Life at a Multi-TeV Muon Collider

NCBJ, Warsaw

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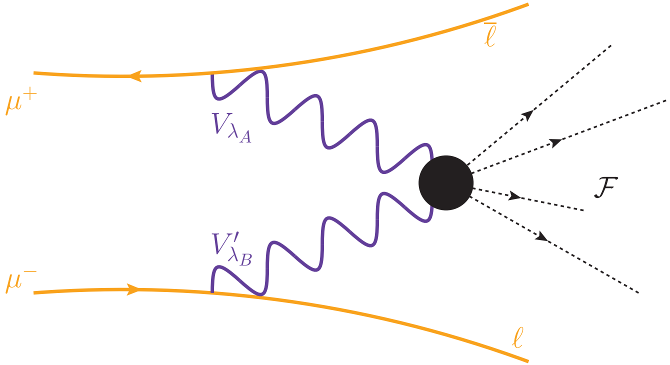
[1] w/ A. Costantini, F. Maltoni, O. Mattelaer, et al [2005.10289; 2111.02442]; and w/ Buarque-Franzosi, Gallinaro, et al [2106.01393]

Thank you for the invitation!

the big picture

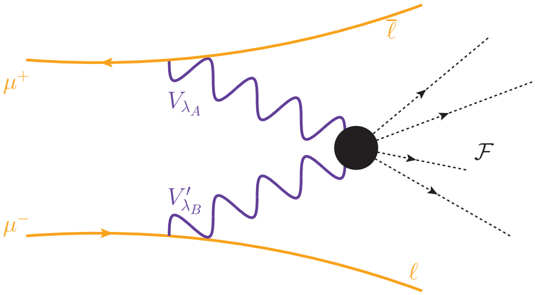
What is life like at a multi-TeV $\mu^+\mu^-$ collider?



Note: for this talk, no substantial difference between e^+e^- and $\mu^+\mu^-$, only collider energy $\sqrt{s}$

Why is this relevant?

Why?[2] Situation where scattering formalism is **theoretically interesting**



Partonic collisions at $Q \sim \mathcal{O}(10)$ TeV explore when **electroweak (EW)** symmetry is nearly restored, i.e., $(M^2_{W/Z/H}/Q^2) \to 0$

See C. Bauer, et al ('16,'17,'18); T. Han, et al ('16,'20,'21); A. Manohar, et al ('14,'18) + others

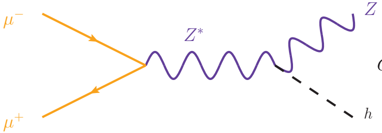
When momentum transfers reach $Q \sim \mathcal{O}(10)$ TeV, vector boson scattering (**VBS/VBF**) **acts a bit... funny**

w/ A. Costantini, et al [2005.10289]

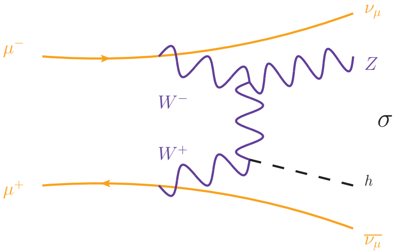
[2] Many motivations, e.g., Al Ali, et al. [2103.14043]; R&D progress as reported in the European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass + US activities

so what are $\mu^+\mu^-$ collisions at many-TeV like anyway?

Quick interlude: s-channel annihilation vs VBF/S



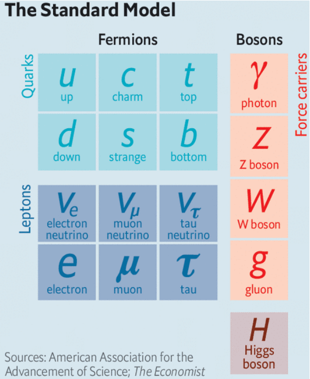
$$\sigma^{s-channel} \sim \frac{g^4}{s}$$



$$\sigma^{VBF} \sim \frac{g^8}{M_{WW}^2} \log^2\left(\frac{M_{WW}^2}{M_W^2}\right)$$

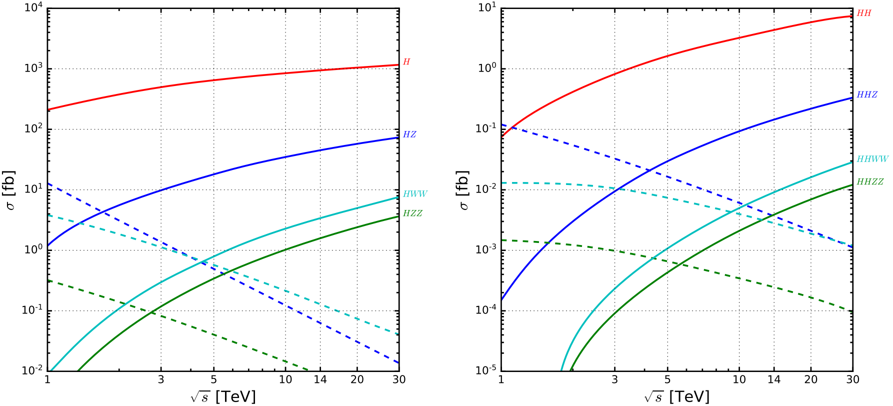
More legs $\implies$ more propagators $\implies$ $\int dk^2/(k^2 - M_W^2) \sim \log(\Lambda^2/M_W^2)$
Larger s $\implies$ larger (M_{WW}^2/M_W^2) $\implies$ collinear V compensate for g

Higgs production[3]



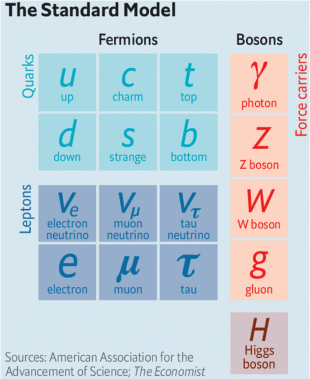
[3] In the following, we use full matrix elements at leading order with MadGraph5_aMC@NLO (and some diagram selection)

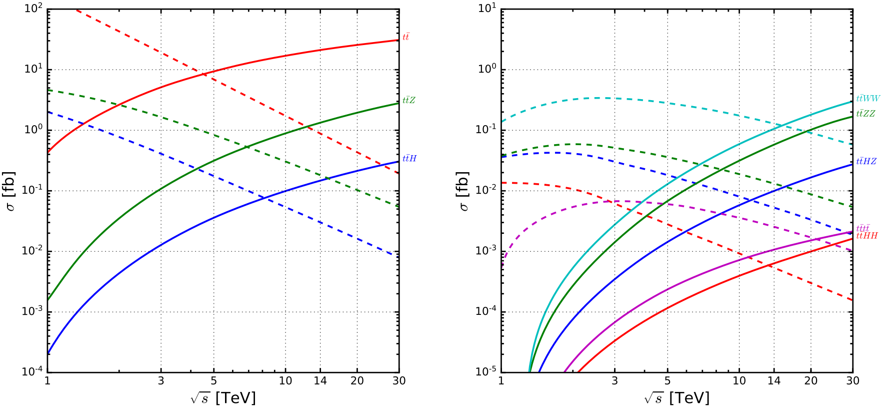
cross sections (σ) vs $\sqrt{s}$ for
s-channel annihilation (dash) vs VBF (solid)



- Eventually, $\sigma^{VBF} > \sigma^{s-channel}$ since
  - $\sigma^{s-channel} \sim 1/s$
  - $\sigma^{VBF} \sim \log^2(M_{VV}^2/M_V^2)/M_{VV}^2$ due to forward emission of $V = W/Z$

Top production

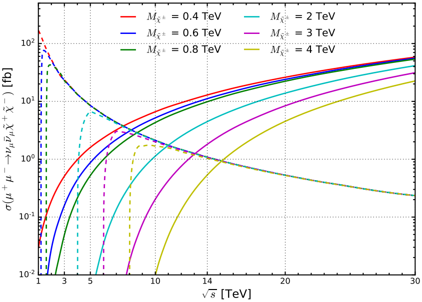


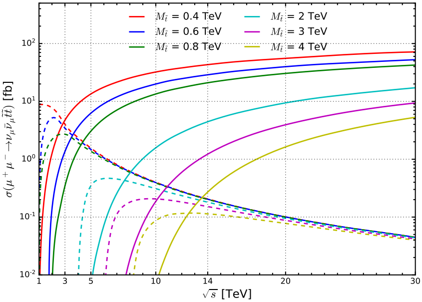


- Do you notice a pattern?

Supersymmetry

(L) chargino pairs (R) stop pairs

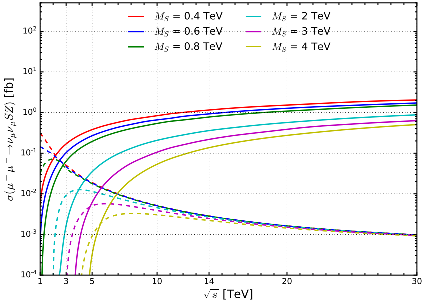


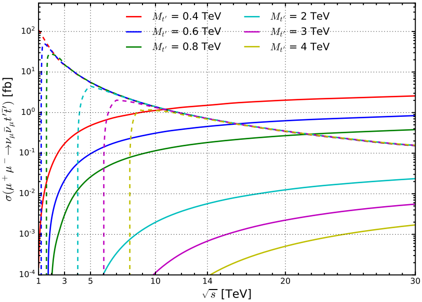


- And now?

Simple Extensions

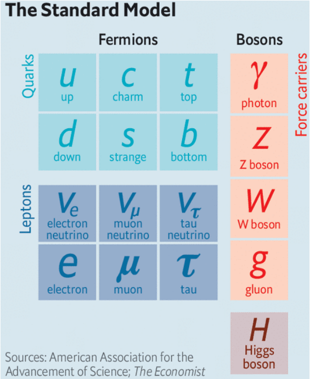
(L) Singlet + Z production (R) vector-like top pair production



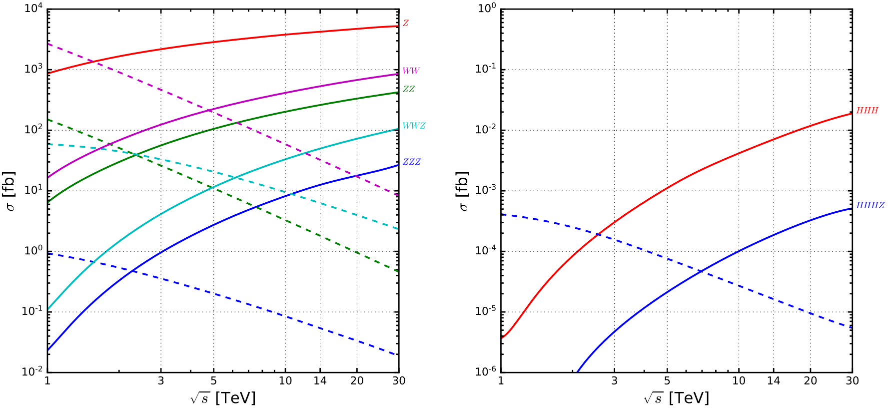


- . . . a little different but a lot of the same

Many-boson production[4]



4 My favorite! I find these processes really neat!



- **VBF is the dominant production vehicle for many processes**

Evidence for trend that VBF/S rates will always exceed s-ch. rates

Is this obvious? (not to me at first!) **Is there intuition for this?** (yes!)

w/ A. Costantini, et al [2005.10289]

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Idea: crudely compare the production of X by writing generically

$$\sigma^{s-ch.} \sim \frac{(s-M_X^2)}{(s-M_V^2)^2} \sim \frac{(s-M_X^2)}{s^2} \quad \leftarrow \text{assumes } s \gg M_V^2$$

$$\frac{d\sigma^{VBF}}{dz_1 dz_2} \sim \underbrace{f_V(z_1) f_{V'}(z_2)}_{\text{“}\mu\text{PDFs"}} \underbrace{\frac{(M_{VV'}^2 - M_X^2)}{(M_{VV'}^2 - M_V^2)^2}}_{M_{VV'}^2 = z_1 z_2 s \gg M_V^2} \sim f_V(z_1) f_{V'}(z_2) \frac{(z_1 z_2 s - M_X^2)}{(z_1 z_2)^2} \frac{\sigma^{s-ch.}}{(s-M_X^2)}$$

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PDFs are largest when $z = E_V/E_\mu \ll 1$ but $E_V \sim \sqrt{s} \gg M_V$

$$\implies f_V(z_i) \sim \frac{g_W^2}{4\pi} \frac{1}{z_i} \log\left(\frac{s}{M_V^2}\right) \quad \leftarrow \text{crude approximation}$$

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Observation: $\sigma^{VBF} = \sigma^{s-ch.} \times \int dz_1 dz_2 \dots$ is solvable for $M_{VV'} \gg M_X$!

Universal behavior: when production of X by VBF and annihilation are driven by same physics, VBF **dominates** when $\sqrt{s}$ satisfies

$$\frac{\sigma^{\text{VBF}}}{\sigma^{s-ch.}} \sim \mathcal{S}\left(\frac{g_W^2}{4\pi}\right)^2 \left(\frac{s}{M_X^2}\right) \log^2 \frac{s}{M_V^2} \log \frac{s}{M_X^2} > 1$$

Scaling estimate not so bad if $M_X \gg M_V$. Difference is about $\mathcal{O}(10\%)$

mass (M_X) [TeV]	SZ (Singlet)	H_2Z (2HDM)	$t'\overline{t'}$ (VLQ)	$\tilde{t}\bar{\tilde{t}}$ (MSSM)	$\tilde{\chi}^0\tilde{\chi}^0$ (MSSM)	$\tilde{\chi}^+\tilde{\chi}^-$ (MSSM)	Scaling (Eq. 7.7)
400 GeV	2.1 TeV	2.1 TeV	11 TeV	2.9 TeV	3.2 TeV	7.5 TeV	1.0 (1.7) TeV
600 GeV	2.5 TeV	2.5 TeV	16 TeV	3.8 TeV	3.8 TeV	8.1 TeV	1.3 (2.4) TeV
800 GeV	2.8 TeV	2.8 TeV	22 TeV	4.3 TeV	4.3 TeV	8.5 TeV	1.7 (3.1) TeV
2.0 TeV	4.0 TeV	4.0 TeV	>30 TeV	7.8 TeV	6.9 TeV	11 TeV	3.7 (6.8) TeV
3.0 TeV	4.8 TeV	4.8 TeV	>30 TeV	10 TeV	9.0 TeV	13 TeV	5.3 (9.8) TeV
4.0 TeV	5.5 TeV	5.5 TeV	>30 TeV	13 TeV	11 TeV	15 TeV	6.8 (13) TeV

Table 9. For representative processes and inputs, the required muon collider energy $\sqrt{s}$ [TeV] at which the VBF production cross section surpasses the s-channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale M_X ($2M_X$).

When $(M^2_{W/Z/H}/M^2_{VV}) \to 0$, qualitatively new behavior emerges:

VBF/S becomes the dominant scattering mechanism of EW states

However, in practice, numerical computations of VBF/S are difficult:

- Final states with **many legs and diagrams**
- Larger $\sqrt{s}$ exacerbates **large gauge cancellations**
- onset of **large soft and collinear logarithms**, e.g.,

$$d\mathcal{P}(\mu \to W\nu \text{ splitting}) \propto \underbrace{\frac{g_W^2}{(4\pi)^2}}_{\approx 2.5\cdot 10^{-3}} \times \underbrace{\log^2(M_{WW}^2/M_W^2)}_{\approx 350-500 \text{ for } M_{WW}=1-5 \text{ TeV}} \approx 0.9-1.3$$

per leg

Historically, **one approach** to studying the **EW theory at high energies** is to treat it like **massless QCD**:

- Electroweak boson PDFs (← rich literature!)
- + EW DGLAP evolution (lots of recent progress →)
- Electroweak parton showers (← more recent progress)
- Electroweak Sudakov resummation (← just super cool!)
- ...

	$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{1+\bar{z}^2}{z}\right)$		$\frac{1}{8\pi^2}\frac{1}{k_T^2}\left(\frac{z}{2}\right)$
	$\to\ V_T\ f_s^{(\prime)}$	$[BW]^0_T\ f_s$	$H^{0(*)}\ f_{\text{-}s}$ or $\phi^\pm\ f'_{\text{-}s}$
$f_{s=L,R}$	$g_V^2(Q^V_{f_s})^2$	$g_1 g_2 Y_{f_s} T^3_{f_s}$	$y^2_{f_R^{(\prime)}}$

[Han, et al ('16)]

Historically, success of **this approach** unclear since computations were difficult to produce and prescriptions varied

This is not necessarily the case today due to modern technology

The Effective W/Z Approximation (EWA)[5]

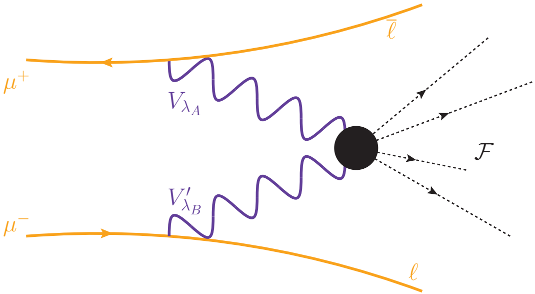
a.k.a. weak boson parton distribution functions

[5] Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

Idea: Treat W/Z in (VV')-scattering as partons when $M_{VV'} \gg M_W, M_Z$,

- PDFs for $\mu \to V_T$ splitting identical to gluons in QCD
- PDFs for $\mu \to V_0$ splitting is "novel" complication ("power-suppressed" PDF)
- To derive PDFs, one expands $\mu \to lV_\lambda$ matrix elements in powers of

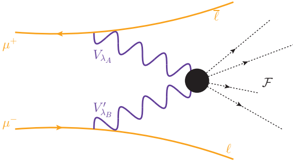
$$\mathcal{O}\left(\frac{p_T^2}{M_{VV}^2}\right) \text{ for } V_T \quad \text{or} \quad \mathcal{O}\left(\frac{p_T^2}{M_{VV}^2}\right) \text{ and } \mathcal{O}\left(\frac{M_V^2}{M_{VV}^2}\right) \text{ for } V_0$$



Expert note: these power-law corrections are universal and quasi-universal!

Idea: one can write the following scattering formula

$$\sigma(\mu^+\mu^- \to \mathcal{F} + \text{ anything}) = f_{i/\mu^+} \otimes f_{j/\mu^-} \otimes \hat{\sigma}_{ij} + \text{uncertainties}$$



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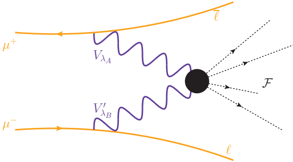
$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dPS_{\mathcal{F}}}_{\text{sum over all configurations / phase space integral}}$$



$$\times \left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f) \; f_{V'_{\lambda_B}/\mu^-}(\xi_2, \mu_f)}_{W^+_\lambda / W^-_\lambda / Z_\lambda / \gamma_\lambda \text{ PDFs at LO}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{"hard scattering" at LO}}$$

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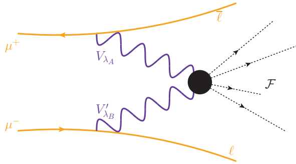

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$$+ \underbrace{\mathcal{O}\left(\frac{M^2_{V_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\frac{p^2_{T,V_k}}{M^2_{VV'}}\right)}_{\text{perturbative power-law corrections}} \quad \longleftarrow \text{(arise from expanding } \mu_\lambda \to V_\lambda l \text{ matrix elements)}$$

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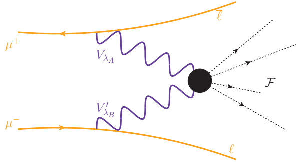

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$$+ \underbrace{\mathcal{O}\left(\log\frac{\mu_f^2}{M_V^2}\right)}_{\text{log corrections}} \quad \leftarrow \text{(due to working with LO/Bare PDFs; } \mu_f \text{ is only an UV regulator here)}$$

We studied the **red** terms

w/ Antonio Costantini, Fabio Maltoni, Olivier Mattelaer [2111.02442]

Now suppose someone implemented this framework into an event generator...

(this is the brief interlude about “new technology”)

MadGraph5_aMC@NLO (mg5amc) in a nutshell

In a Nutshell

MG5aMC is the 5th (or 6th) iteration of the Monte Carlo (MC) event generator `MadisonGraph` (or `MadGraph`) by Stelzer and Long at Wisconsin

[hep-ph/9401258]

- For a given scattering process, generates Feynman graphs and helicity amplitudes (`HELAS` routines) for *fast* numerical evaluation

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  - `ME` also writes phase space points (external momenta!) to file with integration (probability) weight, i.e., `MG+ME` is a MC event generator

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  Doing this efficienctly and robustly is difficult but doable. Maltoni, Stelzer [hep-ph/0208156]

- + arbitrary color structures, + spin correlated decays of resonances (MadSpin), + amplitude support for arbitrary Feynman Rule (ALOHA), +jet matching/merging, + loop-induced processes (MadLoop)

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- Merger with MC@NLO for NLO in QCD [1405.0301] and NLO in EW [1804.10017]

... what exactly did we do?[6]

[6] w/ Antonio Costantini, Fabio Maltoni, Olivier Mattelaer [2111.02442]

Implementing EW boson PDFs in MadGraph5

- **NEW: (Polarized) Effective Vector Boson Approx. (EVA)**
  - Bare (LO) PDFs for helicity-polarized $W_\lambda, Z_\lambda, \gamma_\lambda$ from $\ell^\pm_\lambda$
  - Automatically support PDFs for unpolarized W/Z **(EWA)** from $\ell^\pm_\lambda$
- **KEPT: Improved Weizsäcker-Williams approximation (iWWA)**
  - Unpolairzed γ PDF + power corrections from $\ell^\pm$ (Frixione, et al [hep-ph/9310350])
- **Technicalities:**
  - M_W, M_Z always nonzero in PDFs and matrix elements!
  - static and dyamic μ_f
  - n-point μ_f variation
  - Choice of p_T and q as evolution variable (this gives extra $\log(1-\xi)$ terms in PDFs!)
  - Also enabled **EVA+DIS** collider configuration
- **Technical appendix** rederiving W_λ, Z_λ PDFs to provide standard reference and mapping between different approaches in the literature
  - Publicly released in v3.3.0 (Major milestone for lepton colliders; see Frixione, et al [2108.10261])

PDFs for $e^{\pm}, \mu^{\pm} \to W_\lambda/Z_\lambda/\gamma_\lambda + \ell$ depend on helicities (λ)

- Subtle but important differences if evolving by q^2 of V vs p_T^2 of ℓ

(this can account for some differences between groups!)

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

```
c     /* ******************************************************* *
c     EVA (1/6) for f_L > v_+
      double precision function eva_fL_to_vp(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo                ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

c      print*,'gg2,gL2,mv2,x,mu2,ievo',gg2 !3,gL2,mv2,x,mu2,ievo
      coup2 = gg2*gL2/fourPiSq
      split = (1.d0-x)**2 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vp = coup2*split*xxlog
      return
      end
c     /* ******************************************************* *
c     EVA (2/6) for f_L > v_-
      double precision function eva_fL_to_vm(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo                ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

      coup2 = gg2*gL2/fourPiSq
      split = 1.d0 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vm = coup2*split*xxlog
      return
      end
```

EWA / EVA in MadGraph

Implementing all this was a multi-year, multi-step process:

1. **Automating** matrix elements and cross sections for external partons with fixed helicities D. Buarque Franzosi, O. Mattelaer, RR, S. Shil [1912.01725]
   - Essentially enable $A_\lambda + B_{\lambda_B} \to C_{\lambda_C} + D_{\lambda_D} + \ldots$ (λ_k=helicity)
   - Theoretically easy (after reorganizing Collinear Fact. Thm and defining polarized PDF/parton shower)
   - Dev. tricky since Lorentz invariance is lost (a ref. frame must be specified)
2. **Improving** dPS integration routine (`sde2`) for t-channel mom.

   K. Ostrolenk and O. Mattelaer [2102.00773]
3. **Adding** support for of EVA RR, A. Costantini, F. Maltoni, O. Mattelaer [2111.02442]

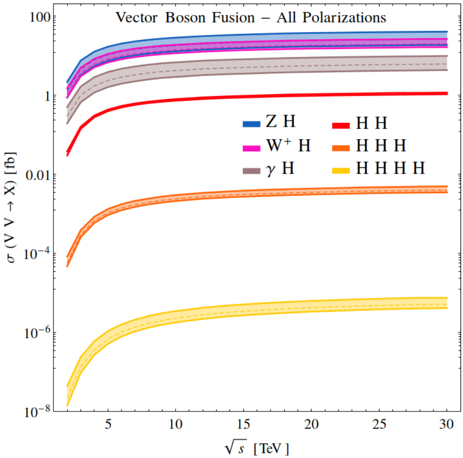
some results on $V_\lambda V'_{\lambda'} \to X$ in $\mu^+\mu^-$ collisions [7]

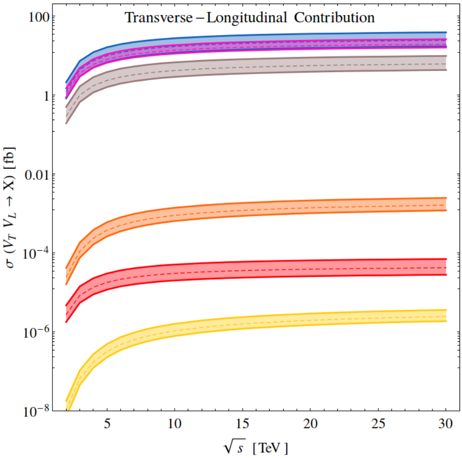
[7] w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2111.02442]

Higgs production in EVA

We then had fun looking into *many* processes

(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to HX$ (R) $V_T V_0 \to HX$

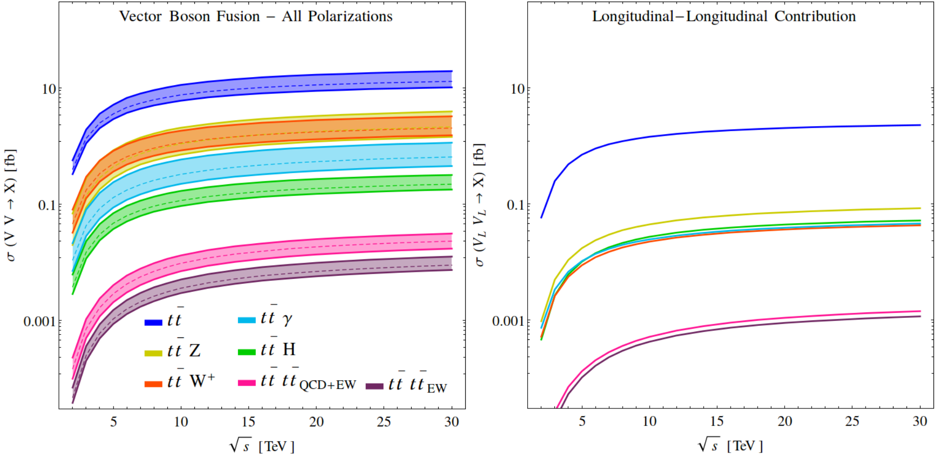




Tops in EVA

… *many* processes

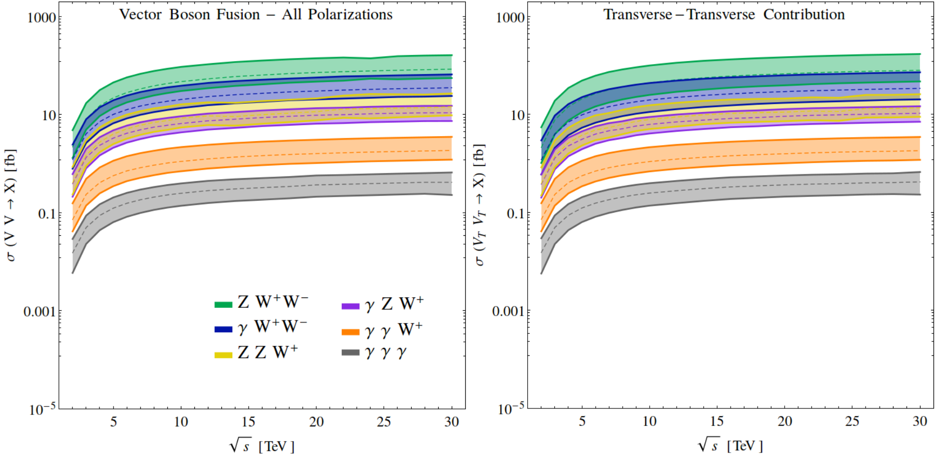
(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to t\bar{t}X$ (R) $V_0 V_0 \to t\bar{t}X$



Triboson in EVA

☺☺☺

(L) $\sum_{\lambda_A,\lambda_B} V_{\lambda_A} V_{\lambda_B} \to 3V$ (R) $V_T V_T \to 3V$



Diboson in EVA

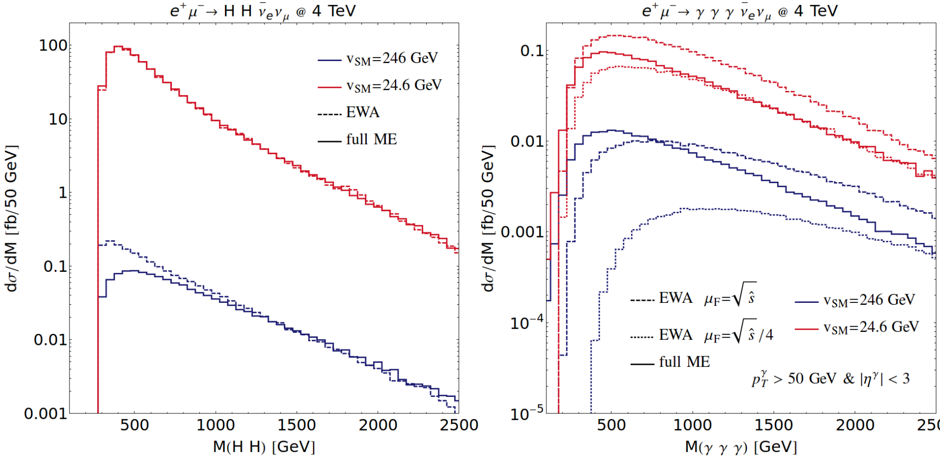
(4 polarization plots + 1 table) × each class of processes

		σ [fb]		
	mg5amc syntax	$\sqrt{s}=3$ TeV	$\sqrt{s}=14$ TeV	$\sqrt{s}=30$ TeV
$\sum V_{\lambda_A}V'_{\lambda_B} \to W^+W^-$	`vp vm > w+ w-`	$2.2\cdot10^{2}\,{}^{+98\%}_{-35\%}$	$7.0\cdot10^{2}\,{}^{+91\%}_{-33\%}$	$8.6\cdot10^{2}\,{}^{+88\%}_{-32\%}$
$V_TV'_T \to W^+W^-$	`vp{T} vm{T} > w+ w-`	$2.0\cdot10^{2}\,{}^{+99\%}_{-35\%}$	$6.6\cdot10^{2}\,{}^{+93\%}_{-34\%}$	$8.0\cdot10^{2}\,{}^{+92\%}_{-33\%}$
$V_0V'_T \to W^+W^-$	`vp{0} vm{T} > w+ w-`	$1.2\cdot10^{1}\,{}^{+54\%}_{-27\%}$	$4.4\cdot10^{1}\,{}^{+50\%}_{-25\%}$	$5.2\cdot10^{1}\,{}^{+49\%}_{-24\%}$
$V_0V'_0 \to W^+W^-$	`vp{0} vm{0} > w+ w-`	$4.2\cdot10^{-1}$	$1.7\cdot10^{0}$	$2.0\cdot10^{0}$
$\sum V_{\lambda_A}V'_{\lambda_B} \to W^+Z$	`vp vm > w+ z`	$5.3\cdot10^{1}\,{}^{+105\%}_{-40\%}$	$1.8\cdot10^{2}\,{}^{+97\%}_{-37\%}$	$2.2\cdot10^{2}\,{}^{+95\%}_{-37\%}$
$V_TV'_T \to W^+Z$	`vp{T} vm{T} > w+ z`	$5.0\cdot10^{1}\,{}^{+111\%}_{-42\%}$	$1.6\cdot10^{2}\,{}^{+103\%}_{-39\%}$	$2.0\cdot10^{2}\,{}^{+100\%}_{-38\%}$
$V_0V'_T \to W^+Z$	`vp{0} vm{T} > w+ z`	$3.4\cdot10^{0}\,{}^{+36\%}_{-18\%}$	$1.4\cdot10^{1}\,{}^{+34\%}_{-17\%}$	$1.7\cdot10^{1}\,{}^{+34\%}_{-17\%}$
$V_0V'_0 \to W^+Z$	`vp{0} vm{0} > w+ z`	$3.9\cdot10^{-2}$	$2.1\cdot10^{-1}$	$2.6\cdot10^{-1}$
$\sum V_{\lambda_A}V'_{\lambda_B} \to ZZ$	`vp vm > z z`	$4.4\cdot10^{1}\,{}^{+164\%}_{-52\%}$	$1.6\cdot10^{2}\,{}^{+144\%}_{-48\%}$	$1.9\cdot10^{2}\,{}^{+143\%}_{-48\%}$
$V_TV'_T \to ZZ$	`vp{T} vm{T} > z z`	$4.0\cdot10^{1}\,{}^{+171\%}_{-54\%}$	$1.4\cdot10^{2}\,{}^{+153\%}_{-50\%}$	$1.7\cdot10^{2}\,{}^{+150\%}_{-49\%}$
$V_0V'_T \to ZZ$	`vp{0} vm{T} > z z`	$4.2\cdot10^{0}\,{}^{+66\%}_{-33\%}$	$1.8\cdot10^{1}\,{}^{+61\%}_{-30\%}$	$2.2\cdot10^{1}\,{}^{+60\%}_{-30\%}$
$V_0V'_0 \to ZZ$	`vp{0} vm{0} > z z`	$1.1\cdot10^{-1}$	$6.0\cdot10^{-1}$	$7.2\cdot10^{-1}$
$\sum V_{\lambda_A}V'_{\lambda_B} \to \gamma Z$	`vp vm > a z`	$1.9\cdot10^{1}\,{}^{+169\%}_{-53\%}$	$7.1\cdot10^{1}\,{}^{+149\%}_{-49\%}$	$8.8\cdot10^{1}\,{}^{+145\%}_{-48\%}$
$V_TV'_T \to \gamma Z$	`vp{T} vm{T} > a z`	$1.8\cdot10^{1}\,{}^{+172\%}_{-54\%}$	$6.8\cdot10^{1}\,{}^{+153\%}_{-50\%}$	$8.4\cdot10^{1}\,{}^{+149\%}_{-49\%}$
$V_0V'_T \to \gamma Z$	`vp{0} vm{T} > a z`	$9.5\cdot10^{-1}\,{}^{+67\%}_{-33\%}$	$4.4\cdot10^{0}\,{}^{+61\%}_{-30\%}$	$5.5\cdot10^{0}\,{}^{+60\%}_{-30\%}$
$V_0V'_0 \to \gamma Z$	`vp{0} vm{0} > a z`	$5.6\cdot10^{-4}$	$4.5\cdot10^{-3}$	$6.5\cdot10^{-3}$
$\sum V_{\lambda_A}V'_{\lambda_B} \to \gamma W^+$	`vp vm > a w+`	$1.1\cdot10^{1}\,{}^{+111\%}_{-42\%}$	$4.0\cdot10^{1}\,{}^{+101\%}_{-39\%}$	$4.9\cdot10^{1}\,{}^{+99\%}_{-38\%}$
$V_TV'_T \to \gamma W^+$	`vp{T} vm{T} > a w+`	$1.1\cdot10^{1}\,{}^{+111\%}_{-42\%}$	$3.9\cdot10^{1}\,{}^{+102\%}_{-39\%}$	$4.8\cdot10^{1}\,{}^{+100\%}_{-38\%}$
$V_0V'_T \to \gamma W^+$	`vp{0} vm{T} > a w+`	$1.6\cdot10^{-2}\,{}^{+62\%}_{-31\%}$	$7.3\cdot10^{-1}\,{}^{+56\%}_{-28\%}$	$9.2\cdot10^{-1}\,{}^{+54\%}_{-27\%}$
$V_0V'_0 \to \gamma W^+$	`vp{0} vm{0} > a w+`	$1.5\cdot10^{-4}$	$1.2\cdot10^{-3}$	$1.7\cdot10^{-3}$
$\sum V_{\lambda_A}V'_{\lambda_B} \to \gamma\gamma$	`vp vm > a a`	$2.1\cdot10^{0}\,{}^{+172\%}_{-54\%}$	$8.5\cdot10^{0}\,{}^{+152\%}_{-50\%}$	$1.1\cdot10^{1}\,{}^{+147\%}_{-48\%}$
$V_TV'_T \to \gamma\gamma$	`vp{T} vm{T} > a a`	$2.1\cdot10^{0}\,{}^{+172\%}_{-54\%}$	$8.5\cdot10^{0}\,{}^{+152\%}_{-50\%}$	$1.1\cdot10^{1}\,{}^{+147\%}_{-48\%}$
$V_0V'_T \to \gamma\gamma$	`vp{0} vm{T} > a a`	$7.8\cdot10^{-4}\,{}^{+70\%}_{-35\%}$	$3.4\cdot10^{-3}\,{}^{+67\%}_{-34\%}$	$4.2\cdot10^{-3}\,{}^{+67\%}_{-33\%}$
$V_0V'_0 \to \gamma\gamma$	`vp{0} vm{0} > a a`	$5.8\cdot10^{-4}$	$4.7\cdot10^{-3}$	$6.8\cdot10^{-3}$

the money plot #1

Plot: M_{WW} for (L) $W_0 W_0 \to HH$ (R) $W_T W_T \to \gamma\gamma\gamma$

solid (dashed) = full ME (EVA); lower (upper)= $\sqrt{2}\langle\Phi\rangle = v_{\rm EW}\left(\frac{v_{\rm EW}}{10}\right)$



EVA works within uncertainties when $(M_V^2/M_{VV}^2) < 10^{-2}$.

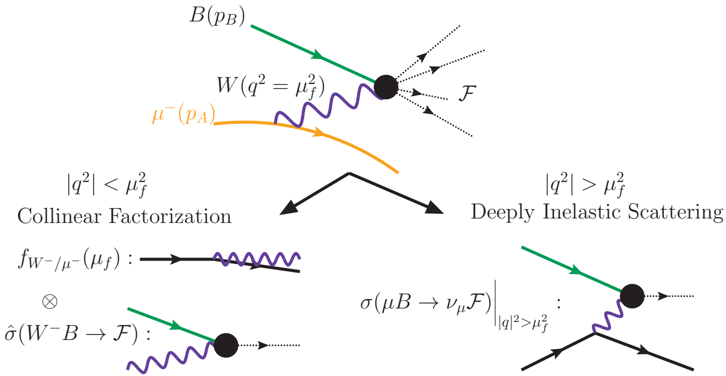
tl;dr: M_V is large $\implies$ M_{VV} must be larger! Numerically consistent with heavy Q factorization!

the money plot #2

Matching EW PDFs to matrix elements

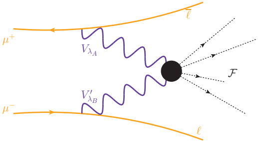
Idea: Total cross section ($\sigma^{\rm total}$) can be split into "collinear" ($\sigma^{\rm collinear}$) and "wide-angle" ($\sigma^{\rm wide-angle}$) bits. Summing *should* recover $\sigma^{\rm total}$

$$\sigma^{\rm total} = \sigma^{\rm collinear} + \sigma^{\rm wide-angle} + \mathcal{O}\left(\frac{M_W^2}{M_{WW}^2}\right) + \underbrace{\mathcal{O}\left(\frac{p_T^{\nu 2}}{M_{WW}^2}\right)}_{\text{How large?}}$$



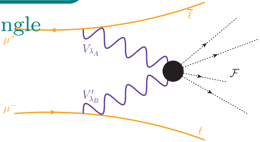
Consider a single μ^- $\to$ $W^-\nu_\mu$ splitting in W^+W^- scattering

$$\sigma^{\rm total} \ = \ \int_0^{M_{WW}} dp_\nu \frac{d\sigma}{dp_\nu}$$



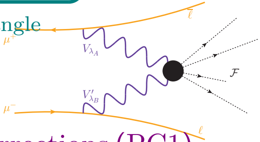
Consider a single $\mu^- \to W^- \nu_\mu$ splitting in $W^+ W^-$ scattering

$$\sigma^{\text{total}} = \int_0^{M_{WW}} dp_\nu \frac{d\sigma}{dp_\nu} = \underbrace{\int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{collinear}} + \underbrace{\int_{\mu_f}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{wide-angle}}$$



Consider a single $\mu^-\to W^-\nu_\mu$ splitting in W^+W^- scattering

$$\sigma^{\rm total} = \int_0^{M_{WW}} dp_\nu \frac{d\sigma}{dp_\nu} = \underbrace{\int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\rm collinear} + \underbrace{\int_{\mu_f}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\rm wide-angle}$$

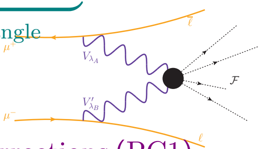


The **collinear part** contains the W PDF:

$$\sigma^{\rm collinear} = \int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\mu_f^2}{M_W^2}\right)}_{\rm PDF} + \underbrace{\text{power corrections (PC1)}}_{\rm neglect}$$

Consider a single $\mu^- \to W^-\nu_\mu$ splitting in W^+W^- scattering

$$\sigma^{\text{total}} = \int_0^{M_{WW}} dp_\nu \frac{d\sigma}{dp_\nu} = \underbrace{\int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{collinear}} + \underbrace{\int_{\mu_f}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{wide-angle}}$$



The **collinear part** contains the W PDF:

$$\sigma^{\text{collinear}} = \int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\mu_f^2}{M_W^2}\right)}_{\text{PDF}} + \underbrace{\text{power corrections (PC1)}}_{\text{neglect}}$$

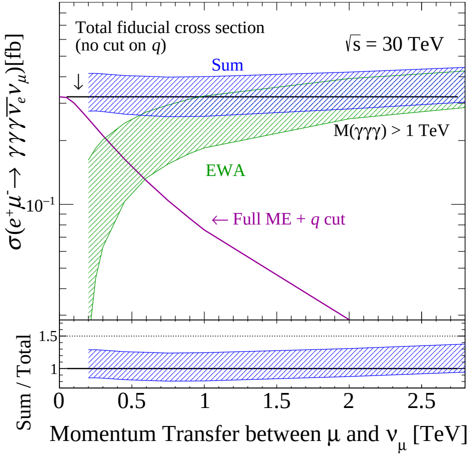
The **wide-angle part** also depends on μ_f:

$$\sigma^{\text{wide-angle}} = \int_{\mu_f}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{M_{WW}^2}{\mu_f^2}\right)}_{\text{same log as in PDF}} + \underbrace{\text{PC2}}_{\text{keep}}$$

Summing $\sigma^{\text{collinear}}$ and $\sigma^{\text{wide-angle}}$ *should* recover σ^{total}, up to **PC1**

ME matching: $\sigma^{sum} = \sigma^{\rm EWA} + \sigma^{\rm wide-angle}$ is independent of μ_f:

Plot: $\sigma(e^+\mu^- \to \gamma\gamma\gamma\overline{\nu_e}\nu_\mu)$ vs matching scale (μ_f)



Take away: Bare PDFs prefer small μ_f, where $\mathcal{O}(p_T^\nu/M_{WW}^2)$ is small!

(otherwise, one is outside the coll. limit; μ_f different for RG-improved PDFs)

When $M^2_{W/Z/H}/Q^2 \to 0$, qualitatively new behavior emerges

Bluntly, a $\mathcal{O}(10)$ TeV $\mu^+\mu^-$ collider behaves more like a high-energy hadron collider than a sub-TeV e^+e^- collider

[2005.10289]

Take-away: EWA/EVA can work (EW theory is a gauge theory!); some historical disagreements can be tied to size of power/log corrections

[2111.02442]

Outlook: EWA/EVA in MadGraph is now available and plans underway to merge parallel Snowmass efforts

For a broader summary about VBF/VBS, see Buarque-Franzosi, Gallinaro, RR, et al [2106.01393]

Thank you!

What is missing?

(A better understanding of high-energy VBF/S)

What is missing? (1/3)

Idea: $W_\lambda/Z_\lambda/\gamma_\lambda$ PDFs are unrenormalized, so replace them!

- ≥ 2 groups have advance-stage PDFs with EW-DGLAP evolution

Old

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

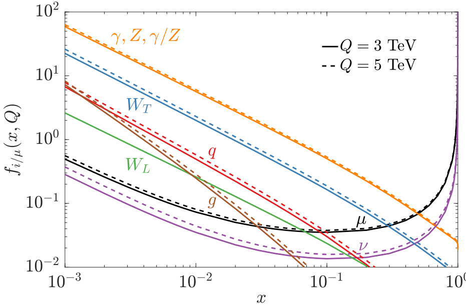
$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

New



(efforts underway to incorporate Han, et al [2007.14300]!)

What is missing? (2/3)

$$\sigma(\mu^+\mu^- \to \mathcal{F} + X) = \Delta \otimes f \otimes f \otimes \hat{\sigma} + \text{uncertainties}$$

$$= \underbrace{\sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^{1} d\xi_1 \int_{\tau_0/\xi_1}^{1} d\xi_2 \int dz \int dPS_{\mathcal{F}}}_{\text{sum over all configs. / phase space integral}} \quad \underbrace{\Delta(z)}_{\text{Sudakov factor (several ways to include this)}}$$

$$\times \left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f) f_{V'_{\lambda_B/\mu^-}}(\xi_2, \mu_f)}_{W^+_\lambda / W^-_\lambda / Z_\lambda / \gamma_\lambda \text{ PDFs at LO}} \right] \times \underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{“hard scattering” at LO}}$$

$$+ \underbrace{\mathcal{O}\left(\frac{M^2_{V_k}}{M^2_{VV'}}\right) + \mathcal{O}\left(\frac{p^2_{T,V_k}}{M^2_{VV'}}\right)}_{\text{perturbative power-law corrections}}$$

$$+ \underbrace{\mathcal{O}\left(\frac{\alpha_W}{M^2_{VV'}} \log \frac{\mu_f^2}{M^2_{VV'}}\right)}_{\text{log corrections}}$$

What is missing? (3/3)

Z_T/γ_T **interference:** When Z_T/γ_T can interfere, in principle, there is a "third PDF" and "third squared matrix element"

- $f_{\gamma_T/\mu}(\xi,\mu)$: from $\mu \to \mu\gamma_T$ ME and paired with $|\mathcal{M}(\gamma_T X \to \mathcal{F})|^2$
- $f_{Z_T/\mu}(\xi,\mu)$: from $\mu \to \mu Z_T$ ME and paired with $|\mathcal{M}(Z_T X \to \mathcal{F})|^2$
- $f_{(\gamma_T * Z_T)/\mu}(\xi,\mu)$: from $\mu \to \mu Z_T/\gamma_T$ interference and paired with the quantity $\Re[\mathcal{M}^*(\gamma_T X \to \mathcal{F})\mathcal{M}(Z_T X \to \mathcal{F})]$

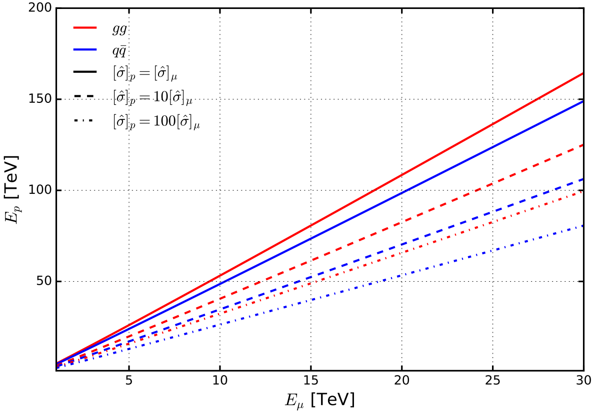
The last is documented in more recent literature (e.g., Han, et al; Manohar, et al) but does not involve scattering of asymptotic states / mass eigenstates

Proton vs muon

Like e^+e^- machines, $\mu^+\mu^-$ machines collide elementary particles

- up to rad. corrections, $\mu\mu$ collisions carry full energy $\implies \sqrt{\hat{s}} = \sqrt{s}$
- pp colliders, e.g., LHC and FCC-hh, need larger $\sqrt{s}$ for same $\sqrt{\hat{s}}$

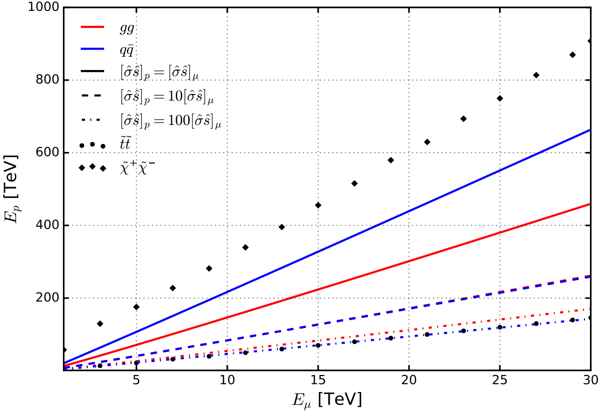
Plot: $\sqrt{s_{pp}}$ needed for $\hat{\sigma}_{pp} = \hat{\sigma}_{\mu\mu}$ in $2 \to 1$ processes at $\sqrt{s_{\mu\mu}}$



$2 \to 1$ processes are only small subset of possibilities (and also special!)

- $2 \to 2$ process help give bigger picture (and variability!)

Plot: $\sqrt{s_{pp}}$ needed for $\hat{s}_{pp}\hat{\sigma}_{pp} = \hat{s}_{\mu\mu}\hat{\sigma}_{\mu\mu}$ in $2 \to 2$ processes at $\sqrt{s_{\mu\mu}}$



- Assumed that $2M = 0.9\sqrt{s_{\mu\mu}}$
- PDF and phase space impact $2 \to 2$ more than the $2 \to 1$ suggests