Asymptotically Safe Quantum Gravity

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Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}]}$$

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- Path integral with gauge fixing, sources

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu}\, e^{-S[\bar{g}+\hat{h}]-S_{\mathrm{gf}}[\bar{g},\hat{h}]-S_{\mathrm{gh}}[\bar{g},\hat{h},\hat{c},\hat{\bar{c}}]+\int_x \sqrt{\bar{g}}\, J^{\mu\nu}(x)\hat{h}_{\mu\nu}(x)}$$

- Gauge fixing requires metric split, e.g., $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance
- Path integral with gauge fixing, sources and matter

$$Z[\bar{g}, J] \sim \int \mathcal{D}\hat{h}_{\mu\nu}\, \mathcal{D}\hat{\phi}\; e^{-S[\bar{g}+\hat{h}]-S_{\text{gf}}[\bar{g},\hat{h}]-S_{\text{gh}}[\bar{g},\hat{h},\hat{c},\hat{\bar{c}}]-S_{\text{SM}}[\bar{g}+\hat{h},\hat{\phi}_{\text{SM}}]+\int_x \sqrt{\bar{g}}\, J\cdot\hat{\phi}}$$

- Gauge fixing requires metric split, e.g., $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- Methods: Perturbation theory, lattice, functional methods, ...

Perturbative quantum gravity

Einstein-Hilbert gravity

$$S_{\text{EH}} = \frac{1}{16\pi G_{\text{N}}} \int_x \sqrt{\det g_{\mu\nu}} (2\Lambda - R(g_{\mu\nu}))$$

- Perturbatively non-renormalisable: $[G_{\text{N}}] = -2$
- Need infinitely many counter terms: No predictivity

['t Hooft, Veltmann '74; Goroff, Sagnotti '85]

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Higher-derivative action

$$S_{\text{HD}} = \int_x \sqrt{\det g_{\mu\nu}} \left(\frac{1}{2\lambda} C^2_{\mu\nu\rho\sigma} - \frac{\omega}{3\lambda} R^2 \right) + S_{\text{EH}}$$

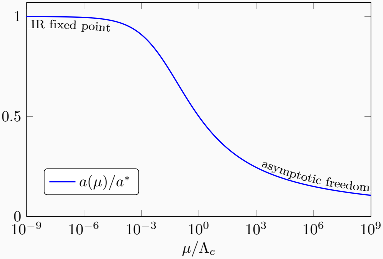
- Perturbatively renormalisable: $[\omega] = [\lambda] = 0$
- Non-unitary [Stelle '74]

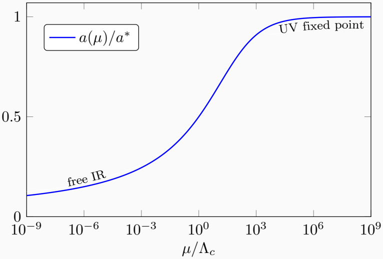
$$G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\text{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

Asymptotic freedom vs safety

The beta function of a coupling describes its change with the energy scale

$$\beta(a) = \beta_1 a^2 + \beta_2 a^3 + \dots$$





- Banks-Zaks
- Wilson-Fisher

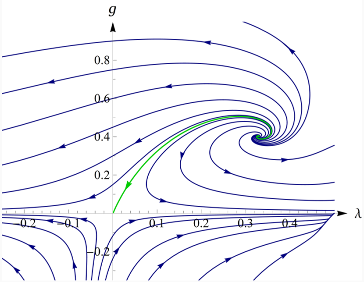
- Litim-Sannino
- Quantum Gravity

Asymptotic safety is a natural generalisation of asymptotic freedom

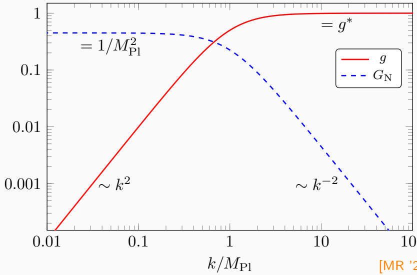
Asymptotically safe quantum gravity

QG could be non-perturbatively renormalisable via an interacting UV FP (motivated by the interacting UV FP in $d = 2 + \varepsilon$ dimensions) [Weinberg '76]

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$$



[Reuter '96; Reuter, Saueressig '01; Picture: Wikipedia]



[MR '20]

Asymptotically safe quantum gravity

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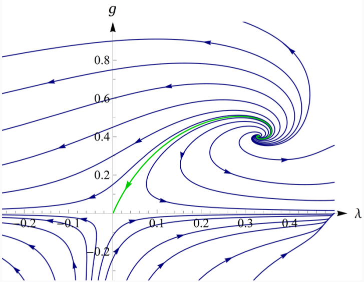

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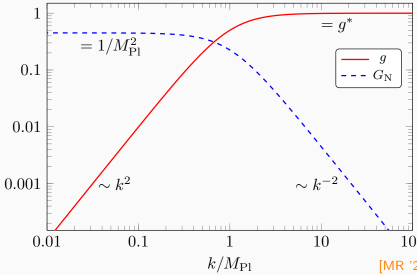


[MR '20]

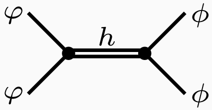
Predictivity ⇔ UV critical hypersurface is finite dimensional
[Denz, Pawlowski, MR '16; Falls, Ohta, Percacci '20; Kluth, Litim '20; Knorr '21; ...]

Unitarity ⇔ Properties of the spectral function, scattering amplitudes, ...
[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; Draper, Knorr, Ripken, Saueressig '20; Platania, Wetterich '20]

Towards testing unitarity

A unitary scattering matrix, $S^\dagger S = 1$, implies

- Well-behaved propagators without ghost or tachyonic instabilities
- Bounds on scattering amplitudes, e.g., violated by GR



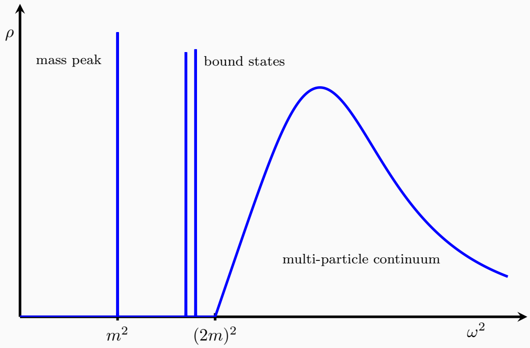
$$\overset{\text{GR}}{\propto} (p_1 + p_2)^2 \not\lesssim 1$$

Need access to correlation functions at time-like momenta

Källén-Lehmann spectral representation

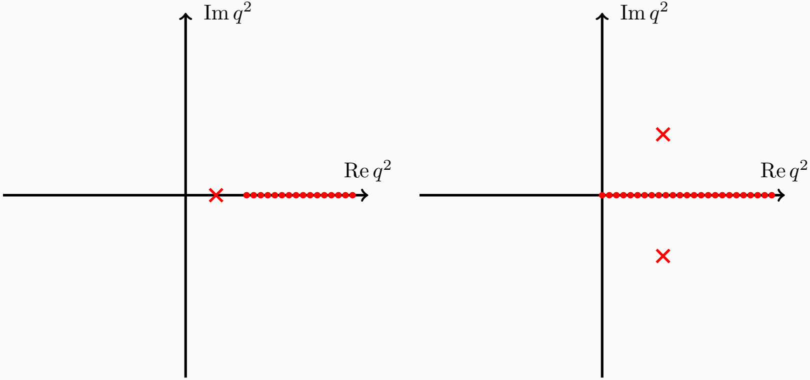
[Källén '52; Lehmann '54]

$$\mathcal{G}(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} \mathcal{G}(\omega^2 + i\varepsilon)$$



with $\rho(\omega^2) > 0$ and $\int \rho(\lambda^2)\mathrm{d}\lambda^2 = 1$

Propagator in the complex plane

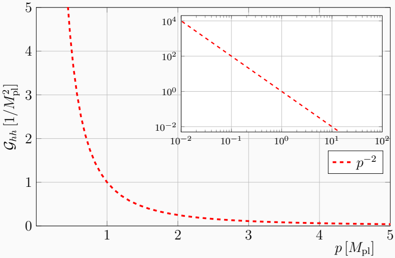


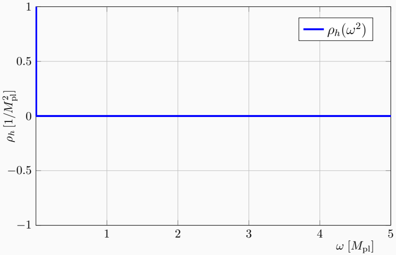
Most non-perturbative methods only provide numerical data for $q^2 < 0$

Classical graviton spectral function

Einstein-Hilbert action: $S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$

Flat Minkowski background: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



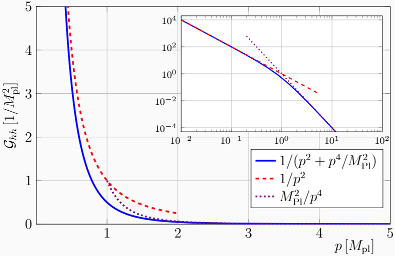


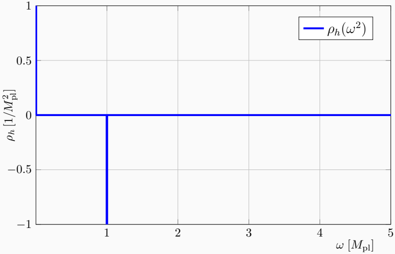
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

Higher-derivative action: $S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$





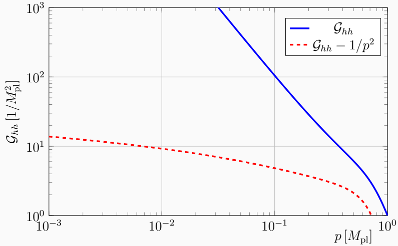
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

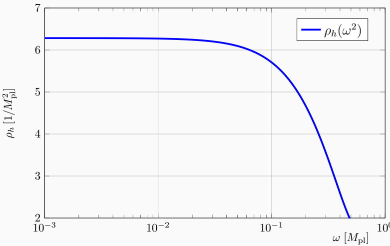
$$\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\text{Pl}}^2)$$

EFT graviton spectral function

One-loop effective action:

$$\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\,(c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma}) + \dots$$

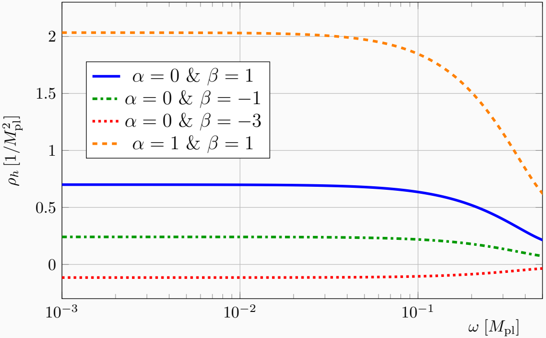




$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)p^4} \qquad \rho_h(\omega^2) \sim \delta(\omega^2) + 2\pi + 4\pi w^2 \ln(w^2) + \dots$$

EFT graviton spectral function

Gauge-fixing $S_{\text{gf}} = \frac{1}{\alpha}\int_x F_\mu^2$ with $F_\mu = \bar{\nabla}^\nu h_{\mu\nu} - \frac{1+\beta}{4}\bar{\nabla}_\mu h^\nu{}_\nu$



Propagator and spectral function are gauge dependent

The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

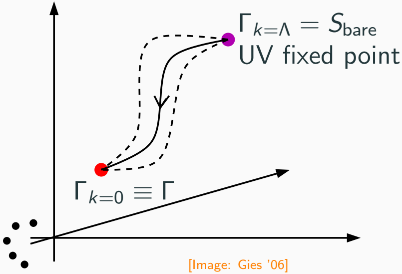
$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right]$$

R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

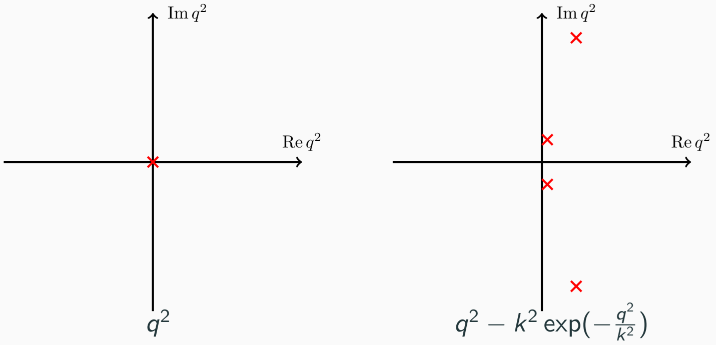
- bare action / UV FP
- quantum effective action Γ
- Wilsonian integrating out of momentum modes



[Image: Gies '06]

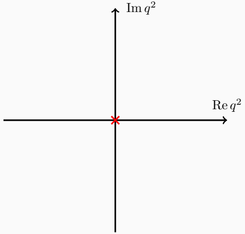
Standard Euclidean formulations

- Modified dispersion $q^2 \to q^2 + R_k(q^2)$ implements Wilsonian integrating out of momentum modes but introduces poles and cuts
- Can not use $\mathcal{G}_{hh}(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{\lambda^2 - q^2}$ at finite k
- Analytic continuation possible at $k = 0$ [Bonanno, Denz, Pawlowski, MR '21]

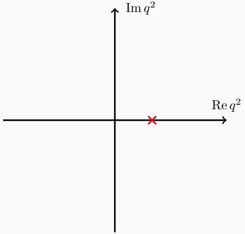


Spectral renormalisation group

- Callan-Symanzik cutoff $R_k \sim k^2$ allows use of spectral representation



q^2



$q^2 - k^2$

Spectral renormalisation group

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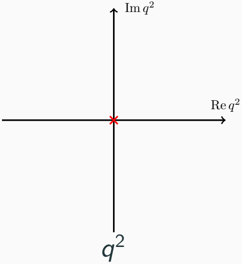
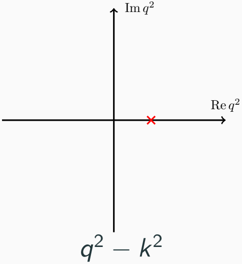




- Dimensional regularisation of UV divergences in $d = 4 - \varepsilon$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k\, \partial_t R_k - \partial_t S_{\mathrm{ct},k}$$

[Braun, Chen, Fu, Geißel, Horak, Huang, Ihssen, Pawlowski, MR, Rennecke, Tan, Töpfel, Wessely, Wink '22]

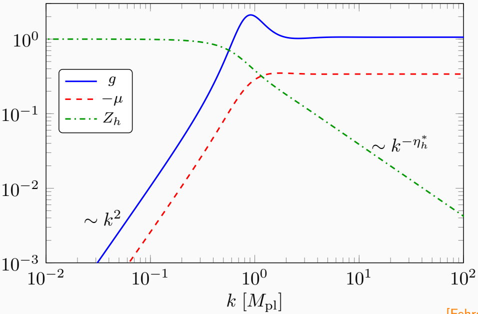
- Directly compute flow of spectral function with $m_h^2 = k^2(1 + \mu(k))$

$$\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$$

UV-IR trajectories

$$(g,\,\eta_h,\,\mu)\Big|_* = (1.06,\,0.96,\,-0.34)$$

$$\theta = 2.49 \pm 3.17\,i$$

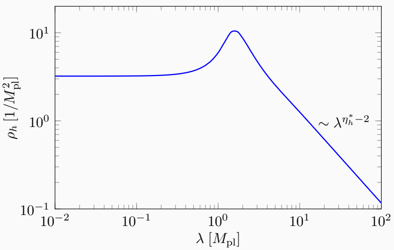


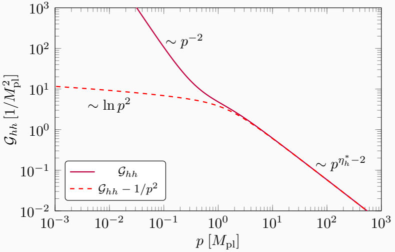
[Fehre, Litim, Pawlowski, MR '21]

$$G_N(k) = g(k)/k^2 \xrightarrow{k\to 0} G_N$$

$$-2\Lambda(k) = k^2\mu(k) \xrightarrow{k\to 0} -2\Lambda = 0$$

Graviton spectral function

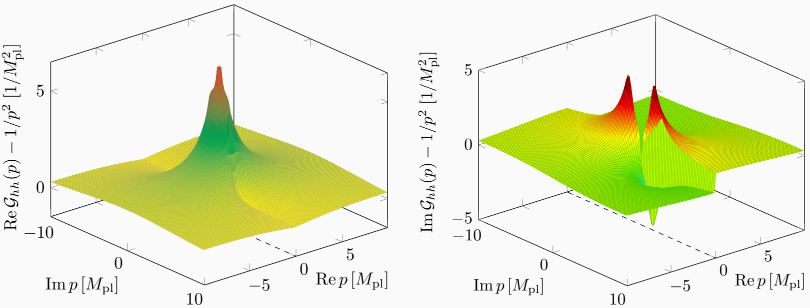




[Fehre, Litim, Pawlowski, MR '21]

- Massless graviton delta-peak with positive multi-graviton continuum
- Approximately matches EFT below $M_{\rm pl}$ (f_h feedback neglected)
- Asymptotically safe scaling above $M_{\rm pl}$ with $\eta_h^* = 0.96$
- Good agreement with reconstruction result [Bonanno, Denz, Pawlowski, MR '21]

Graviton propagator in the complex plane

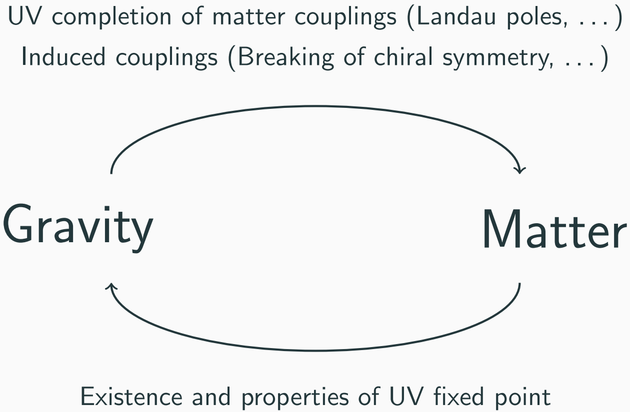


[Fehre, Litim, Pawlowski, MR '21]

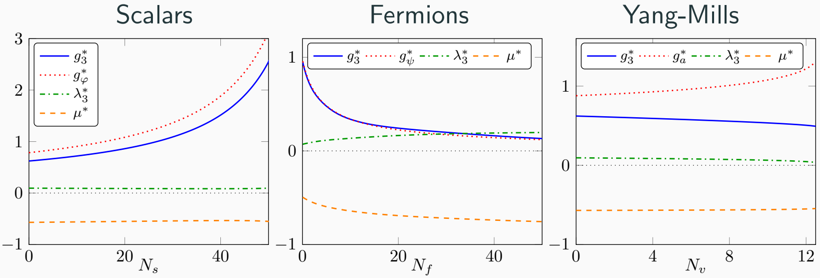
- Satisfies all necessary conditions for unitarity & causality [see also Platania '22]
- Existence of poles and cuts in the complex plane might be gauge invariant [Kluth, Litim, MR '22]

Gravity & matter

Interplay between matter and gravity



UV fixed point with matter



[Eichhorn, Labus, Pawlowski, MR '18] [Eichhorn, Lippoldt, Schiffer '18] [Christiansen, Litim, Pawlowski, MR '17]

Fixed point stable under (at least) SM matter inclusion

Avoiding Landau poles

Parameterisation of $U(1)$ beta function: $\beta_{g_1} = \beta_{g_1,\text{matter}} - f_g g_1$

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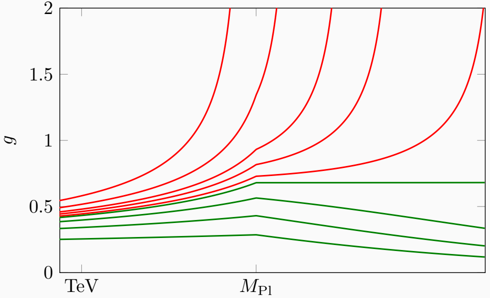

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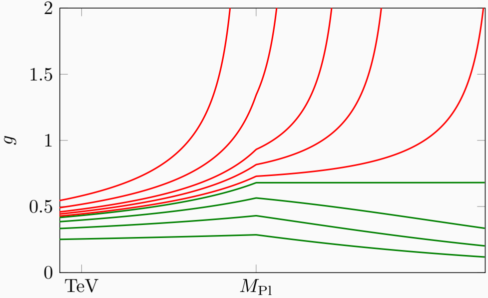
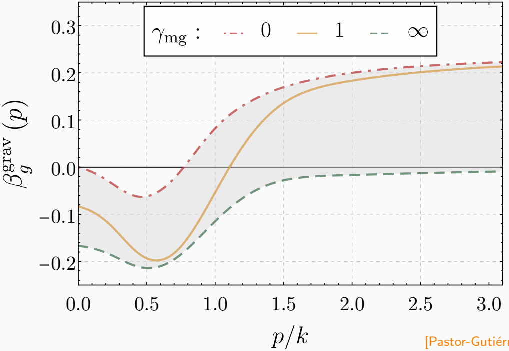


Avoiding SM Landau poles:

- $f_g \geq 9.8 \cdot 10^{-3}$ for $U(1)_Y$ [Eichhorn, Versteegen '17]
- $f_y \geq 10^{-4}$ for quark masses [Eichhorn, Held '18, Alkhofer, Eichhorn, Held, Nieto, Percacci, Schröfl '20]

Gravity contribution to the gauge couplings

$$\beta_g = \beta_g^{\text{matter}} + \beta_g^{\text{grav}}$$



[Pastor-Gutiérrez, Pawlowski, MR '22]

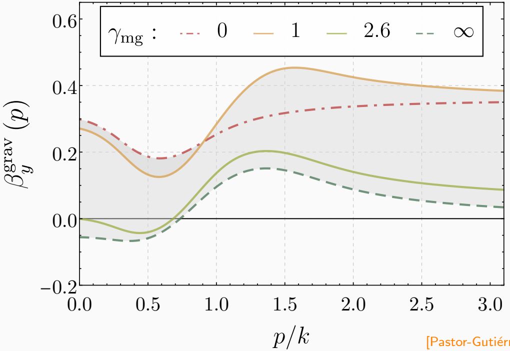
- γ_{mg} parameterises regulator dependence
- Negative or vanishing contribution at $p = 0$ due to kinematic identity

[Folkerts, Litim, Pawlowski '11; Christiansen, Litim, Pawlowski, MR '17]

Gravity assists asymptotic freedom in gauge sector

Gravity contribution to the Yukawa couplings

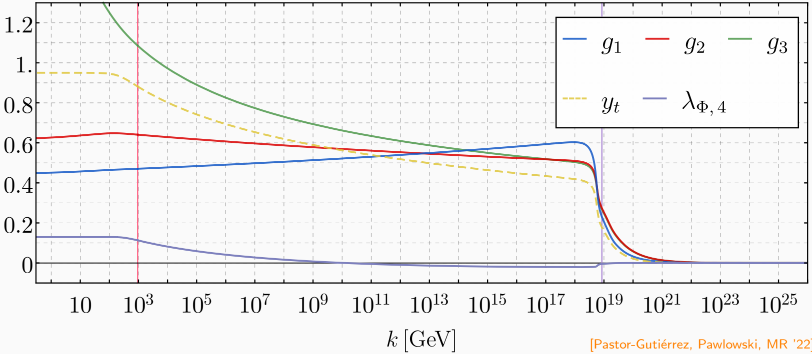
$$\beta_y = \beta_y^{\text{matter}} + \beta_y^{\text{grav}}$$



[Pastor-Gutiérrez, Pawlowski, MR '22]

- Positive and negative contributions possible, $\gamma_{\text{mg}}^{\text{stab}} = 2.6$
- Need higher-order terms [Eichhorn, Held '17] or momentum dependences

RG running of the ASSM



[Pastor-Gutiérrez, Pawlowski, MR '22]

- Assume f_y positive
- Free fixed point for all matter couplings
- Usual metastability scale $k_{\text{meta}} = 1.2 \cdot 10^{10}$ GeV
- Massive particles decouple from flow due to threshold effects

Matching of Higgs mass

Beta function of quartic scalar coupling

$$\beta_\lambda = \beta_{\lambda,\text{matter}} - f_\lambda \lambda$$

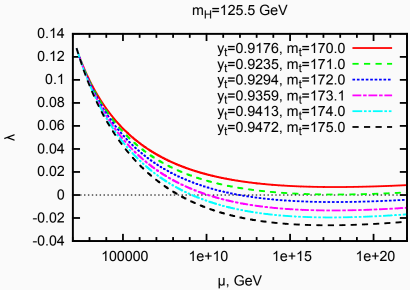
$f_\lambda \leq 0$ is negative and the UV fixed point $\lambda^* = 0$ is UV repulsive

[Percacci, Perini '03; Narain, Percacci '09; Eichhorn, Hamada, Lumma, Yamada '17; Pawlowski, MR, Wetterich, Yamada '18; . . .]

Higgs mass prediction from Gaußian FP at one-loop

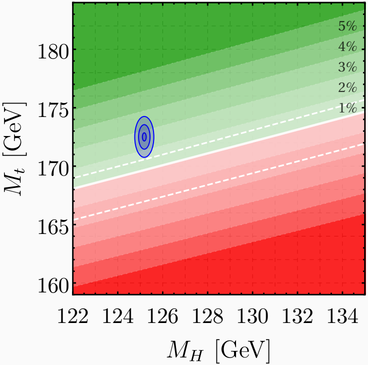
$$m_h = 126\,\text{GeV}$$

[Shaposhnikov, Wetterich '09]



[Bezrukov, Rubio, Shaposhnikov '14]

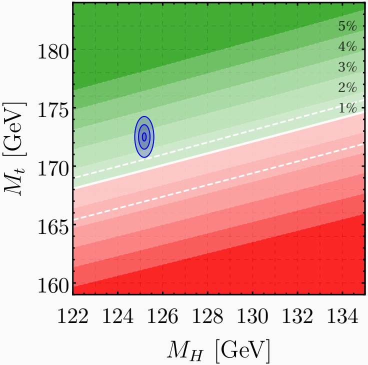
Higgs-top plane



[Pastor-Gutiérrez, Pawlowski, MR '22]

- Main theory uncertainties
    - Relation of Euclidean top mass to top pole mass
    - Matching of strong coupling
- Measured Higgs/top mass at 5σ outside of predicted values

Higgs-top plane



[Pastor-Gutiérrez, Pawlowski, MR '22]

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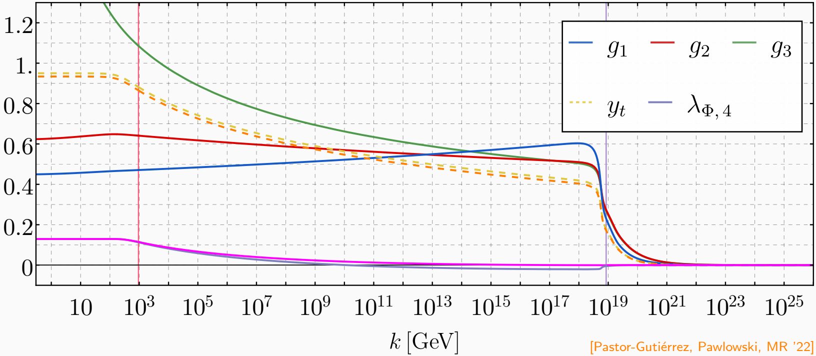
Potential solutions

- New physics, dark matter, etc. [MR, Smirnov '19; Eichhorn, Pauly '20; . . .]
- New non-trivial fixed point in green region [Pastor-Gutiérrez, Pawlowski, MR '22]

RG running of the ASSM



[Pastor-Gutiérrez, Pawlowski, MR '22]

The standard Gaußian trajectory leads to a small but significant mismatch of the top quark mass

Summary

- Asymptotic safety is a strong contender for the fundamental theory of quantum gravity
- Lorentzian signature and unitarity
  - New technical setup for direct Lorentzian computations
  - First direct computation of graviton spectral function
  - Positive indications for unitarity
- Interplay of gravity and matter
  - UV fixed point exists a wide range of matter fields
  - Landau poles are removed (problems in Yukawa sector?)
  - First complete non-perturbative running of the SM from UV to IR
  - Small but significant distance of Higgs mass from Gaußian UV FP

Thank you for your attention