EFT Matching with Functional Methods

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Based on work with J. Fuentes-Martín, M. König, J. Pagès, and F. Wilsch

MATCHETE

University of Basel

Warsaw physics seminars
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EFTs in BSM physics

Why EFTs play a role in the search for new physics

Direct searches for new physics

ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2022

ATLAS Preliminary

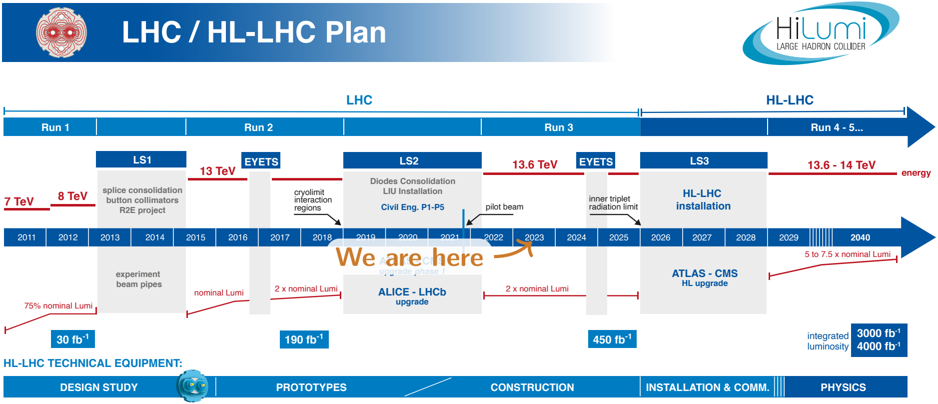
$\int \mathcal{L}\, dt = (3.6 - 139)\ \mathrm{fb}^{-1}$ $\sqrt{s} = 8, 13$ TeV

	Model	ℓ, γ	Jets†	E_T^{miss}	$\int \mathcal{L}\, dt[\mathrm{fb}^{-1}]$	Limit			Reference
Extra dimensions	ADD $G_{KK} + g/q$	$0\ e, \mu, \tau, \gamma$	1 – 4 j	Yes	139	M_D	11.2 TeV	$n = 2$	2102.10874
	ADD non-resonant $\gamma\gamma$	$2\ \gamma$	–	–	36.7	M_S	8.6 TeV	$n = 3$ HLZ NLO	1707.04147
	ADD QBH	–	2 j	–	37.0	M_{th}	8.9 TeV	$n = 6$	1703.09127
	ADD BH multijet	–	≥3 j	–	3.6	M_{th}	9.55 TeV	$n = 6$, $M_D = 3$ TeV, rot BH	1512.02586
	RS1 $G_{KK} \to \gamma\gamma$	$2\ \gamma$	–	–	139	G_{KK} mass	4.5 TeV	$k/\overline{M}_{Pl} = 0.1$	2102.13405
	Bulk RS $G_{KK} \to WW/ZZ$	multi-channel			36.1	G_{KK} mass	2.3 TeV	$k/\overline{M}_{Pl} = 1.0$	1808.02380
	Bulk RS $G_{KK} \to WV \to \ell\nu qq$	$1\ e, \mu$	2 j / 1 J	Yes	139	G_{KK} mass	2.0 TeV	$k/\overline{M}_{Pl} = 1.0$	2004.14636
	Bulk RS $g_{KK} \to tt$	$1\ e, \mu$	≥1 b, ≥1J/2j	Yes	36.1	g_{KK} mass	3.8 TeV	$\Gamma/m = 15\%$	1804.10823
	2UED / RPP	$1\ e, \mu$	≥2 b, ≥3 j	Yes	36.1	KK mass	1.8 TeV	Tier (1,1), $\mathcal{B}(A^{(1,1)} \to tt) = 1$	1803.09678
Gauge bosons	SSM $Z' \to \ell\ell$	$2\ e, \mu$	–	–	139	Z' mass	5.1 TeV		1903.06248
	SSM $Z' \to \tau\tau$	$2\ \tau$	–	–	36.1	Z' mass	2.42 TeV		1709.07242
	Leptophobic $Z' \to bb$	–	2 b	–	36.1	Z' mass	2.1 TeV		1805.09299
	Leptophobic $Z' \to tt$	$0\ e, \mu$	≥1 b, ≥2 J	Yes	139	Z' mass	4.1 TeV	$\Gamma/m = 1.2\%$	2005.05138
	SSM $W' \to \ell\nu$	$1\ e, \mu$	–	Yes	139	W' mass	6.0 TeV		1906.05609
	SSM $W' \to \tau\nu$	$1\ \tau$	–	Yes	139	W' mass	5.0 TeV		ATLAS-CONF-2021-025
	SSM $W' \to tb$	–	≥1 b, ≥1 J	–	139	W' mass	4.4 TeV		ATLAS-CONF-2021-043
	HVT $W' \to WZ \to \ell\nu qq$ model B	$1\ e, \mu$	2 j / 1 J	Yes	139	W' mass	4.3 TeV	$g_V = 3$	2004.14636
	HVT $W' \to WZ \to \ell\nu\, \ell'\ell'$ model C	$3\ e, \mu$	2 j (VBF)	Yes	139	W' mass	340 GeV	$g_V c_H = 1$, $g_f = 0$	ATLAS-CONF-2022-005
	HVT $W' \to WH$ model B	$0\ e, \mu$	≥1 b, ≥2 J		139	W' mass	3.2 TeV	$g_V = 3$	2007.05293
	LRSM $W_R \to \mu N_R$	$2\ \mu$	1 J	–	80	W_R mass	5.0 TeV	$m(N_R) = 0.5$ TeV, $g_L = g_R$	1904.12679
CI	CI $qqqq$	–	2 j	–	37.0	Λ	21.8 TeV	η_{LL}^{-}	1703.09127
	CI $\ell\ell qq$	$2\ e, \mu$	–	–	139	Λ	35.8 TeV	η_{LL}^{-}	2006.12946
	CI $eebs$	$2\ e$	1 b	–	139	Λ	1.8 TeV	$g_* = 1$	2105.13847
	CI $\mu\mu bs$	$2\ \mu$	1 b	–	139	Λ	2.0 TeV	$g_* = 1$	2105.13847
	CI $tttt$	≥1 e,μ	≥1 b, ≥1 j	Yes	36.1	Λ	2.57 TeV	$\lvert C_{4t} \rvert = 4\pi$	1811.02305
DM	Axial-vector med. (Dirac DM)	$0\ e, \mu, \tau, \gamma$	1 – 4 j	Yes	139	m_{med}	2.1 TeV	g_q=0.25, g_χ=1, $m(\chi)$=1 GeV	2102.10874
	Pseudo-scalar med. (Dirac DM)	$0\ e, \mu, \tau, \gamma$	1 – 4 j	Yes	139	m_{med}	376 GeV	g_q=1, g_χ=1, $m(\chi)$=1 GeV	2102.10874
	Vector med. Z'-2HDM (Dirac DM)	$0\ e, \mu$	2 b	Yes	139	m_{med}	3.1 TeV	$\tan\beta$=1, g_Z=0.8, $m(\chi)$=100 GeV	2108.13391
	Pseudo-scalar med. 2HDM+a	multi-channel			139	m_{med}	560 GeV	$\tan\beta$=1, g_χ=1, $m(\chi)$=10 GeV	ATLAS-CONF-2021-036
LQ	Scalar LQ 1st gen	$2\ e$	≥2 j	Yes	139	LQ mass	1.8 TeV	$\beta = 1$	2006.05872
	Scalar LQ 2nd gen	$2\ \mu$	≥2 j	Yes	139	LQ mass	1.7 TeV	$\beta = 1$	2006.05872
	Scalar LQ 3rd gen	$1\ \tau$	2 b	Yes	139	LQ_3^u mass	1.2 TeV	$\mathcal{B}(LQ_3^u \to b\tau) = 1$	2108.07665
	Scalar LQ 3rd gen	$0\ e, \mu$	≥2 j, ≥2 b	Yes	139	LQ_3^u mass	1.24 TeV	$\mathcal{B}(LQ_3^u \to t\nu) = 1$	2004.14060
	Scalar LQ 3rd gen	≥2 e, μ, ≥1 τ	≥1 j, ≥1 b	–	139	LQ_3^d mass	1.43 TeV	$\mathcal{B}(LQ_3^d \to t\tau) = 1$	2101.11582
	Scalar LQ 3rd gen	$0\ e, \mu$, ≥1 τ	0 – 2 j, 2 b	Yes	139	LQ_3^d mass	1.26 TeV	$\mathcal{B}(LQ_3^d \to b\nu) = 1$	2101.12527
	Vector LQ 3rd gen	$1\ \tau$	2 b	Yes	139	LQ_3^V mass	1.77 TeV	$\mathcal{B}(LQ_3^V \to b\tau) = 0.5$, Y-M coupl.	2108.07665
Heavy quarks	VLQ $TT \to Zt + X$	$2e/2\mu/\geq 3e,\mu$	≥1 b, ≥1 j	–	139	T mass	1.4 TeV	SU(2) doublet	ATLAS-CONF-2021-024
	VLQ $BB \to Wt/Zb + X$	multi-channel			36.1	B mass	1.34 TeV	SU(2) doublet	1808.02343
	VLQ $T_{5/3} T_{5/3} \vert T_{5/3} \to Wt + X$	2(SS)/≥3 e,μ	≥1 b, ≥1 j	Yes	36.1	$T_{5/3}$ mass	1.64 TeV	$\mathcal{B}(T_{5/3} \to Wt)$= 1, $c(T_{5/3} Wt)$= 1	1807.11883
	VLQ $T \to Ht/Zt$	$1\ e, \mu$	≥1 b, ≥3 j	Yes	139	T mass	1.8 TeV	SU(2) singlet, κ_T= 0.5	ATLAS-CONF-2021-040
	VLQ $Y \to Wb$	$1\ e, \mu$	≥1 b, ≥1 j	Yes	36.1	Y mass	1.85 TeV	$\mathcal{B}(Y \to Wb)$= 1, $c_R(Wb)$= 1	1812.07343
	VLQ $B \to Hb$	$0\ e,\mu$	≥2b, ≥1j, ≥1J	–	139	B mass	2.0 TeV	SU(2) doublet, κ_B= 0.3	ATLAS-CONF-2021-018
Excited fermions	Excited quark $q^* \to qg$	–	2 j	–	139	q^* mass	6.7 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1910.08447
	Excited quark $q^* \to q\gamma$	$1\ \gamma$	1 j	–	36.7	q^* mass	5.3 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1709.10440
	Excited quark $b^* \to bg$	–	1 b, 1 j	–	36.1	b^* mass	2.6 TeV		1805.09299
	Excited lepton ℓ^*	$3\ e, \mu$	–	–	20.3	ℓ^* mass	3.0 TeV	$\Lambda = 3.0$ TeV	1411.2921
	Excited lepton ν^*	$3\ e, \mu, \tau$	–	–	20.3	ν^* mass	1.6 TeV	$\Lambda = 1.6$ TeV	1411.2921
Other	Type III Seesaw	2,3,4 e, μ	≥2 j	Yes	139	N^0 mass	910 GeV		2202.02039
	LRSM Majorana ν	$2\ \mu$	2 j	–	36.1	N_R mass	3.2 TeV	$m(W_R) = 4.1$ TeV, $g_L = g_R$	1809.11105
	Higgs triplet $H^{\pm\pm} \to W^\pm W^\pm$	2,3,4 e, μ (SS)	various	Yes	139	$H^{\pm\pm}$ mass	350 GeV	DY production	2101.11961
	Higgs triplet $H^{\pm\pm} \to \ell\ell$	2,3,4 e, μ (SS)	–	–	139	$H^{\pm\pm}$ mass	1.08 TeV	DY production	ATLAS-CONF-2022-010
	Higgs triplet $H^{\pm\pm} \to \ell\tau$	$3\ e, \mu, \tau$	–	–	20.3	$H^{\pm\pm}$ mass	400 GeV	DY production, $\mathcal{B}(H_L^{\pm\pm} \to \ell\tau) = 1$	1411.2921
	Multi-charged particles	–	–	–	36.1	multi-charged particle mass	1.22 TeV	DY production, $\lvert q \rvert = 5e$	1812.03673
	Magnetic monopoles	–	–	–	34.4	monopole mass	2.37 TeV	DY production, $\lvert g \rvert = 1 g_D$, spin 1/2	1905.10130

$\sqrt{s}$ = 8 TeV | $\sqrt{s}$ = 13 TeV partial data | $\sqrt{s}$ = 13 TeV full data

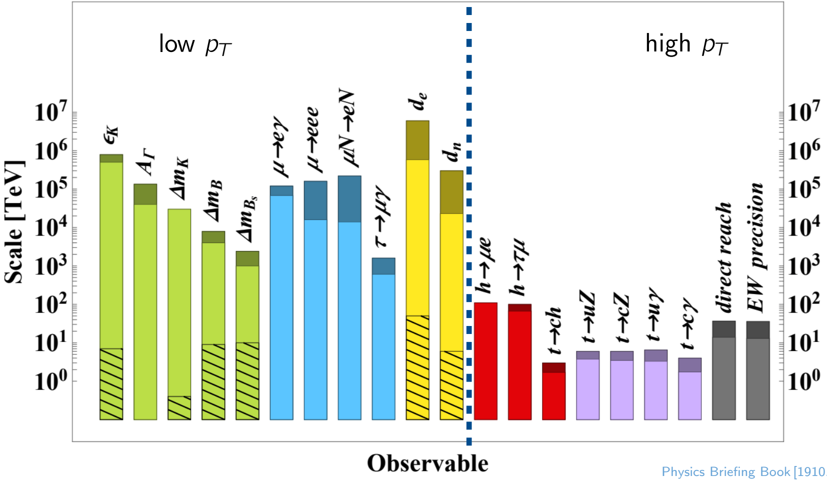
10^{-1} 1 10 **Mass scale [TeV]**

Lots of luminosity



- Marginal increase in energy, but **$\sim 20\times$ more luminosity!**
- Rather than looking for resonances, we can look for **traces of new physics**

Probing high-scales through precision



Physics Briefing Book [1910.11775]

Effective theories

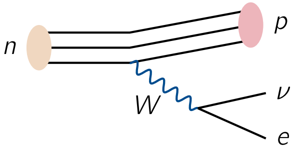
Effective theories are ubiquitous in physics:

- GR → Newtonian gravity
- QCD → nuclear physics
- Charge distribution → multi-pole expansion

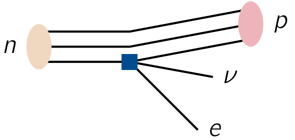
ETs effectively separates energy scales

- ETs can be formulated independently of the full theory
- In QFT the freeze out of heavy fields is formalized by **the decoupling theorem**, which is the foundation of EFTs
  Appelquist, Carazzone '75
- All theories break down eventually, so everything is an ET

Electroweak theory



Fermi theory



Effective field theory

High-energy physics manifests as contact interactions in EFTs

UV Physics

$$\mathcal{L}_{\mathrm{EFT}}(\phi) = \mathcal{L}_{d=4}(\phi) + \sum_{d=5}^{\infty} \sum_{k} \frac{C_{d,k}}{\Lambda^{d-4}} \mathcal{O}_{d,k}(\phi)$$

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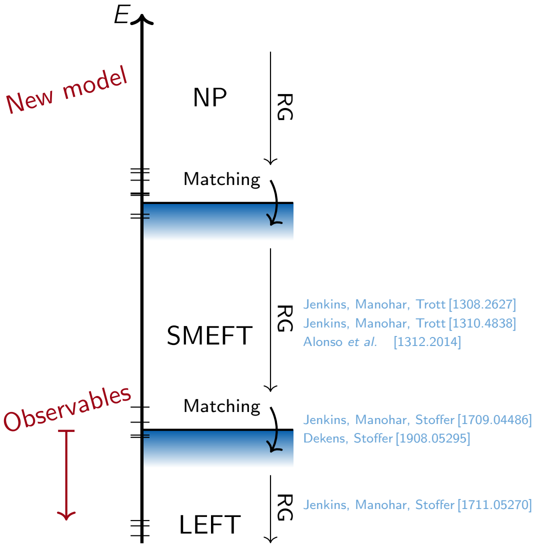
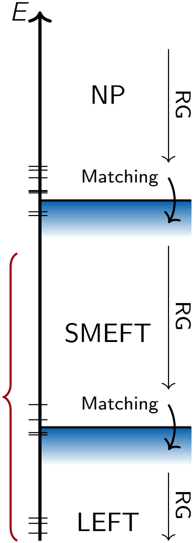
- **Bottom–up:**
  - The use of EFTs allow for a **model-comprehensive** ("model-independent") analysis of deviations from the SM, quantifying possible deviations as an expansion in E/Λ
- **Top–down:**
  - **Precision computations** necessitates the use of EFTs to separate the large scales introduced in BSM physics and avoid large logs
  - Many BSM models results in the same EFT, ensuring that computation are **reusable**: you only need to compute once in the EFT

Top–down EFT workflow



The repetitive nature of EFT computations call for **automated tools!**

Top–down EFT workflow



Jenkins, Manohar, Trott [1308.2627]
Jenkins, Manohar, Trott [1310.4838]
Alonso *et al.* [1312.2014]

Jenkins, Manohar, Stoffer [1709.04486]
Dekens, Stoffer [1908.05295]

Jenkins, Manohar, Stoffer [1711.05270]

Included in codes
wilson and **DsixTools**
Aebischer, Kumar, Straub [1804.05033]
Cellis *et al.* [1704.04504]
Fuentes-Martín *et al.* [2010.16341]

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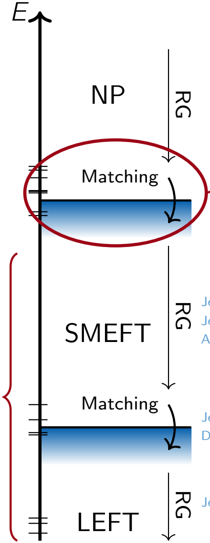

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Carmona *et al.* [2112.10787]; Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

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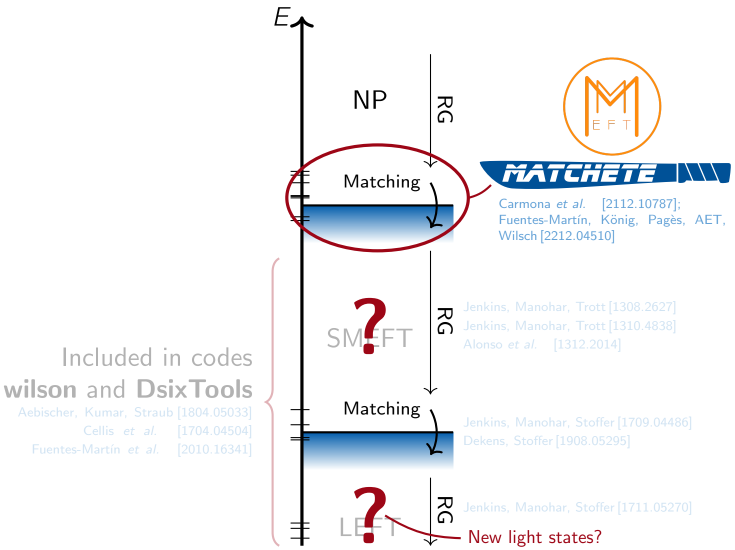
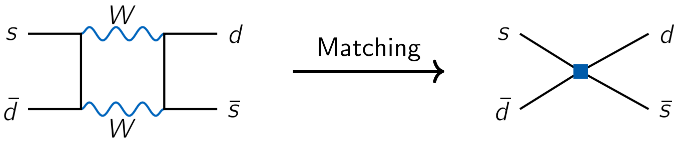


The repetitive nature of EFT computations call for **automated tools!**

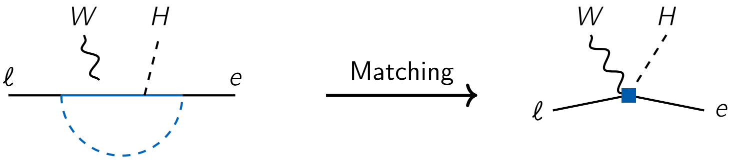
Matching at 1-loop order

1-loop matching is often the **leading contribution** from high-scale physics

- FCNCs in the SM



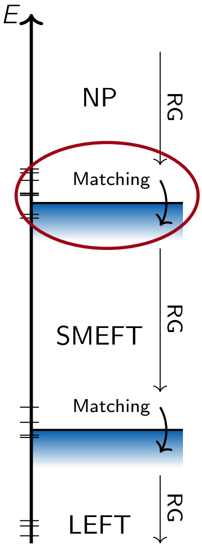
- In BSM models: dipoles, FCNCs, EW precision, ...



Matching EFTs at 1-Loop

The tools of the trade

Matching weakly coupled theories

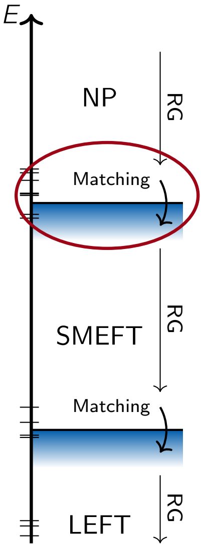


$\mathcal{L}_{\text{EFT}}$ should reproduce the physics of $\mathcal{L}_{\text{UV}}$ at energies $E \ll \Lambda$:

$$\mathcal{L}_{\text{UV}}(\Phi,\, \phi) \xrightarrow{\text{Matching}} \mathcal{L}_{\text{EFT}}(\phi)$$

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{d=4}(\phi) + \sum_{d=5}^{\infty} \sum_{\ell=0}^{\infty} \sum_{k} \underbrace{\frac{C_{d,k}^{(\ell)}}{(16\pi^2)^{\ell}\, \Lambda^{d-4}}}_{\text{double expansion}} \mathcal{O}_{d,k}^{(\ell)}(\phi)$$

Matching weakly coupled theories



$\mathcal{L}_{\mathrm{EFT}}$ should reproduce the physics of $\mathcal{L}_{\mathrm{UV}}$ at energies $E \ll \Lambda$:

$$\begin{tikzcd}
\mathcal{L}_{\mathrm{UV}}(\Phi,\, \phi) \arrow[rr, "\text{Matching}"] \arrow[d] & & \mathcal{L}_{\mathrm{EFT}}(\phi) \arrow[d] \\
\Gamma_{\mathrm{UV}}\big[\widehat{\Phi}(\phi),\, \phi\big] & \simeq \arrow[ur, "\text{Diagramatic}"] & \Gamma_{\mathrm{EFT}}[\phi]
\end{tikzcd}$$

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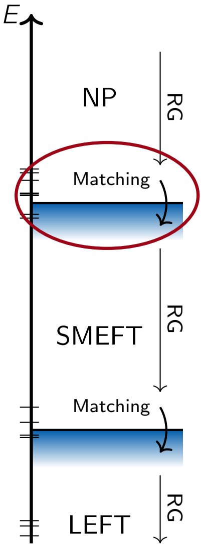
Matching weakly coupled theories



$\mathcal{L}_{\text{EFT}}$ should reproduce the physics of $\mathcal{L}_{\text{UV}}$ at energies $E \ll \Lambda$:

$$
\begin{tikzcd}
\mathcal{L}_{\text{UV}}(\Phi,\,\phi) \arrow[rr, "\text{Matching}"] \arrow[d] & & \mathcal{L}_{\text{EFT}}(\phi) \arrow[d] \\
\Gamma_{\text{UV}}\big[\widehat{\Phi}(\phi),\,\phi\big] \arrow[urr, "\text{Func.}"] & \simeq \arrow[ur] & \Gamma_{\text{EFT}}[\phi]
\end{tikzcd}
$$

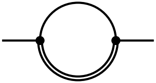
$$
\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}_{d=4}(\phi) + \sum_{d=5}^{\infty} \sum_{\ell=0}^{\infty} \sum_{k} \underbrace{\frac{C_{d,k}^{(\ell)}}{(16\pi^2)^{\ell}\,\Lambda^{d-4}}}_{\text{double expansion}} \mathcal{O}_{d,k}^{(\ell)}(\phi)
$$

Advantages of functional matching:

- Does not require prior knowledge of EFT basis
- Well-suited for algorithmic approach
- Computations are manifestly gauge covariant

Expansion by regions

With expansion by regions we can **separate scales in loop integrals**, e.g., a 2-point function with $p^2, m^2 \ll M^2$:

$$I =$$  $$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{1}{k^2 - M^2}$$

Expansion by regions

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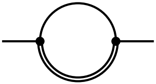
$I =$  $= \displaystyle\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{1}{k^2 - M^2}$

$k^2 \gtrsim M^2$

$I_h =$  $= \displaystyle\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^2 - M^2} + \dots$

Expansion by regions

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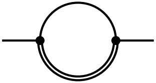
$$I =$$



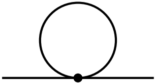
$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{1}{k^2 - M^2}$$

$$I_h =$$

$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^2 - M^2} + \ldots$$

$k^2 \ll M^2$

$$I_s =$$



$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{-1}{M^2} + \ldots$$

Expansion by regions

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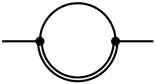
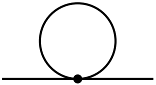
$$I = $$


$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{1}{k^2 - M^2}$$

$$I_h = $$


$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^2 - M^2} + \dots$$

$$I_s = $$


$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k+p)^2 - m^2} \frac{-1}{M^2} + \dots$$

In dimensional regularization, integrals equal the sum of their **hard** and **soft** parts

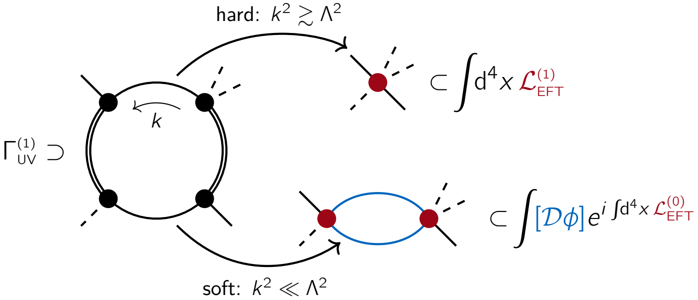
Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$I = I_h + I_s$$

The regions I_h and I_s are **systematically improvable** power series in $1/M^2$

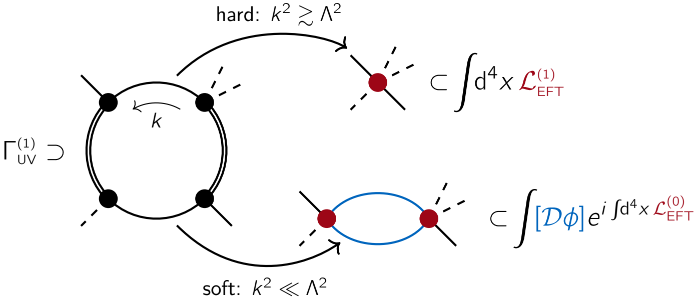
Separation of scales

Mixed (heavy–light) loop example:



Separation of scales

Mixed (heavy–light) loop example:



The quantum effective action of the UV theory is split in a hard and a soft part:

$$\Gamma^{(1)}_{\text{UV}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}} + \Gamma^{(1)}_{\text{UV}}\Big|_{\text{soft}}$$

Heavy field loops

Mixed field loops

Light field loops

Separation of scales

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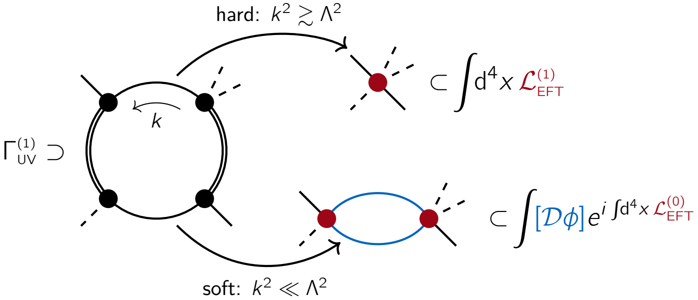

- $\Gamma^{(1)}_{\text{UV}}\big|_{\text{soft}}$: long-distance contributions included in 1-loop matrix elements of tree-level EFT operators

$$\Gamma^{(1)}_{\text{UV}}\big|_{\text{soft}} = \Gamma^{(1)}_{\text{EFT}}$$

- $\Gamma^{(1)}_{\text{UV}}\big|_{\text{hard}}$: short-distance contributions going into the EFT operators

Fuentes-Martin *et al.* [1607.02142]; Zhang [1610.00710]

$$\Gamma^{(1)}_{\text{UV}}\big|_{\text{hard}} = \int d^d x\, \mathcal{L}^{(1)}_{\text{EFT}}$$

Functional matching

The theory is expanded around the classical fields, $\hat{\eta}$:

$$\mathcal{L}_{\text{UV}}[\eta + \hat{\eta}] = \underbrace{\mathcal{L}_{\text{UV}}[\hat{\eta}]}_{\text{classical piece}} \; + \; \eta_i \underbrace{\frac{\delta \mathcal{L}_{\text{UV}}}{\delta \eta_i}[\hat{\eta}]}_{\text{EOM} \to 0} \; + \; \tfrac{1}{2}\eta_i \eta_j \underbrace{\frac{\delta^2 \mathcal{L}_{\text{UV}}}{\delta \eta_i \, \delta \eta_j}[\hat{\eta}]}_{\text{fluctuation operator } \mathcal{Q}_{ij}[\hat{\eta}]} + \ldots$$

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By saddlepoint approximation, the effective action is

$$e^{i\,\Gamma_{\text{UV}}[\hat{\eta}]} = e^{iS_{\text{UV}}[\hat{\eta}]} \int \mathcal{D}\eta \; \exp\left(i \int \mathrm{d}^d x \, \tfrac{1}{2} \eta_i \mathcal{Q}_{ij}[\hat{\eta}] \eta_j + \ldots \right)$$

$$\implies \; \Gamma_{\text{UV}}[\hat{\eta}] = S_{\text{UV}}[\hat{\eta}] + \frac{i}{2} \text{STr} \log \mathcal{Q}[\hat{\eta}] + \ldots$$

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Master formula for 1-loop matching

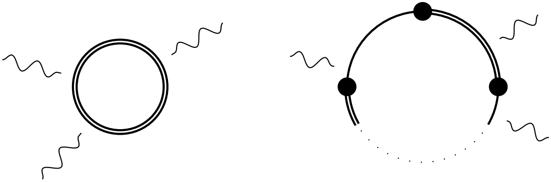
$$\int \mathrm{d}^d x\,\mathcal{L}^{(1)}_{\rm EFT} = \frac{i}{2}\,\mathrm{STr}\,\log\,\Delta^{-1}\Big|_{\rm hard} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\big[(\Delta X)^n\big]\Big|_{\rm hard}$$

where $\dfrac{\delta^2\mathcal{L}_{\rm UV}}{\delta\eta\,\delta\eta}[\hat{\eta}] = \Delta^{-1}(i\hat{D},\,M) - X(i\hat{D},\,\hat{\eta}),\qquad \Lambda^{1(2)} \sim \Delta^{-1} \gg X$

Cohen, Lu, Zhang [2011.02484] [2012.07851]; Fuentes-Martín, König, Pagès, AET, Wilsch [2012.08506]

Covariant derivative expansion

The traces are evaluated **gauge covariantly** with the CDE:



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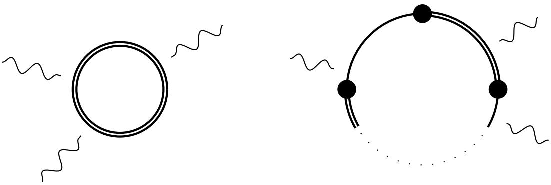

Example: $\mathrm{STr}[\Delta X]\big|_{\text{hard}}$ in a scalar theory with $\mathcal{L}_{\text{int}} = -\frac{\lambda}{2}(\Phi^\dagger\Phi)\phi^2$

$$\mathrm{STr}[\Delta X] = \int_x \int_k \frac{1}{(k_\mu + i\underbrace{D_\mu}_{\text{Open covariant derivate}})^2 - M_\Phi^2}(\lambda\phi^2)$$

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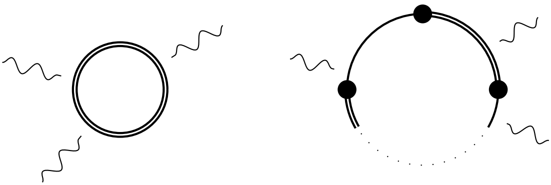

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$e^{-i\,D\cdot\partial_k}$ $e^{i\,D\cdot\partial_k}$

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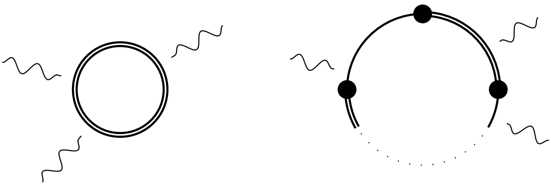

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$$= \int_x\int_k \frac{1}{(k_\mu + i\widetilde{G}_{\mu\nu}\partial_k^\nu)^2 - M_\Phi^2}(\lambda\phi^2)$$

$$\sum_{n=0}^{\infty} \frac{(-i)^n}{(n+2)n!}(D_{\alpha_1}\cdots D_{\alpha_n}G_{\mu\nu})\partial_k^{\alpha_1}\cdots\partial_k^{\alpha_n}$$

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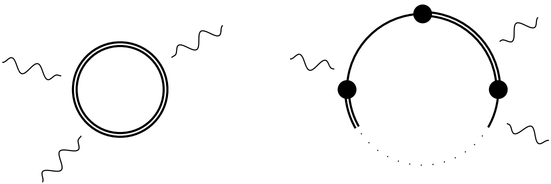

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$$\begin{aligned}\mathrm{STr}[\Delta X] &= \int_x \int_k \frac{1}{(k_\mu + iD_\mu)^2 - M_\Phi^2}(\lambda\phi^2) \\ &= \int_x \int_k \frac{1}{(k_\mu + i\widetilde{G}_{\mu\nu}\partial_k^\nu)^2 - M_\Phi^2}(\lambda\phi^2) \\ &= \int_x \int_k \frac{1}{k^2 - M_\Phi^2} \sum_{n=0}^{\infty} \left[\left((\widetilde{G}_{\mu\nu}\partial_k^\nu)^2 - i\{k^\mu, \widetilde{G}_{\mu\nu}\}\partial_k^\nu \right) \frac{1}{k^2 - M_\Phi^2} \right]^n (\lambda\phi^2)\end{aligned}$$

Truncate according to EFT order

Simplification and basis reduction

Number of SMEFT generators (1 gen., dim. 6):

$$80 \quad (1986) \quad \longrightarrow \quad 59 \quad (2017)$$

Buchmüller, Wyler '86 — Grzadkowski *et al.* [1008.4884]

Simplification and basis reduction

$$\mathcal{L} = -\tfrac{1}{2}\phi\partial^2\phi - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

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Exact simplification (linear):
IBP, Dirac identities, group identities, commutation relations,...

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On-shell equivalence (non-linear):

$$\text{Field redefinition:} \qquad \phi \longrightarrow \phi + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3$$

$$\mathcal{L} \longrightarrow -\tfrac{1}{2}\phi\partial^2\phi - \tfrac{1}{2}m^2\phi^2 - \left(\frac{\lambda}{24} + \frac{(3C_2 - C_3)m^2}{3\Lambda^2}\right)\phi^4 + \frac{18C_1 - \lambda(3C_2 - C_3)}{18\Lambda^2}\phi^6$$

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Removal of evanescent operators: (in application of fermion Fierz identities)
We will return to this point

Linear simplifications

Example: Integrating out heavy fermion in the fundamental representation of SU(3)

In[12]:=
```
LEFT // NiceForm
```

Out[12]//NiceForm=

$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

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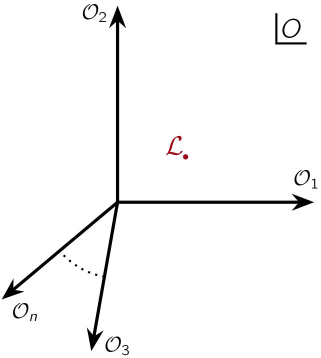
$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$



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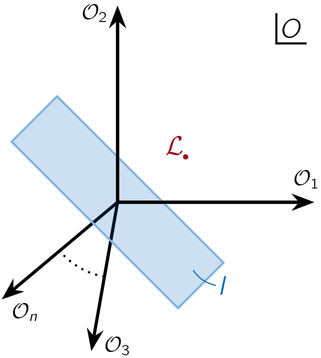

$I \subseteq O$ is the space of all operator identities, e.g., IBP relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

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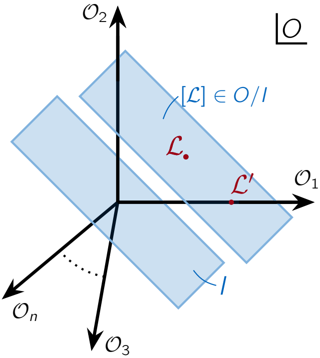

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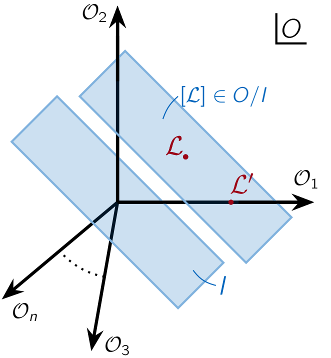
With **linear algebra** on the basis of I we find a simple representative element for $[\mathcal{L}_{\text{EFT}}] \in O/I$:

In[13]:=
```
LEFT // GreensSimplify // NiceForm
```
Out[13]//NiceForm=

$$-\frac{1}{15}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



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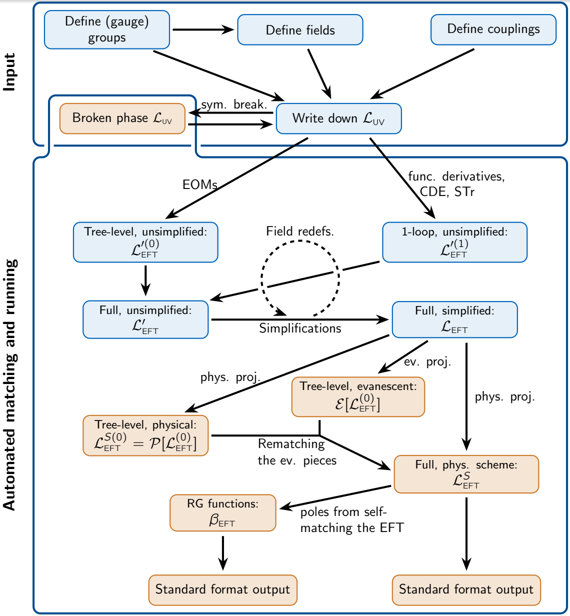
MATCHETE

To make your way through the BSM jungle

Automated EFT matching



Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

- **Matchete v0.1** is a Mathematica package
- Matching of **any model** with heavy scalars/fermions
- Simple and intuitive input/output
- Handles all group theory
- Simplifies to EFT basis*

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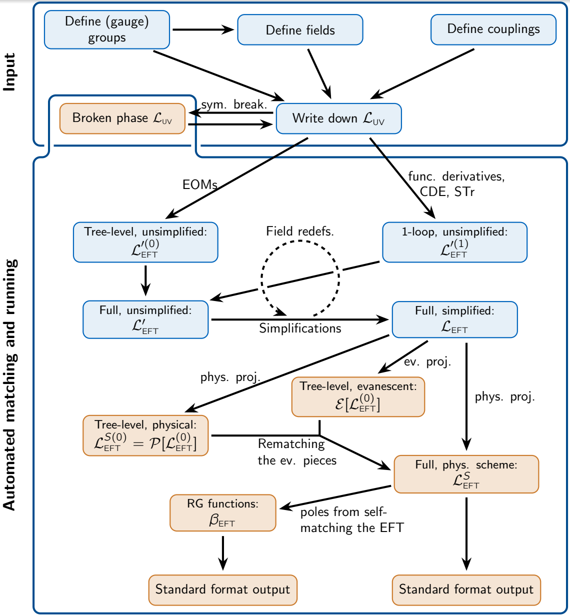

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Future plans:

- Handling of evanescent contribution
- SSB and heavy vectors
- Interface with EFT tool chain
- 1-loop RG computations

Example: SM + Vector-like lepton

Setup

SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_L\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_R\cdot EE\right)$$

Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_\mu\bar{H}_i\,D_\mu H^i+\mu^2\,\bar{H}_i\,H^i+i\left(\bar{d}^p_a\cdot\gamma_\mu\,P_R\cdot D_\mu d^{ap}\right)+i\left(\bar{e}^p\cdot\gamma_\mu\,P_R\cdot D_\mu e^p\right)+$$
$$i\left(\overline{EE}\cdot\gamma_\mu\cdot D_\mu EE\right)-ME\left(\overline{EE}\cdot EE\right)+i\left(\bar{l}^p_i\cdot\gamma_\mu\,P_L\cdot D_\mu l^{ip}\right)+i\left(\bar{q}^p_{ai}\cdot\gamma_\mu\,P_L\cdot D_\mu q^{aip}\right)+i\left(\bar{u}^p_a\cdot\gamma_\mu\,P_R\cdot D_\mu u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_i\,\bar{H}_j\,H^i\,H^j-\overline{Yd}^{pr}\,\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)-Ye^{pr}\,H^i\left(\bar{l}^p_i\cdot P_R\cdot e^r\right)-Yd^{pr}\,H^i\left(\bar{q}^p_{ai}\cdot P_R\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_i\left(\bar{q}^p_{aj}\cdot P_R\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}\,H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}-\overline{yE}^p\,\bar{H}_i\left(\overline{EE}\cdot P_L\cdot l^{ip}\right)-yE^p\,H^i\left(\bar{l}^p_i\cdot P_R\cdot EE\right)$$

Example: SM + Vector-like lepton

Matching

In[10]:=
```
LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;
```

In[11]:=
```
LEFTOnShell = LEFT // EOMSimplify;
Length@%
```

EOMSimplify: The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

Out[12]= 67

In[13]:=
```
SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[13]//NiceForm=

$$\frac{i}{360}\hbar\frac{1}{ME^2}\left(48\,gY^4\,\delta^{pr}+5\,\overline{yE}^{s}\left(3\,yE^{t}\,\overline{Ye}^{tr}\,Ye^{sp}\left(1+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)-2\,yE^{s}\,gY^2\left(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\overline{H}_i\,H^i\left(\overline{e}^{r}\cdot\gamma_\mu P_R\cdot e^{p}\right)+\overline{H}_i\,D_\mu H^i\left(\overline{e}^{r}\cdot\gamma_\mu P_R\cdot e^{p}\right)\right)$$

$$Q_{He}^{pr}=(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$$

Example: SM + Vector-like lepton

```
LEFTOnShell // NiceForm
```

liceForm=

$$
\begin{aligned}
&-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+\left(-\frac{1}{4}-\frac{1}{3}\hbar\, gY^2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)B^{\mu\nu 2}+D_\mu\bar{H}_i\,D_\mu H^i+\\
&\Big(cHH+\frac{1}{6}\hbar\,\overline{yE}^p\,yE^p\,cHH\,\frac{1}{ME^2}\Big(2\,cHH-3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\bar{H}_i H^i+i\left(\bar{d}^r_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap}\right)\delta^{pr}+\\
&i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot D_\mu e^p\right)\delta^{pr}+i\left(\bar{l}^r_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}\right)\delta^{pr}+i\left(\bar{q}^r_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}\right)\delta^{pr}+i\left(\bar{u}^r_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap}\right)\delta^{pr}+\\
&\Big(-\frac{1}{2}\lambda+\hbar\Big(-\frac{1}{2}\overline{yE}^p\Big(4\,yE^r\,\overline{Ye}^{rs}\,Ye^{ps}\Big(1+\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)-yE^p\Big(-2\,\overline{yE}^r\,yE^r\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]+\lambda\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)-\\
&\qquad\frac{1}{180}cHH\,\frac{1}{ME^2}\Big(12\,gY^4-5\,\overline{yE}^p\,yE^p\,gY^2\Big(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)+\\
&\qquad\quad 5\,\overline{yE}^p\Big(-12\left(\overline{yE}^r\,yE^p\,yE^r+6\,yE^r\,\overline{Ye}^{rs}\,Ye^{ps}-2\,yE^p\lambda\right)+yE^p\,gL^2\Big(5+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)\Big)\bar{H}_i\bar{H}_j H^i H^j+\\
&\Big(-\overline{Yd}^{pr}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,\overline{Yd}^{pr}\,\frac{1}{ME^2}\Big(-4\,cHH+3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)+\\
&\Big(-\overline{Ye}^{pr}+\frac{1}{24}\hbar\,yE^s\,\frac{1}{ME^2}\Big(-3\,\overline{yE}^p\,\overline{Ye}^{sr}\left(2\,cHH-ME^2\right)\Big(3+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)+2\,\overline{yE}^s\,\overline{Ye}^{pr}\Big(-4\,cHH+3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)\\
&\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)+\\
&\Big(-Ye^{rp}+\frac{1}{24}\hbar\,\overline{yE}^s\,\frac{1}{ME^2}\Big(3\,ME^2\Big(2\,yE^s\,Ye^{rp}\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)+yE^r\,Ye^{sp}\Big(3+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)-\\
&\qquad 2\,cHH\Big(4\,yE^s\,Ye^{rp}+3\,yE^r\,Ye^{sp}\Big(3+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)H^i\left(\bar{l}^r_i\cdot P_R\cdot e^p\right)+\\
&\Big(-Yd^{rp}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,Yd^{rp}\,\frac{1}{ME^2}\Big(-4\,cHH+3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)H^i\left(\bar{q}^r_{ai}\cdot P_R\cdot d^{ap}\right)+\\
&\Big(-Yu^{rp}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,Yu^{rp}\,\frac{1}{ME^2}\Big(-4\,cHH+3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\bar{H}_i\left(\bar{q}^r_{aj}\cdot P_R\cdot u^{ap}\right)\varepsilon^{ji}+\\
&\Big(-\overline{Yu}^{pr}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,\overline{Yu}^{pr}\,\frac{1}{ME^2}\Big(-4\,cHH+3\,ME^2\Big(1+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}+\\
&\frac{1}{180}\hbar\,\frac{1}{ME^2}\Big(12\,\lambda\,gY^4+\\
&\quad 5\,\overline{yE}^p\Big(12\,\overline{yE}^r\,yE^p\left(\overline{yE}^s\,yE^r\,yE^s+6\,yE^s\,\overline{Ye}^{st}\,Ye^{rt}-yE^r\lambda\right)-72\,yE^r\,\overline{Ye}^{rs}\Big(Ye^{ps}\lambda+\overline{Ye}^{tu}\,Ye^{pu}\,Ye^{ts}\Big(1+\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)+\\
&\qquad yE^p\lambda\Big(12\,\lambda+gL^2\Big(5+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)-gY^2\Big(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\Big)\Big)\Big)\Big)\bar{H}_i\bar{H}_j\bar{H}_k H^i H^j H^k+
\end{aligned}
$$

Evanescent Operators

Why can't QFT just play nice?

EFT from a 2HDM

Example: SM + leptophilic Higgs, $\Phi \sim (\mathbf{1},\, \mathbf{2})_{1/2}$:

$$\mathcal{L} \supset \mathcal{L}_{\mathsf{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M_\Phi^2\, \Phi^\dagger \Phi - \left(y_{\Phi e}^{pr}\, \bar{\ell}_p \Phi e_r + \text{h.c.} \right) + \ldots$$

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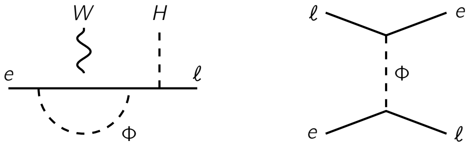

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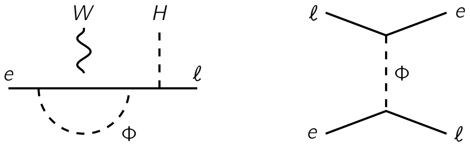

Below the scale $M_\Phi \gg v_{\text{EW}}$

$$\mathcal{L}_{\text{EFT}} \supset C_{eW}^{pr} Q_{eW}^{pr} + C_{\ell e}^{prst} R_{\ell e}^{prst}$$

But the tree-level operator $R_{\ell e}$ is not part of the Warsaw basis

Changing basis in an EFT

In $d = 4$ dimensions, $\mathcal{L}_{\text{EFT}} = \widetilde{\mathcal{L}}_{\text{EFT}}$, where

$$\mathcal{L}_{\text{EFT}} \supset C^{pr}_{eW} Q^{pr}_{eW} + C^{prst}_{\ell e} R^{prst}_{\ell e} \qquad R^{prst}_{\ell e} = (\bar{\ell}_p e_r)(\bar{e}_s \ell_t)$$

$$\widetilde{\mathcal{L}}_{\text{EFT}} \supset C^{pr}_{eW} Q^{pr}_{eW} - \tfrac{1}{2} C^{ptsr}_{\ell e} Q^{prst}_{\ell e} \qquad Q^{prst}_{\ell e} = (\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r)$$

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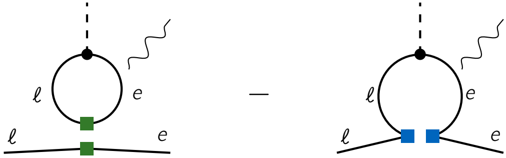
But the 1-loop EFT amplitudes are different!

$$i\big(\mathcal{A}_{eH\to\ell W} - \widetilde{\mathcal{A}}_{eH\to\ell W}\big) = \frac{g_2}{64\pi^2}[C_{\ell e}]^{prst} y_e^{ts} \big(\bar{u}\tau^I \sigma_{\mu\nu} P_R u\big) q^\mu \varepsilon^{*\nu}$$



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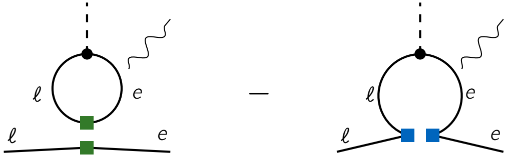

For $d \neq 4$, there is an **evanescent operator:**

$$R_{\ell e}^{prst} = -\tfrac{1}{2} Q_{\ell e}^{ptsr} + E_{\ell e}^{prst}, \qquad E_{\ell e}^{prst} \xrightarrow{d\to 4} 0$$

Evanescent operators

An **evanescent operator** E is an operator satisfying

$$\mathrm{rank}(E) = \epsilon \xrightarrow{d \to 4} 0$$

Evanescent contributions have long been accounted for in the LEFT (Weak Effective Hamiltonian). Not so much in BSM context

Buras, Weisz '90; Dugan, Grinstein '91; Herrlich, Nierste [hep-ph/9412375];...

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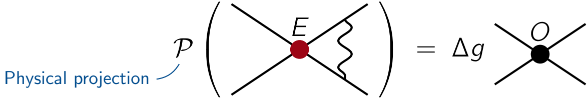
The physical contributions from evanescent operators are **finite and local**



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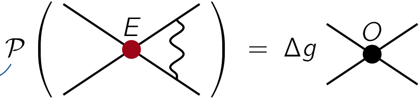
Physical projection



e.g., in the 2HDM example

$$E_{\ell e}^{prst} \longrightarrow -\frac{g_L y_e^{ts}}{128\pi^2} Q_{eW}^{pr} + [\text{many other contributions}]$$

The physical projector

Choosing a set of identities allows for defining the **physical projector $\mathcal{P}$**:

$$O_d = \underbrace{\mathcal{P}\, O_d}_{\text{phys. part}} + \underbrace{\overbrace{\mathcal{E}_{\mathcal{P}}}^{\text{id} - \mathcal{P}} O_d}_{\text{ev. part}}$$

- Reduction of Dirac structures for 4-fermion operators, e.g.,

$$(\gamma^\mu \gamma^\nu \gamma^\lambda P_L) \otimes [\gamma_\lambda \gamma_\nu \gamma_\mu P_L] = 4(1 - \overbrace{2\epsilon}^{\text{Compatibility with NDR}})\,(\gamma^\mu P_L) \otimes [\gamma_\mu P_L] + E^{[3]}_{LL}$$

- Fierz identities for 4-fermion operators, e.g.,

$$(P_R) \otimes [P_L] = -\tfrac{1}{2}(\gamma_\mu P_L] \otimes [\gamma_\mu P_R) + E_{\text{Fierz}}(P_R, P_L)$$

- Other identities involving γ_5 and/or the Levi-Civita tensor, e.g.,

$$\varepsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = 2i\sigma_{\mu\nu}\gamma_5 + E^{(\varepsilon\cdot\sigma)}_{\mu\nu}$$

Evanescence-free schemes

For an EFT Lagrangian $\mathcal{L} = \bar{g}_a O^a + \bar{\eta}_i E^i$, the 1-loop effective action is

1-loop diagrams, tree-level couplings

$$\Gamma = \int_x \left(\bar{g}_a O^a + \bar{\eta}_i E^i \right) + \bar{\Gamma}(g, \eta).$$

bare couplings

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$$\Gamma = \int_x \Big(\underbrace{\bar{g}_a}_{\text{bare couplings}} O^a + \bar{\eta}_i E^i \Big) + \overbrace{\bar{\Gamma}}^{\text{1-loop diagrams, tree-level couplings}}(g, \eta).$$

Scheme	$\overline{\text{MS}}$
Action: $\mathcal{P} : O^a$	$\bar{g}_a = g_a + \delta g_a$
Action: $\mathcal{E}_{\mathcal{P}} : E^i$	$\bar{\eta}_i = \eta_i + \delta \eta_i$
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Scheme		$\overline{\text{MS}}$	Compensated
Action	$\mathcal{P} : O^a$	$\bar{g}_a = g_a + \delta g_a$	$(\bar{g}_a + \Delta g_a) - \Delta g_a$
	$\mathcal{E}_{\mathcal{P}} : E^i$	$\bar{\eta}_i = \eta_i + \delta\eta_i$	$\delta\eta_i + \eta_i$
Phys. eff. action $\mathcal{P}\Gamma$		$\int_x \bar{g}_a O^a + \mathcal{P}\bar{\Gamma}(g, \eta)$	$\int_x (\bar{g}_a + \Delta g_a) O^a + \mathcal{P}\bar{\Gamma}(g, \eta) - \int_x \Delta g_a O^a$

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The evanescent contribution is defined by

$$\int_x \Delta g_a O^a \equiv \mathcal{P} \Big[\overbrace{\overline{\Gamma}(g, \eta) - \overline{\Gamma}(g, 0)}^{\text{local, finite}} \Big]$$

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Action $\mathcal{P}: O^a$	$\bar{g}_a = g_a + \delta g_a$	$(\bar{g}_a + \Delta g_a) - \boxed{\Delta g_a}$	$\bar{g}_a + \Delta g_a$
Action $\mathcal{E}_\mathcal{P}: E^i$	$\bar{\eta}_i = \eta_i + \delta\eta_i$	$\delta\eta_i \quad + \boxed{\eta_i}$	$\delta\eta_i$
Phys. eff. action $\mathcal{P}\Gamma$	$\int_x \bar{g}_a O^a + \mathcal{P}\overline{\Gamma}(g,\eta)$	$\int_x (\bar{g}_a + \Delta g_a) O^a + \mathcal{P}\overline{\Gamma}(g, 0)$	$\int_x (\bar{g}_a + \Delta g_a) O^a + \mathcal{P}\overline{\Gamma}(g, 0)$

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Handling evanescent contributions means computing Δg

Application in the SMEFT

Tree-level BSM matching to the SMEFT can produce **49 different, redundant four-fermion operators**, which will result in non-trivial evanescent contribution at 1-loop order, e.g.,

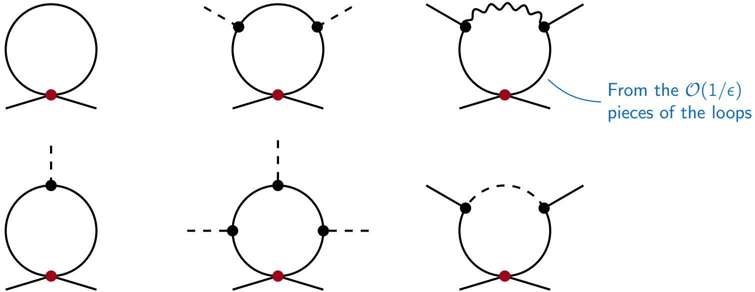
$$R_{\ell e} = (\bar{\ell} e)(\bar{e}\ell) \qquad R^{(8)}_{qu} = (\bar{q} T^A u)(\bar{u} T^a q) \qquad R_{u^c e l q^c} = (\bar{u}^c e)(\bar{l} q^c)$$

Application in the SMEFT

Tree-level BSM matching to the SMEFT can produce **49 different, redundant four-fermion operators**, which will result in non-trivial evanescent contribution at 1-loop order, e.g.,

$$R_{\ell e} = (\bar{\ell} e)(\bar{e}\ell) \qquad R_{qu}^{(8)} = (\bar{q}T^A u)(\bar{u}T^a q) \qquad R_{u^c e l q^c} = (\bar{u}^c e)(\bar{l} q^c)$$

For dimension-6 SMEFT, evanescent operators contribute through 6 covariant trace topologies



Fuentes-Martín, König, Pagès, AET, Wilsch [2211.09144]

Matematica interface

Filter: Redundant | SMEFT | All

$R_{\ell e}^{prst}$ $R_{\ell u}^{prst}$ $R_{\ell d}^{prst}$ R_{qe}^{prst} $R_{qu}^{(1)prst}$ $R_{qu}^{(8)prst}$ $R_{qd}^{(1)prst}$ $R_{qd}^{(8)prst}$ $R_{\ell uqe}^{prst}$ $R_{\ell^c\ell}^{prst}$ $R_{q^cq}^{prst}$ $R_{q^cq}^{\prime prst}$ $R_{q^c\ell}^{prst}$ $R_{q^c\ell}^{\prime prst}$ $R_{e^ce}^{prst}$ $R_{u^cu}^{prst}$ $R_{d^cd}^{prst}$

$R_{e^cu}^{prst}$ $R_{e^cd}^{prst}$ $R_{u^cd}^{prst}$ $R_{u^cd}^{\prime prst}$ $R_{u^cdqq^c}^{prst}$ $R_{u^ce\ell q^c}^{prst}$ $R_{q^cqq^c\ell}^{prst}$ $R_{u^cud^ce}^{prst}$ $R_{\ell\ell}^{(3)prst}$ $R_{qq}^{(1,8)prst}$ $R_{qq}^{(3,8)prst}$ $R_{\ell q}^{(1)prst}$ $R_{\ell q}^{(3)prst}$ $R_{uu}^{(8)prst}$ $R_{dd}^{(8)prst}$

R_{eu}^{prst} R_{ed}^{prst} $R_{ud}^{(1)prst}$ $R_{ud}^{(8)prst}$ $R_{\ell qde}^{prst}$ $R_{\ell^ce}^{prst}$ $R_{\ell^cu}^{prst}$ $R_{\ell^cd}^{prst}$ $R_{q^ced\ell^c}^{prst}$ $R_{q^ce}^{prst}$ $R_{q^cu}^{prst}$ $R_{q^cu}^{\prime prst}$ $R_{q^cd}^{prst}$ $R_{q^cd}^{\prime prst}$ $R_{d^c\ell q^cu}^{prst}$ $R_{u^c\ell q^cd}^{prst}$ $R_{q^ceu^cq}^{prst}$

Operator definition:

$$R_{\ell qde}^{prst} = (\bar{\ell}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu e_t)$$

Reduces to:

$Q_{\ell edq}^{prst}$, $Q_{quqd}^{(1)prst}$, Q_{dW}^{pr}, Q_{dB}^{pr}, Q_{dH}^{pr}, Q_{yd}^{pr}, Q_{eB}^{pr}, Q_{ed}^{prst}, Q_{eH}^{pr}, Q_{eW}^{pr},
$Q_{\ell d}^{prst}$, $Q_{\ell e}^{prst}$, $Q_{\ell equ}^{(1)prst}$, $Q_{\ell equ}^{(3)prst}$, $Q_{\ell q}^{(1)prst}$, $Q_{\ell q}^{(3)prst}$, $Q_{qd}^{(1)prst}$, $Q_{qd}^{(8)prst}$, Q_{qe}^{prst}, Q_{ye}^{pr}

Reduction Identity:

$$\begin{aligned}
R_{\ell qde}^{prst} = &- 2Q_{\ell edq}^{ptsr} + \frac{1}{16\pi^2}\Big(\frac{1}{6}\overline{y_e^{pt}}\; y_d^{uv} Q_{qd}^{(1)ursv} + \frac{1}{4} g_Y y_d^{rs} Q_{eB}^{pt} \\
&+ \frac{3}{4} g_Y \overline{y_e^{pt}}\; \overline{Q_{dB}^{rs}} + Q_{eH}^{pt}\Big(6\overline{y_d^{uv}}\; y_d^{rv} y_d^{us} - 3\lambda y_d^{rs}\Big) \\
&+ Q_{\ell equ}^{(1)ptuv}\Big(\frac{3}{4} y_d^{us} y_u^{rv} + 3 y_d^{rs} y_u^{uv}\Big) + \overline{y_e^{pt}}\; y_d^{uv} Q_{qd}^{(8)ursv} \\
&+ \frac{3}{2}\overline{y_e^{uv}}\; y_d^{rs} Q_{\ell e}^{puvt} + 2\overline{y_e^{pu}}\; \overline{y_e^{vt}}\; y_e^{vu} \overline{Q_{dH}^{rs}} - \frac{1}{16} y_d^{us} y_u^{rv} Q_{\ell equ}^{(3)ptuv} \\
&- \frac{1}{4} g_L \overline{y_e^{pt}}\; \overline{Q_{dW}^{rs}} - \frac{1}{4}\overline{y_e^{ut}}\; y_d^{vs} Q_{\ell q}^{(1)puvr} - \frac{1}{4}\overline{y_e^{ut}}\; y_d^{vs} Q_{\ell q}^{(3)puvr} \\
&- \frac{1}{2}\overline{y_e^{ut}}\; y_d^{rv} Q_{\ell d}^{pusv} - \frac{1}{2}\overline{y_e^{pu}}\; y_d^{vs} Q_{qe}^{vrut} - \frac{3}{4} g_L y_d^{rs} Q_{eW}^{pt} \\
&- \overline{y_e^{pt}}\; \overline{y_u^{uv}}\; \overline{Q_{quqd}^{(1)uvrs}} - \lambda \overline{y_e^{pt}}\; \overline{Q_{dH}^{rs}} - \mu^2 \overline{y_e^{pt}}\; \overline{Q_{yd}^{rs}} \\
&- \overline{y_e^{pu}}\; y_d^{rv} Q_{ed}^{utsv} - \overline{y_e^{pt}}\; y_e^{uv} Q_{\ell edq}^{uvsr} - 3\overline{y_d^{uv}}\; y_d^{rs} Q_{\ell edq}^{ptvu} \\
&- 3\mu^2 y_d^{rs} Q_{ye}^{pt}\Big)
\end{aligned}$$

› TeX

Outlook

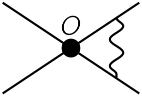
- (Automatic) EFT matching is crucial to BSM phenomenology
- Functional matching provides a direct approach to automated matching
- One must carefully account for evanescent operators in computations
- **Matchete** is a public code for EFT matching. It already greatly simplifies the matching task and many more features are planned!

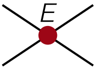
https://gitlab.com/matchete/matchete

MATCHETE

Backup

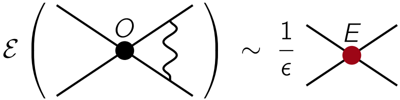
RG in evanescent schemes

$\mathcal{E}\Big($ 

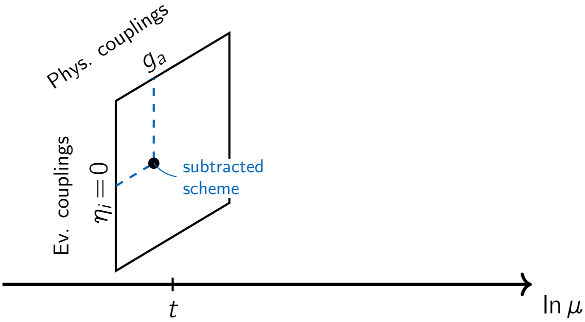
$\Big) \sim \frac{1}{\epsilon}$ 

$\implies \delta\eta(g) \neq 0$

RG in evanescent schemes

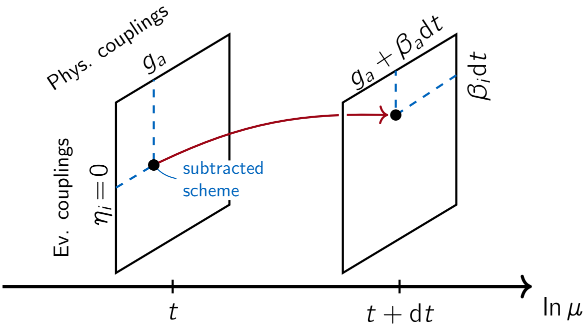


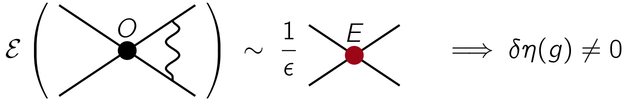
$$\Longrightarrow \quad \delta\eta(g) \neq 0$$



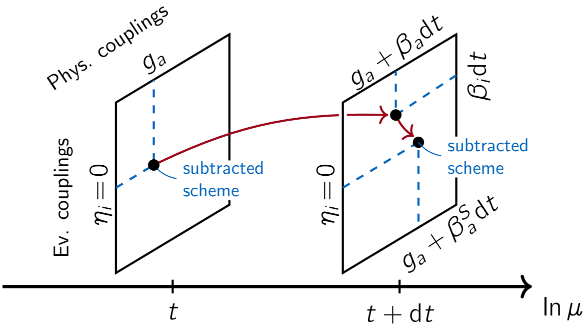
RG in evanescent schemes

$$\mathcal{E}\left(O\right) \sim \frac{1}{\epsilon} E \implies \delta\eta(g) \neq 0$$



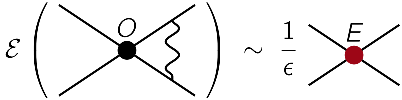


$\mathcal{E}$
O
$\sim \frac{1}{\epsilon}$
E
$\Longrightarrow \delta\eta(g) \neq 0$

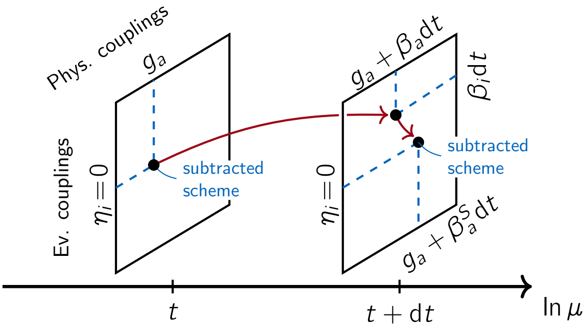


Phys. couplings
g_a
Ev. couplings
$\eta_i = 0$
subtracted scheme
$g_a + \beta_a \mathrm{d}t$
$\beta_i \mathrm{d}t$
$\eta_i = 0$
subtracted scheme
$g_a + \beta_a^S \mathrm{d}t$
t
$t + \mathrm{d}t$
$\ln \mu$

RG in evanescent schemes



$$\implies \delta\eta(g) \neq 0$$



In the subtracted evanescent scheme

$$\frac{\mathrm{d}g_a}{\mathrm{d}t} = \beta_a^S = \beta_a + \overbrace{\beta_i \left.\frac{\partial \Delta g_a}{\partial \eta_i}\right|_{\eta=0}}^{\text{2-loop}}$$

Matchete demonstration (SM implementation)

Gauge Groups

```
DefineGaugeGroup[SU3c, SU@3, gs, G,
    FundAlphabet → CharacterRange["a", "f"],
    AdjAlphabet → CharacterRange["A", "F"]]
DefineGaugeGroup[SU2L, SU@2, gL, W,
    FundAlphabet → CharacterRange["i", "n"],
    AdjAlphabet → CharacterRange["I", "N"]]
DefineGaugeGroup[U1Y, U@1, gY, B]
```

Generation index

```
DefineFlavorIndex[Flavor, 3,
    IndexAlphabet → {"p", "r", "s", "t", "u", "v"}]
```

Fermions

```
DefineField[q, Fermion,
    Indices → {SU3c@fund, SU2L@fund, Flavor},
    Charges → {U1Y[1/6]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[u, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[2/3]},
    Chiral → RightHanded,
    Mass → 0]
DefineField[d, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[-1/3]},
    Chiral → RightHanded,
    Mass → 0]
```

```
DefineField[l, Fermion,
    Indices → {SU2L@fund, Flavor},
    Charges → {U1Y[-1/2]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[e, Fermion,
    Indices → {Flavor},
    Charges → {U1Y[-1]},
    Chiral → RightHanded,
    Mass → 0]
```

Higgs

```
DefineField[H, Scalar,
    Indices → {SU2L@fund},
    Charges → {U1Y[1/2]},
    Mass → 0]
```

Couplings

```
DefineCoupling[λ, SelfConjugate → True]
DefineCoupling[μ, SelfConjugate → True,
  EFTorder → 1];
DefineCoupling[Ye,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yu,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yd,
    Indices → {Flavor, Flavor}]
```

Matchete demonstration (SM implementation)

Lagrangian

```
ℒSM = FreeLag[] +
     -μ[]^2 Bar@H[i] × H[i] -
     λ[]/2 Bar@H[i] × H[i] × Bar@H[j] × H[j] +
     PlusHc[
          -Yu[p, r] × CG[eps@SU2L, {i, j}] ×
         Bar@H@i × Bar@q[a, j, p] ** u[a, r]
          - Yd[p, r] × H@i × Bar@q[a, i, p] ** d[a, r]
          - Ye[p, r] × H@i × Bar@l[i, p] ** e[r]
     ] // RelabelIndices;
```

```
ℒSM // NiceForm
```

Form=

$$
\begin{aligned}
&-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i\, D_\mu H^i - \\
&\mu^2 \bar{H}_i H^i + i\left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i\left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + \\
&i\left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i\left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + \\
&i\left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2} \lambda \bar{H}_i \bar{H}_j H^i H^j - \\
&\overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - \\
&Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - \\
&Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ij} - \overline{Yu}^{pr} H^i \left(\bar{u}^r_a \cdot P_L \cdot q^{ajp}\right) \bar{\varepsilon}_{ij}
\end{aligned}
$$