Technische Universität München
T31 SFB1258 Projekt D04
„Neutrinos and Dark Matter in Astro- and Particle Physics“

NLO Electric field correlators for the Dark Matter relic abundance

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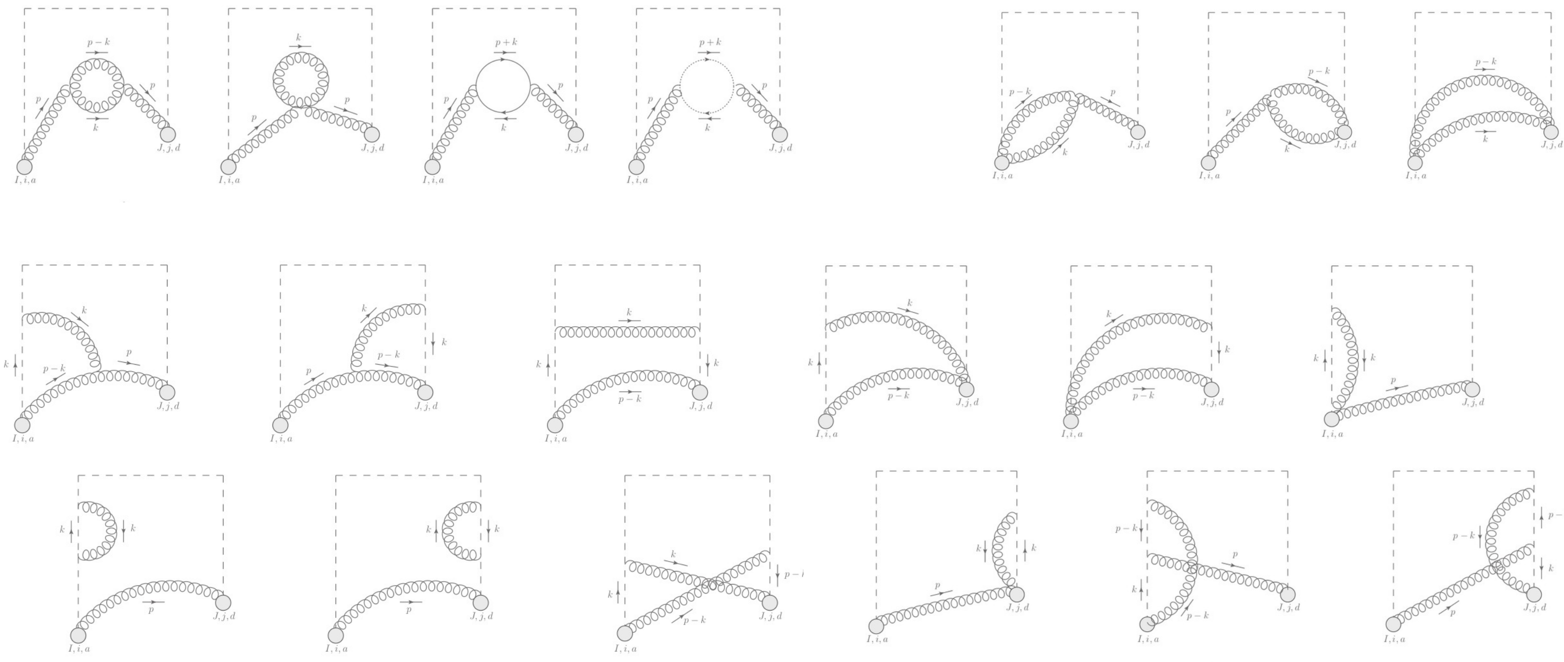
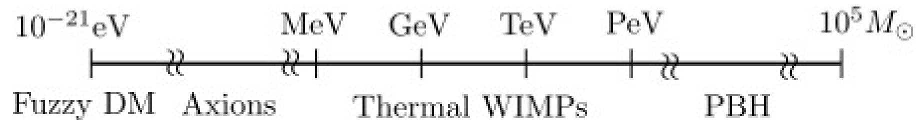
21th March 2023

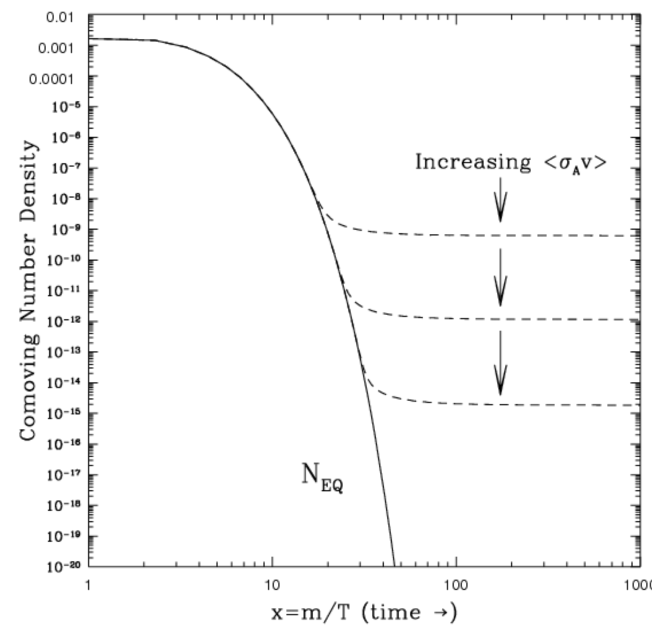


@ HECA seminar, Warsaw

Introduction



- WIMPs produced via freeze-out mechanism can explain all of the present DM energy fraction
- $\gtrsim$ TeV mass scale:
  - less constrained, attractive models
  - e.g., EW charged DM, colored coannihilation
  - Expirience long-range force effects: Sommerfeld enhancement + bound states typically allow for larger DM masses
- How do bound states actually form and get dissociated inside the early Universe plasma? (NLO vs. previous LO effects)



Overview

- Long-range force effects in vacuum (LO):
  - positronium example
  - pNRQED
- Dark Matter examples
  - EW charged DM: Wino, higher reps.
  - colored coannihilation
- Bound-state formation/dissociation at NLO zero and finite temperature
  - Collinear divergences (original motivation)
  - Combining pNREFT and thermal field theory
  - NLO electric field correlator results for U(1) and SU(N)
  - In context with EW charged DM, colored coannihilation, and quarkonium transport inside the QGP

Positronium example

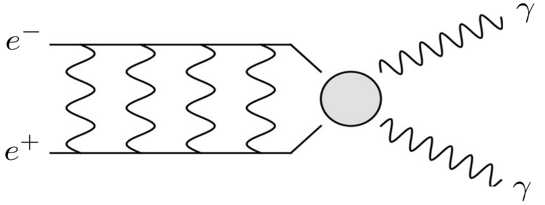
Bound-state decay and Sommerfeld enhancement:

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

Pirenne & Wheeler 1946

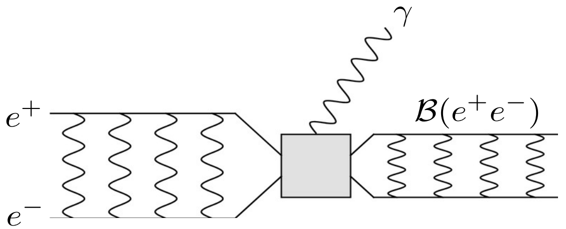
$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v) \, , \text{ for } v \lesssim \alpha .$$

Sakharov 1948 (Sommerfeld 1931)



Bound-state formation (recombination):

$$(\sigma v)_{nl} = \frac{4\alpha}{3} | \langle \psi_{nl} | \, \mathbf{r} \, | \psi_v \rangle |^2 \Delta E^3$$
$$\sim 3 \times \text{annihilation}, \text{ for } v \lesssim \alpha .$$



pNRQED [Pineda&Soto 1999, Brambilla et al. 2000, 2005]

- All these effects are contained in *potential* non-relativistic (pNR) effective field theory:

$$\mathcal{L}^{\mathrm{pNRQED}} \supset \int \mathrm{d}^3 r \; S^\dagger(\mathbf{x},\mathbf{r},t)\left[i\partial_t + \frac{\nabla_{\mathbf{x}}^2}{4m_e} + \frac{\nabla_{\mathbf{r}}^2}{m_e} - V(r) + i2\frac{\pi\alpha^2}{m_e^2}\delta^3(\mathbf{r}) + \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)\right] S(\mathbf{x},\mathbf{r},t)$$

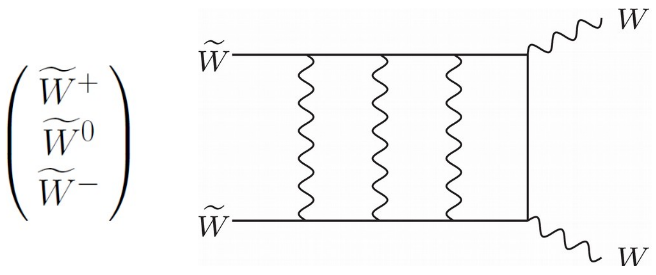
$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$
$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$

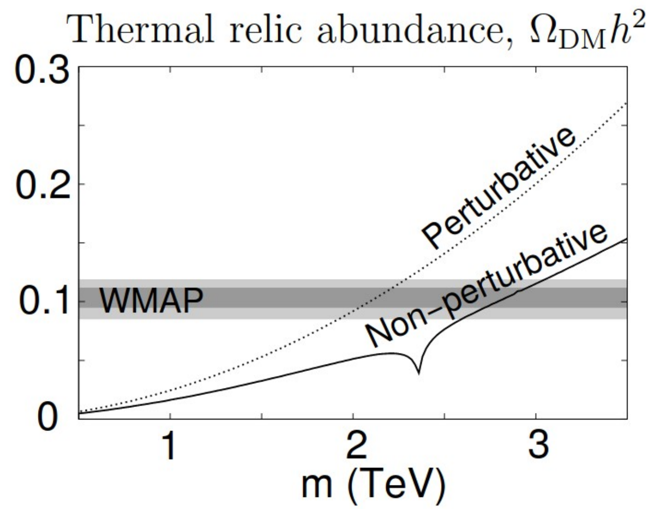
$$(\sigma v)_{nl} = \frac{4\alpha}{3} |\langle \psi_{nl}| \mathbf{r} |\psi_v\rangle|^2 \Delta E^3$$

- Later, pNREFT and thermal field theory are combined to obtain BSF @ NLO

Wino Dark Matter

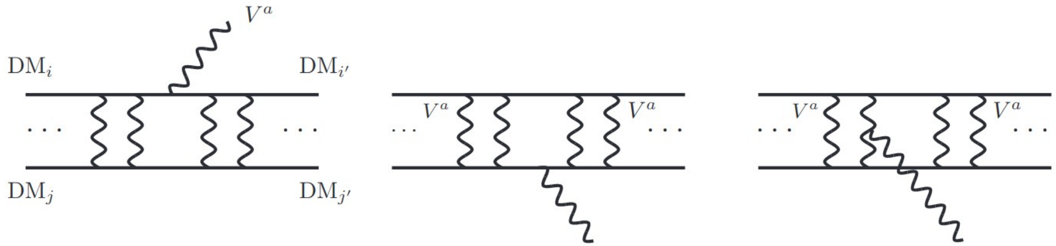
- Majorana Fermion, SU(2) Triplet, zero Hypercharge („most minimal WIMP“)
- Sommerfeld-enhanced annihilation („non-perturbative“) allows for heavier Wino masses
- Indirect detection signal sensitive to predicted Wino mass. Currently, on the edge of being excluded/confirmed.
- Uncertainties:
  - Mass splitting
    [Ibe et al. 12]
  - Potential + final state corrections
    [Beneke et al. 19, 20]
  - Astrophysical (e.g. J-factor)
  - Bound states (next slide)
- CTA will probe Wino



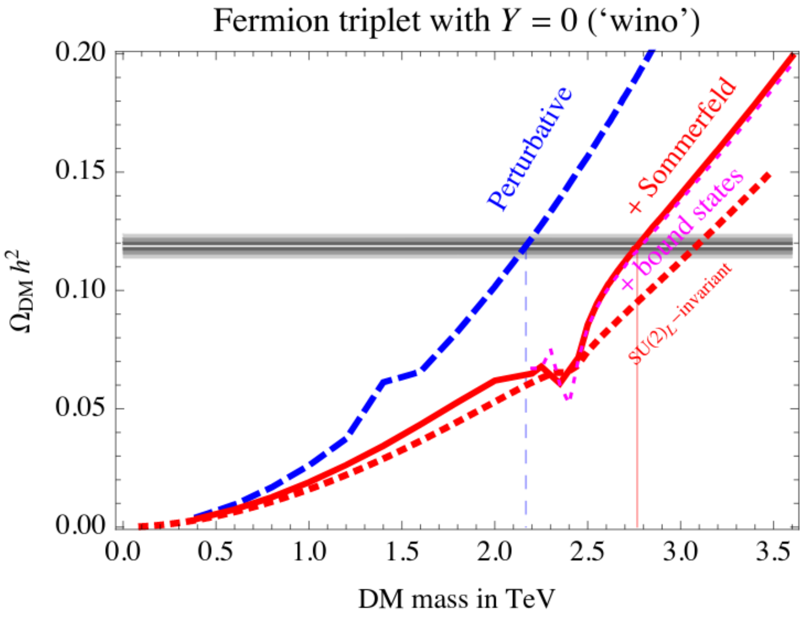


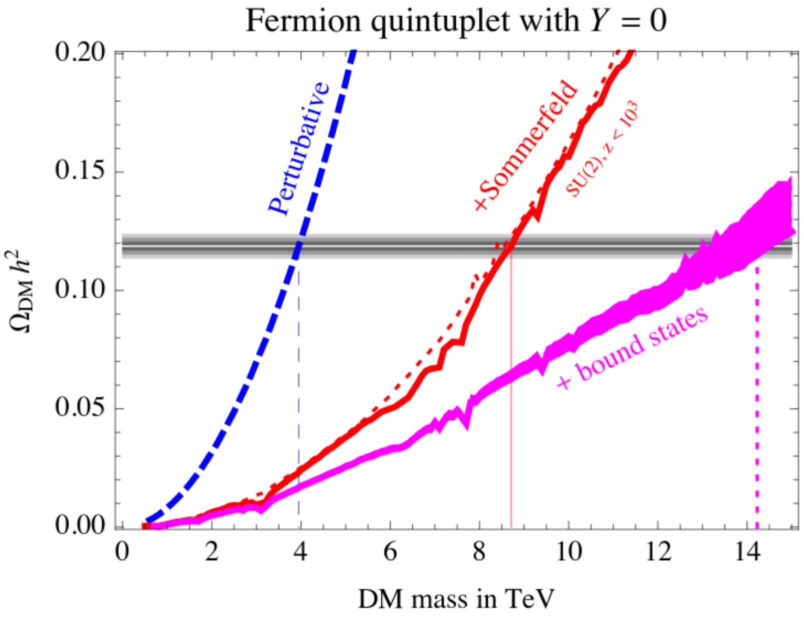
[Hisano et al. 03,05,06]

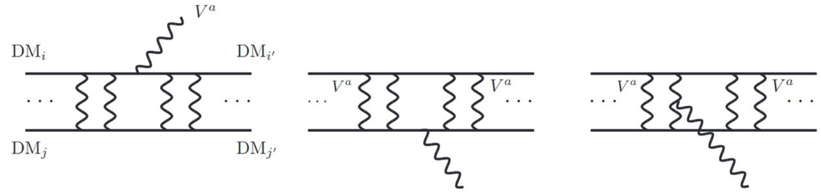
Minimal Dark Matter



[Mitridate *et al.* 17]







Colored coannihilation

- **I.e., coannihilating partner charged under SM SU(3)**
- **Longe-range effects impact** $(\Delta m_\chi, m_\chi)$ **- plane**
  - Squark (scalar triplet)

    [Ellis *et al.* 15, Liew & Luo 16, Mitridate *et al.* 17]
  - Gluino (fermion octet)
- **+ Higgs (+ correct sign)**

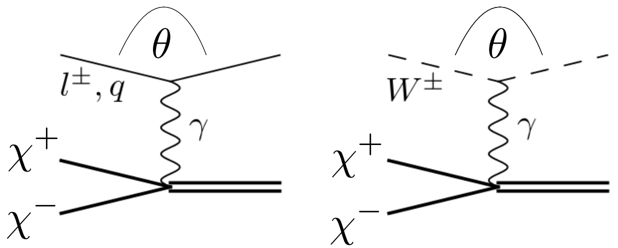
  [Harz & Petraki 18,19]
  - Additional attractive contribution
  - (squark) octet can be bounded
- **Non-perturbative effects**

  [Gross *et al.* 18, Fukuda & Luo & Shirai 18]
  - Relevant for mass splittings below QCD confining scale
  - Enormously large corrections from post-confining effects

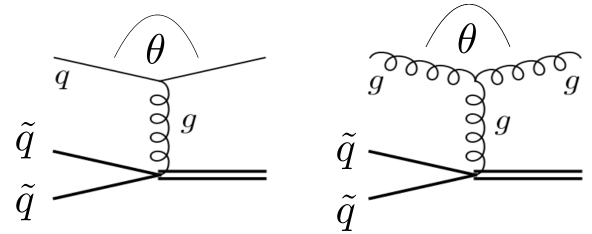
Bound-state formation at NLO

- BSF via bath particle scattering
- Divergent in forward scattering direction $\theta \to 0$
- HTL resummation not applicable for all temperature regimes
- → Thermal field theory analysis needed (cancellation of temperature dependent real and virtual contributions)

EW charged DM



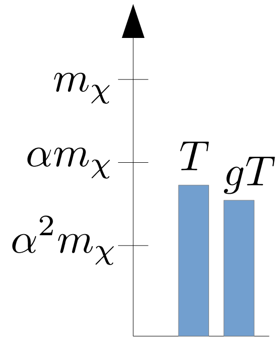
Colored coannihilation



Combining pNREFT with thermal field theory

- QED toy-model for electromagnetically charged coannihilating partners:

$$\mathcal{L}_{\rm int} = g\bar{\chi}\gamma^\mu\chi A_\mu + g\bar{\psi}\gamma^\mu\psi A_\mu$$



- Assuming temperature much smaller than Bohr-momentum, we can utilize pNREFT:

$$\mathcal{L}^{\rm pNR} = \int {\rm d}^3r \ {\rm Tr}\{O^\dagger(\mathbf{x},\mathbf{r},t)\,[i\partial_t - h + \ \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)]\,O(\mathbf{x},\mathbf{r},t)\} + \mathcal{L}^{\rm env}[A,\psi]$$

Contact with environment through the electric dipole operator.

- pNREFT → collision term → cross sections in terms of correlation functions:
  - Via Liouville equation
  - Via Open-quantum system framework
  - Via CTP formalism

Factorization

- Bound-state formation cross section & de-excitation rate *factorize* :

$$(\sigma v)_{nl} \sim g^2 |\langle \psi_{nl}| \mathbf{r} |\psi_v\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

$$\Gamma_{nl}^{n'l'} \sim g^2 |\langle \psi_{nl}| \mathbf{r} |\psi_{n'l'}\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

Recursive relation for *all* dipole transitions avaible

[M. Garny, J. Heisig, PRD, 2021]

Contact with plasma environment encoded in the *Electric Field Correlator*:

$D(x,y) \equiv \langle T_{\mathcal{C}} E(x) E(y)\rangle$, where

$\langle ... \rangle \propto \mathrm{Tr}[e^{-H_{\mathrm{env}}/T} ...].$

Computation of Electric field correlator

- Kubo-Martin-Schwinger relation:

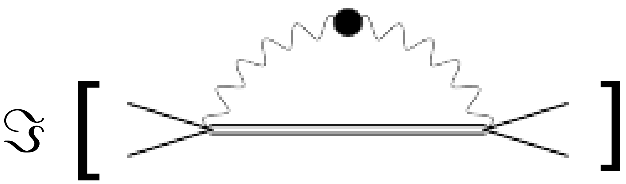
$$D^{-+}(\Delta E, \mathbf{p}) = \left[1 + f_B^{\mathrm{eq}}(\Delta E)\right] D^{\rho}(\Delta E, \mathbf{p})$$

- Spectral function and retarded correlator:

$$D^{\rho} = 2\Im\left[iD^{R}\right]$$

- Dyson equation:

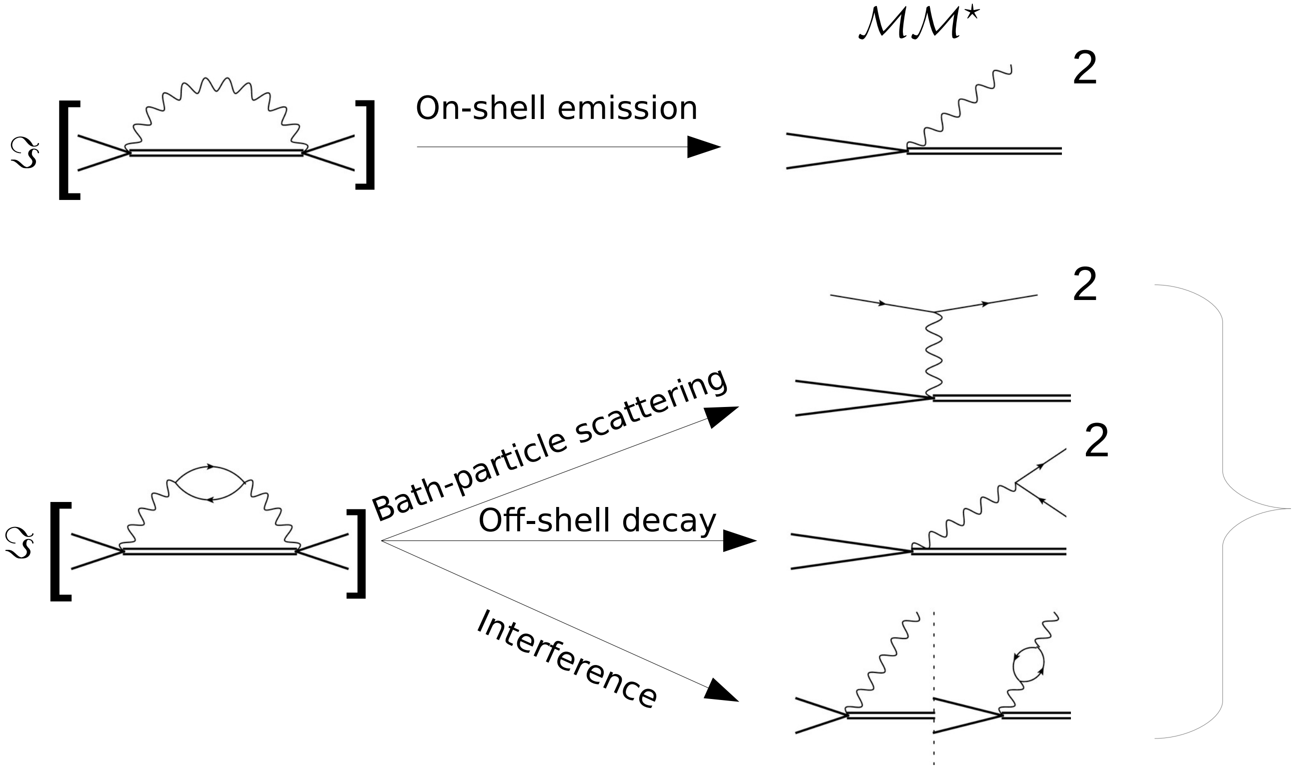
$$D^{R} = D^{R,0} + D^{R,0}\Pi_{R}D^{R,0} + ...$$



CTP diagram

- For gT < E fixed order sufficient, otherwise resummation is needed (only log corrections, in practice less important).

Leading and next-to-leading order

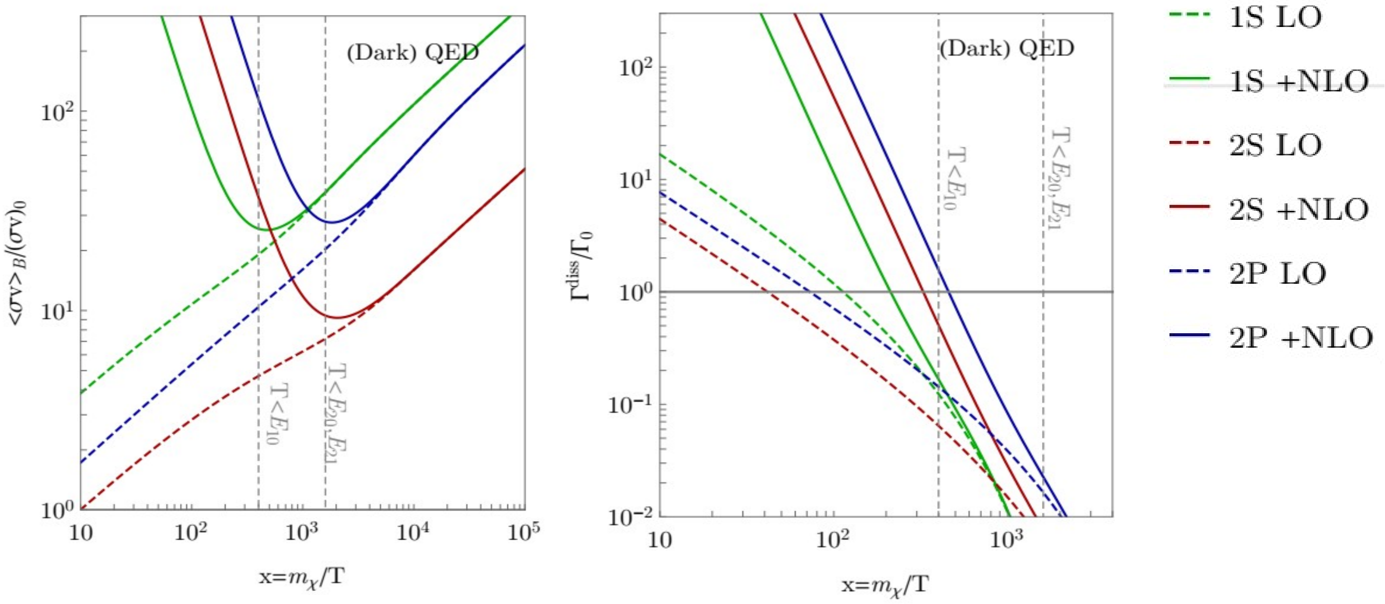


Collinear divergences cancel in the sum over all contributions.

→ Bloch-Nordsieck theorem

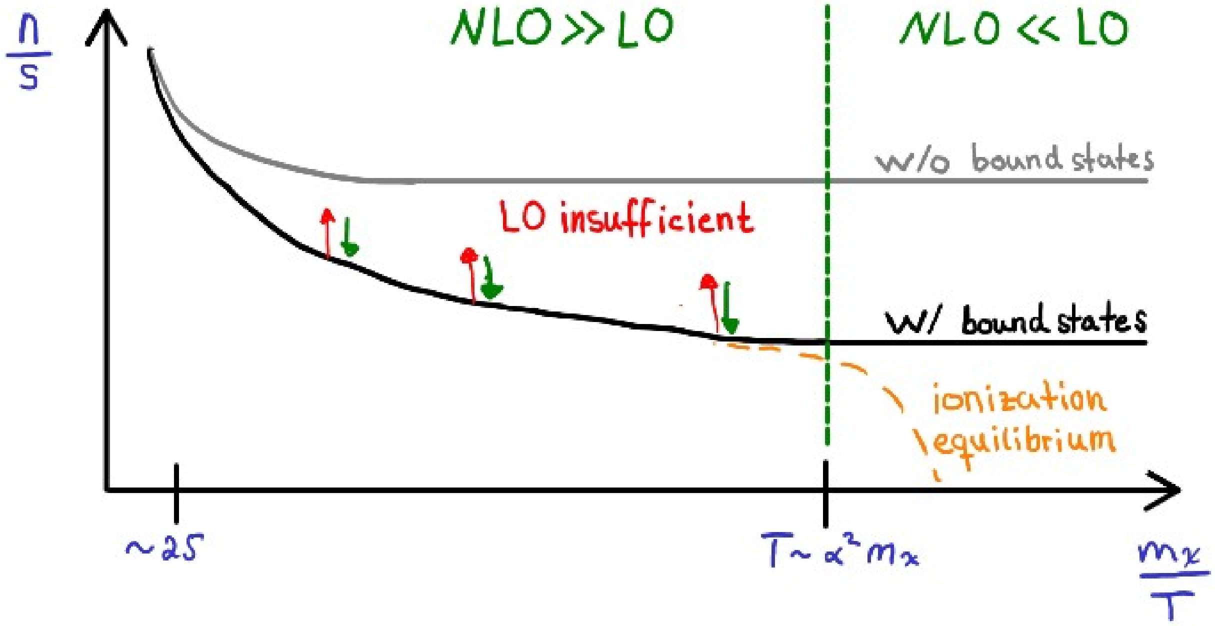
(for finite temperature real and virtual parts somewhat non-trivial)

Results U(1)



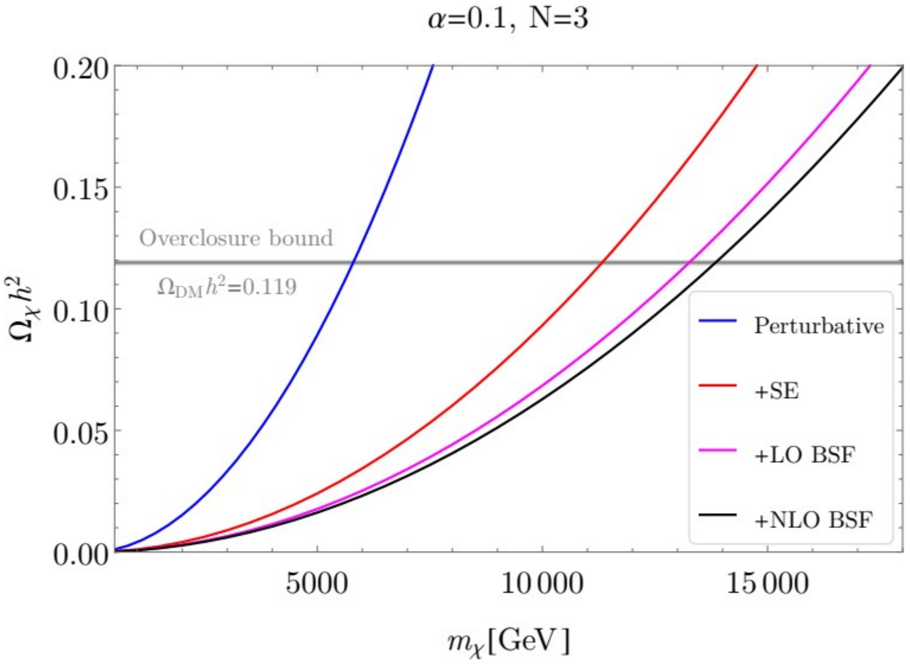
- Strong enhancement for $T \gtrsim E$ due to transitions via bath-particle scattering
- Flipped rate hierarchy
- Leads to ionization equilibrium (maximum depletion)

Freeze-out implications of NLO effects



→ NLO contributions relevant if LO insufficient to maintain ionization equilibrium until T ~ E.

Results U(1)



(ground state only)

[TB, B. Blobel, J. Harz, K. Mukaida, JHEP, 2020]

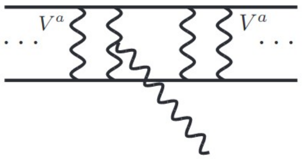
Non-abelian case

- Pair consisting of SU(N) representation (R) and its conjugate:
  - General decomposition: $\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$
  - Focus on adjoint-singlet transitions (singlet tightest bound state)
  - Fully captures $3 \otimes \bar{3} = 1 \oplus 8$ (squark, quarkonium)
  - Subset in gluino scenarios $(8 \otimes 8 = 1 \oplus 8_A \oplus ...)$

- pNREFT for adjoint-singlet interactions:

$$\mathcal{L}_{\text{pNREFT}} \supset \int \mathrm{d}^3r \; \mathrm{Tr}\left[\mathrm{S}^\dagger(i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger(iD_0 - H_{\text{adj}})\mathrm{Adj} \; - V_A(\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\mathrm{S} + \text{h.c.})\right.$$

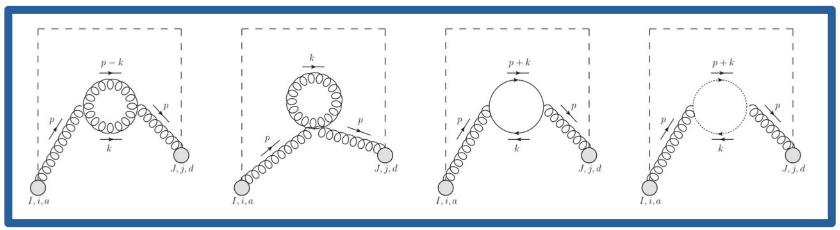
contains



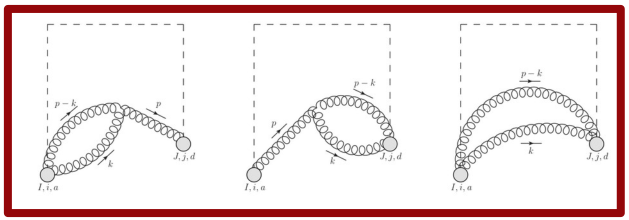
- „kinetic“ term of the adjoint crucial for gauge invariance
  - Redefinition of adjoint field leads to canonical form
  - → Wilson lines appear in the Electric field correlator

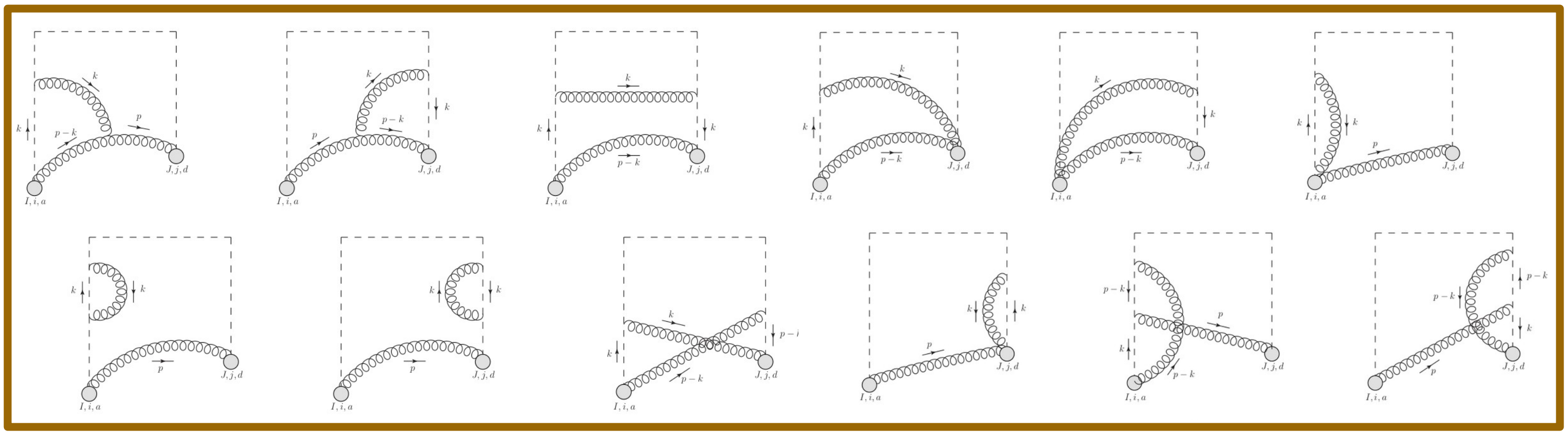
SU(N) electric field correlator @ NLO

Self-energy



Non-linear





Wilson lines

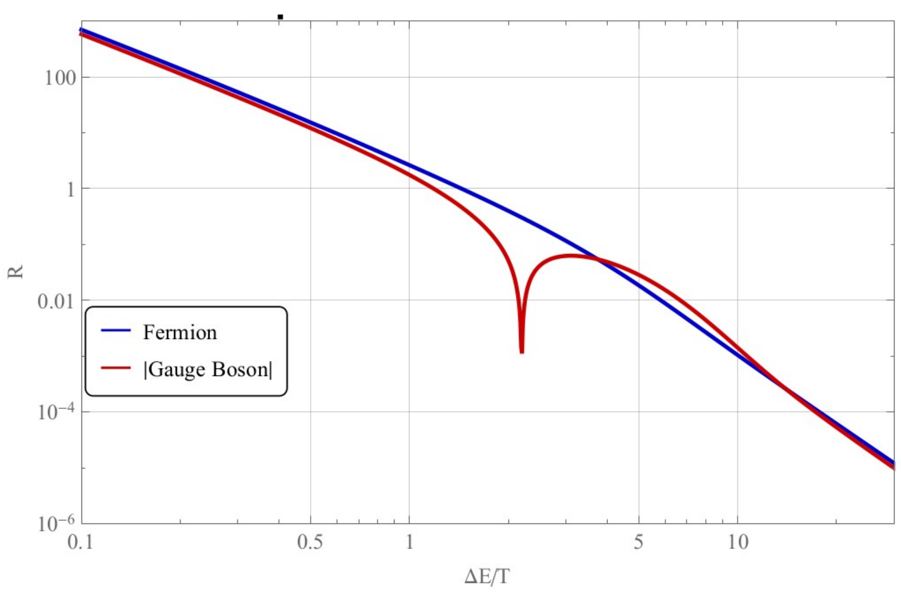
Gauge invariant, infrared & collinear finite.

[TB, K. Mukaida, B. Scheihing-Hitschfeld, X. Yao, JHEP, 2022]

Results SU(N)

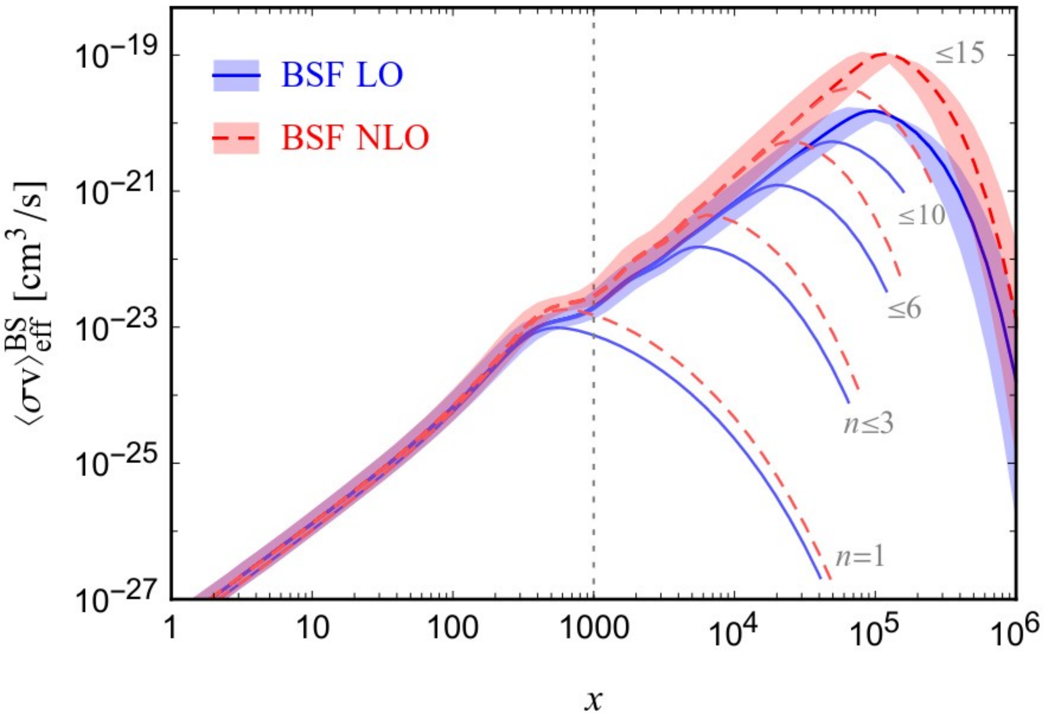
$$(\sigma v_{\rm rel})^{\rm LO+NLO}_{\mathscr{B}} = (\sigma v_{\rm rel})^{\rm LO}_{\mathscr{B}} \times \Big[1 + \alpha N_c R_g^{T=0}(\mu/\Delta E) + \alpha N_c R_g^{T\neq 0}(\Delta E/T) + \alpha N_f R_f^{T=0}(\mu/\Delta E) + \alpha N_f R_f^{T\neq 0}(\Delta E/T)\Big]$$

- Analytic expressions for temperature independent part
- One-integral expressions for finite temperature part
- Can be used for all BSF and level transition (factorization)



Example of application: Fundamental SU(3) scalar („squark“)

- x<1000: ionization equilibrium maintained already at LO
- x>1000 vacuum parts dominate
- (no transition limit)
- → NLO effects only marginally relevant here



[M. Garny, J. Heisig, PRD, 2021]

Remark

- Our results for the non-abelian electric field correlator:
  - are consistent with the *finite temperature part* of the heavy quark diffusion coefficient (different Wilson line structure) [Y. Burnier, M. Laine, J. Langelage and L. Mether, JHEP, 2010]
  - are consistent with the *zero temperature part* of the field strength correlator (same Wilson line structure)

    [M. Eidemuller and M. Jamin, PLB, 1998]
- For EW charged DM, more effort is needed.

Summary & Conclusion

- Precise prediction of relic abundance can sometimes be important
- Especially if experimental probes are sensitive to model parameters (correct mass or mass splitting compatible with cosmology)
- For gaining precision, computed U(1) and SU(N) electric field correlator @ NLO, showing collinear finiteness and gauge invariance. Applicable to all (dipole) transitions thanks to factorization.
- Bound-state formation inside plasma can be dominated by higher-order effects for certain temperature range
- NLO DM bound-state formation effects only marginally relevent so far
- For quarkonium transport, Wilson line structure seems to matter for the zero temperature NLO pieces

Wino Dark Matter, Indirect Detection

- Predicted flux for ID sensitive to the precise Wino mass value („10% variantion in the mass result in 100% change in the flux“).
- Uncertainties:
  - Potential + final state corrections

    [Beneke et al. 19, 20]
  - Mass splitting

    [Ibe et al. 12]
  - Astrophysical (J-factor)
  - Bound states (next slide)
- Cherenkov Telescope Array (CTA) can probe thermal Wino DM.