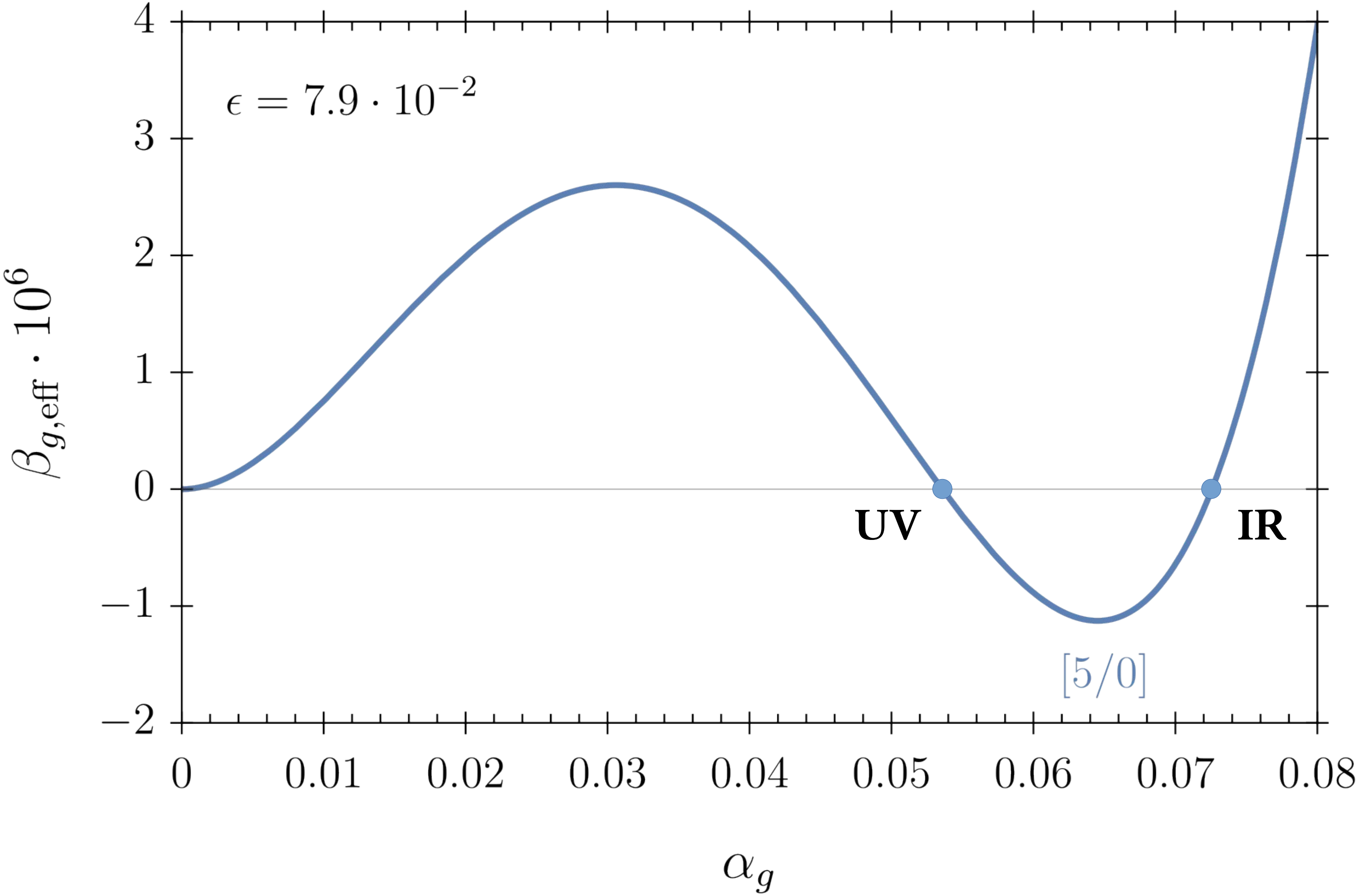
→ complete ϵ dependence up to A_4

→ more consistency than $\beta_{g,y,u,v} = 0$

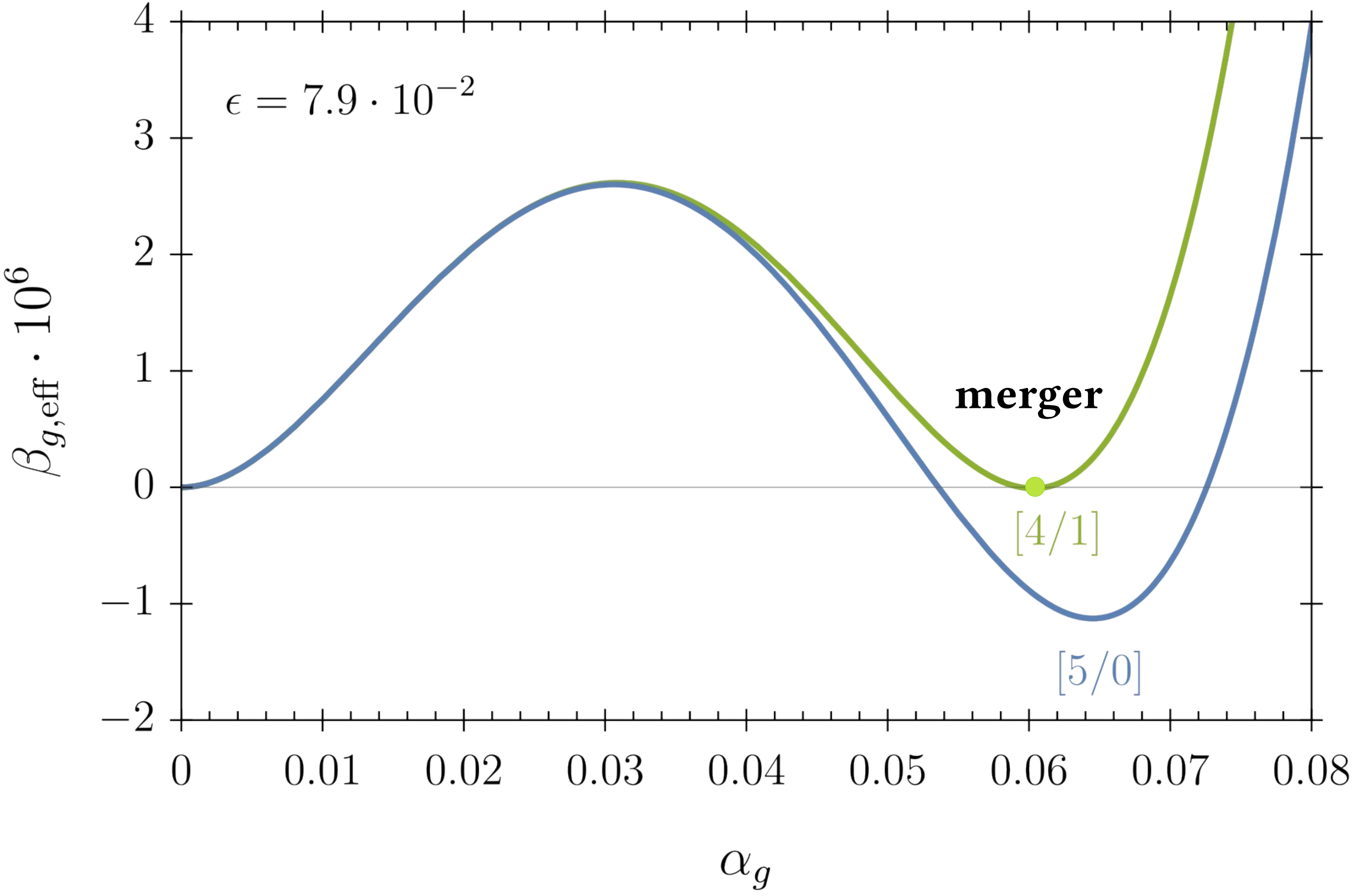
→ Padè and Padè-Borel resummation in α_g

→ requires 432 RGEs

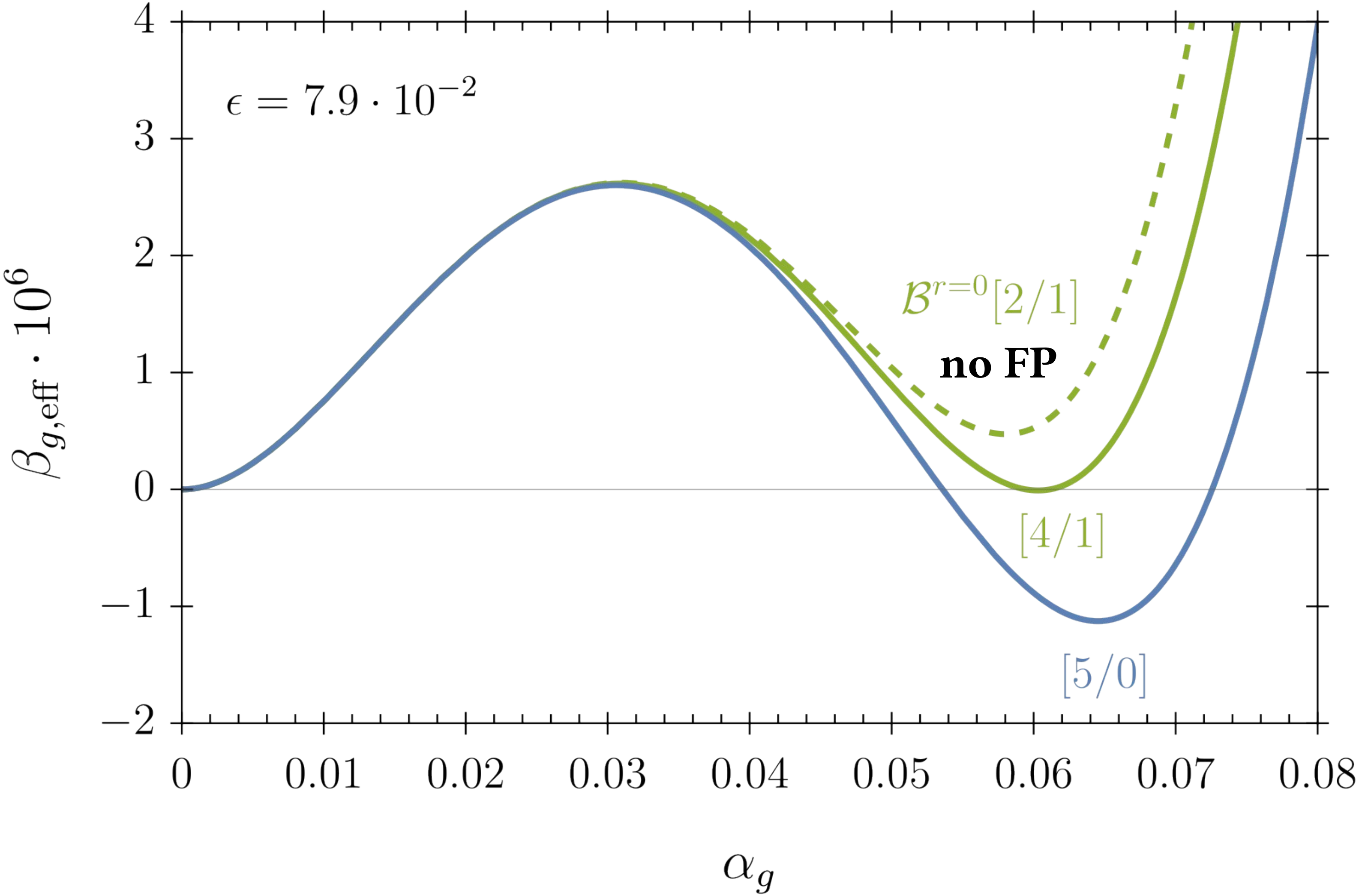
Relevant Separatrix



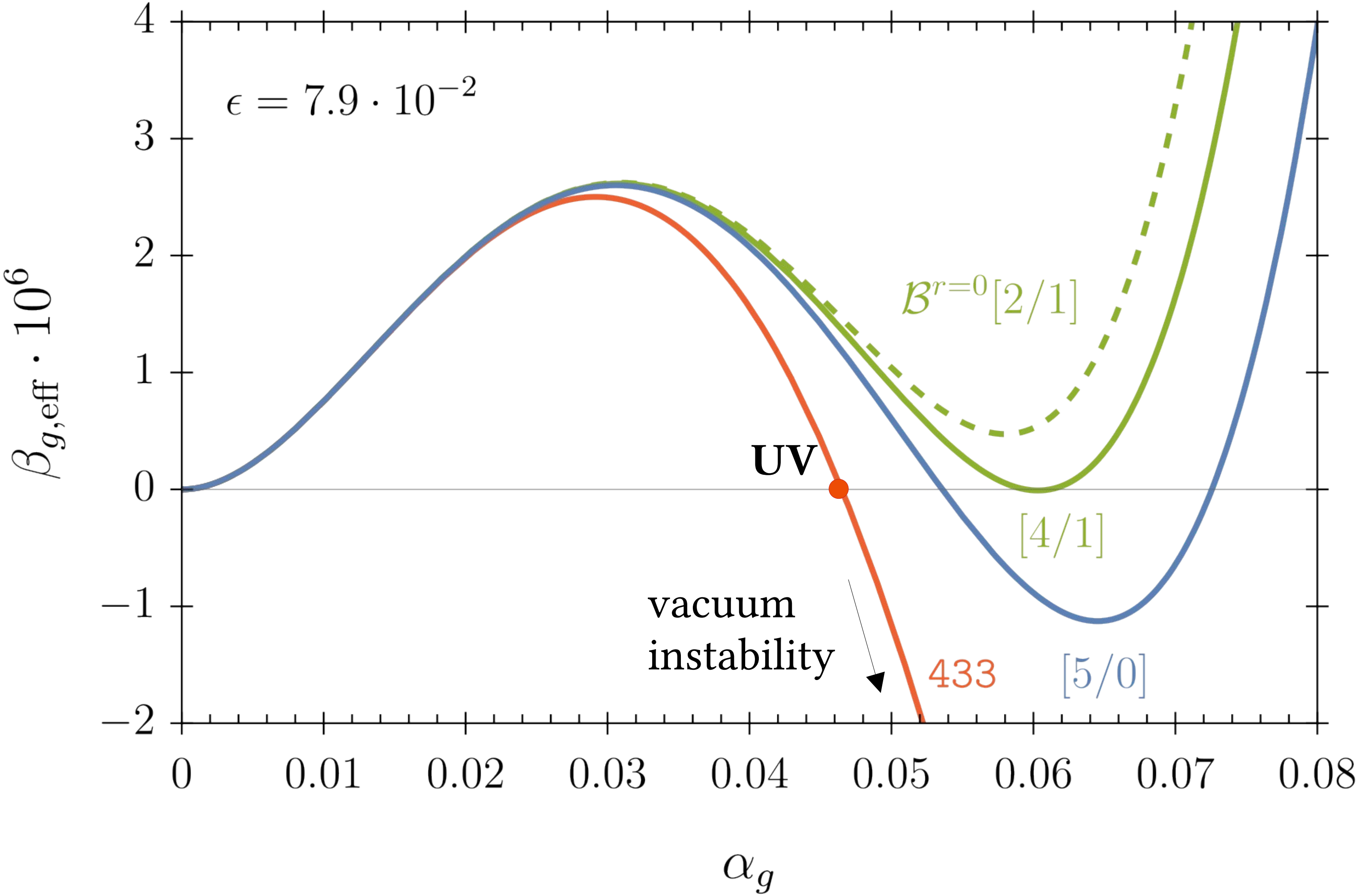
Relevant Separatrix



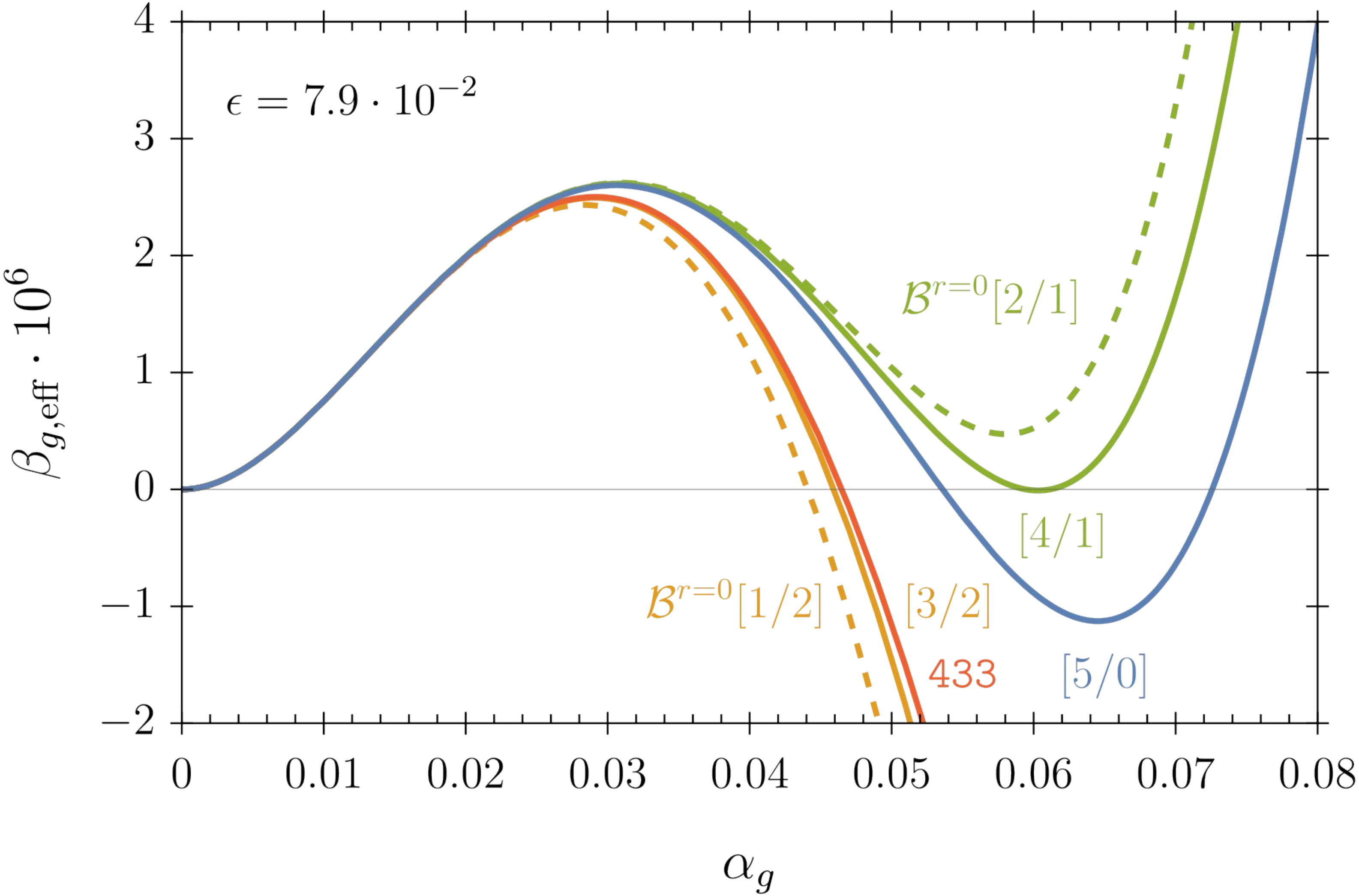
Relevant Separatrix



Relevant Separatrix



Relevant Separatrix



Summary

- conformal window in weakly coupled regime

Summary

- conformal window in weakly coupled regime
- conformal window around same size for 322 and 433 RGEs
- increased consistency between conformal expansion and solving RGEs

Summary

- conformal window in weakly coupled regime
- conformal window around same size for 322 and 433 RGEs
- increased consistency between conformal expansion and solving RGEs
- double trace merger after loss of vacuum stability

Summary

- conformal window in weakly coupled regime
- conformal window around same size for 322 and 433 RGEs
- increased consistency between conformal expansion and solving RGEs
- double trace merger after loss of vacuum stability
- vacuum instability or single trace merger

$$\epsilon_{\text{max}} \approx 0.08 - 0.10$$

- result valid for LiSa & Majorana models

Summary

- conformal window in weakly coupled regime
- conformal window around same size for 322 and 433 RGEs
- increased consistency between conformal expansion and solving RGEs
- double trace merger after loss of vacuum stability
- vacuum instability or single trace merger

$$\epsilon_{\max} \approx 0.08 - 0.10$$

- result valid for LiSa & Majorana models

$$(N_c,\, N_f) = (5, 26),\ (7, 39),\ (9, 50),\ (11, 61),\ \ldots$$