Towards a Non-Local S-Matrix Theory

Dimitrios Karamitros

Manchester U.

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DK, A. Pilaftsis [arXiv:2208.10425]

Outline

Introduction

Why non-local S-matrix?

Some reasons for non-locality/finite volume:

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[1] J. S. Bell, Physics Physique Fizika 1, 195.

[2] J. E. Campagne, Phys. Lett. B **400** (1997), 135-144, A. Ioannisian and A. Pilaftsis, Phys. Rev. D **59** (1999), 053003 [arXiv:hep-ph/9809503 [hep-ph]].

[3] K. Melnikov and V. G. Serbo, Nucl. Phys. B **483** (1997), 67-82 [arXiv:hep-ph/9601290 [hep-ph]], B. Grzadkowski, M. Iglicki and S. Mrówczyński, Nucl. Phys. B **984** (2022), 115967 [arXiv:2108.01757 [hep-ph]].

[4] B. Holdom, J. Terning and K. Verbeek, Phys. Lett. B **232** (1989), 351-356.

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- Other possibilities: QFT picture of diffraction, effective interactions (e.g. non-local chiral ET[4]), hadronization beyond the Lund model, etc.

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Local S-matrix

For concreteness, assume a toy model with

$$\mathcal{L}_{int}(x) = \lambda\, S_1(x)\chi_1(x)\ \Phi(x) + g\, S_2(x)\chi_2(x)\ \Phi(x)\ .$$

The amplitude for $S_1(p_1)\ \chi_1(p_2) \to S_2(k_1)\ \chi_2(k_2)$:

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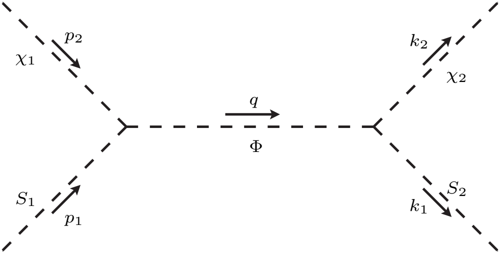

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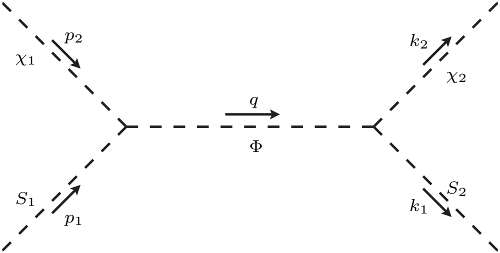

$$T(p,k) = -\lambda \, g \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_\Phi^2 + i\epsilon} \overbrace{\int d^4x \; d^4y \; e^{-i(p-q)\cdot x} e^{i(k-q)\cdot y}}^{\delta^{(4)}(p-q)\delta^{(4)}(k-q)} \; ,$$

with $p = p_1 + p_2, \; k = k_1 + k_2$

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$$e^{-[(x-\langle x\rangle)\cdot\delta p]^2}$$

$$e^{-[(y-\langle y\rangle)\cdot\delta k]^2}$$

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Simplifications

We can simplify the calculations by assuming *steady state* and *spherical symmetry*; $\delta p^0, \delta k^0 \rightarrow 0$, $\delta p^i = 1/\delta x$, and $\delta k^i = 1/\delta y$.

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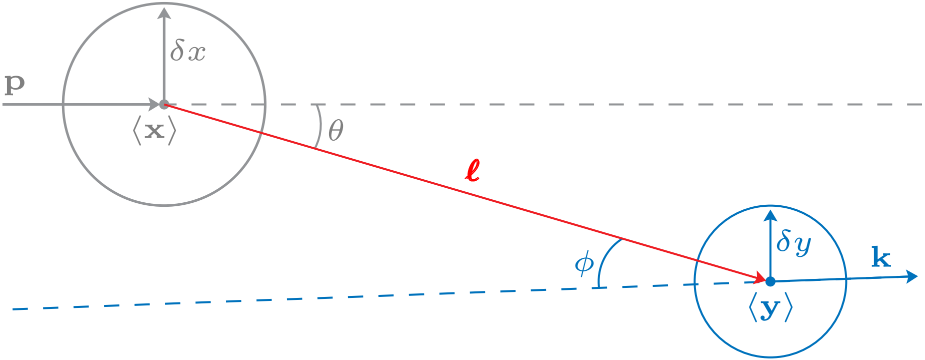
$$\begin{aligned} T(p,k;\boldsymbol{\ell},\delta x,\delta y) = & -(2\pi)\,\delta(p^0-k^0)\,\lambda\,g \\ & \int d^3\boldsymbol{x}\; e^{i\boldsymbol{p}\cdot\boldsymbol{x}-\boldsymbol{x}^2/\delta x^2} \\ & \int d^3\boldsymbol{y}\; e^{-i\boldsymbol{k}\cdot\boldsymbol{y}-\boldsymbol{y}^2/\delta y^2} \\ & \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\;\frac{e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y}-\boldsymbol{\ell})}}{\tilde{q}^2-|\boldsymbol{q}|^2+i\epsilon}\,, \end{aligned}$$

with $\boldsymbol{\ell} = \langle\boldsymbol{y}\rangle - \langle\boldsymbol{x}\rangle$ and $\tilde{q}^2 = p^{0\,2} - m_\Phi^2$.

Physical representation



Analytical results

First analytical result

After some algebra, the amplitude becomes

$$T(p,k;\boldsymbol{\ell},\delta x,\delta y)\sim\delta(p^0-k^0)\,\frac{\delta x^3\,\delta y^3}{|\boldsymbol{L}|}\,e^{-\left[\left(|\boldsymbol{p}|^2+\tilde{q}^2\right)\delta x^2+\left(|\boldsymbol{k}|^2+\tilde{q}^2\right)\delta y^2\right]/4}$$
$$\left[e^{i\tilde{q}|\boldsymbol{L}|}\,\mathrm{Erfc}\left(z_-\right)-e^{-i\tilde{q}|\boldsymbol{L}|}\,\mathrm{Erfc}\left(z_+\right)\right]\ ,$$

where $z_\pm=-\frac{i}{2}\tilde{q}\,\delta\ell\pm\frac{|\boldsymbol{L}|}{\delta\ell}$, $\boldsymbol{L}=\boldsymbol{\ell}-\frac{i}{2}\left(\boldsymbol{p}\,\delta x^2+\boldsymbol{k}\,\delta y^2\right)$,
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From this form we observe that:

- Amplitude is finite in the physical region.
- Momentum is not conserved, and its distribution depends on $\delta x,\delta y$.
- In the limit $\delta x,\delta y\to 0$, we recover the "inverse square law" and complete decoupling between $\boldsymbol{p}$ and $\boldsymbol{k}$; $T\sim\delta x^3\delta y^3\ e^{i\tilde{q}|\boldsymbol{\ell}|}/|\boldsymbol{\ell}|\ \delta(p^0-k^0)$.

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Note:

Momenta at each vertex suffer from uncertainties.
Introduce matrix element as usual

$$M(\boldsymbol{p}, \tilde{q}; \boldsymbol{\ell}, \delta\ell) \sim \frac{\delta\ell^3}{|\boldsymbol{L}|} e^{-\delta\ell^2\left(|\boldsymbol{p}|^2+\tilde{q}^2\right)/2} \left[e^{i\tilde{q}|\boldsymbol{L}|}\,\mathrm{Erfc}\,(z_-) - e^{-i\tilde{q}|\boldsymbol{L}|}\,\mathrm{Erfc}\,(z_+)\right]\ .$$

Limits

Far-field–$|\boldsymbol{\ell}| \gg |\boldsymbol{p}|\ \delta\ell^2$

In the far-field (*Fraunhofer*) region we recover the inverse-square law (similar to $\delta\ell \to 0$). The matrix element becomes:

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Observations:

- Inverse square law (expected and testable).[5]
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- Oscillations of mixed mediators (expected and testable).
- Off-shell mediator may be the most probable outcome (testable).

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Observations:

- Inverse square law (expected and testable).[5]
- Finite.
- Oscillations of mixed mediators (expected and testable).
- Off-shell mediator may be the most probable outcome (testable).
- Suppressed backwards direction.

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Near-field–$|\boldsymbol{\ell}| \ll |\boldsymbol{p}|\ \delta\ell^2$: Subregions

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I	$\big\lvert\lvert\boldsymbol{p}\rvert-\tilde{q}\big\rvert\,\delta\ell \gg \max\big(1,\ \lvert\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}\rvert/\delta\ell\big)$
II	$\lvert\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}\rvert \ll \delta\ell$ and $\big\lvert\lvert\boldsymbol{p}\rvert-\tilde{q}\big\rvert\,\delta\ell \ll 1$
III	$\lvert\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}\rvert \gg \delta\ell$ and $\big\lvert\lvert\boldsymbol{p}\rvert-\tilde{q}\big\rvert\,\delta\ell^2 \ll \lvert\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}\rvert$

$$M \sim \ \delta\ell^2 \times \begin{cases} \dfrac{e^{i\boldsymbol{p}\cdot\boldsymbol{\ell}-(|\boldsymbol{\ell}|/\delta\ell)^2}}{\left(|\boldsymbol{p}|^2-\tilde{q}^2\right)\delta\ell^2\ +\ 4\left[i\boldsymbol{p}\cdot\boldsymbol{\ell}-\left(|\boldsymbol{\ell}|/\delta\ell\right)^2\right]}\ , & \textbf{I} \\[2ex] \dfrac{1}{|\boldsymbol{p}|\delta\ell}\left[1+\frac{2}{\sqrt{\pi}}\ \dfrac{\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}}{\delta\ell}\right]e^{i\tilde{q}\,\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}-(|\boldsymbol{p}|-\tilde{q})^2\delta\ell^2/4}\ , & \textbf{II} \\[2ex] \pm\dfrac{e^{-(|\boldsymbol{p}|\mp\tilde{q})^2\delta\ell^2/4\ \pm i\tilde{q}\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}}}{|\boldsymbol{p}|\ \delta\ell+2i\,\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}/\delta\ell}-\dfrac{i}{2\sqrt{\pi}}\dfrac{e^{i\boldsymbol{p}\cdot\boldsymbol{\ell}-(|\boldsymbol{\ell}|/\delta\ell)^2}}{i\boldsymbol{p}\cdot\boldsymbol{\ell}\ -\ (\hat{\boldsymbol{p}}\cdot\boldsymbol{\ell}/\delta\ell)^2}\ , & \textbf{III} \end{cases}$$

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   - Radial dependence
   - Spatial pattern–on-shell
   - Spatial pattern–off-shell
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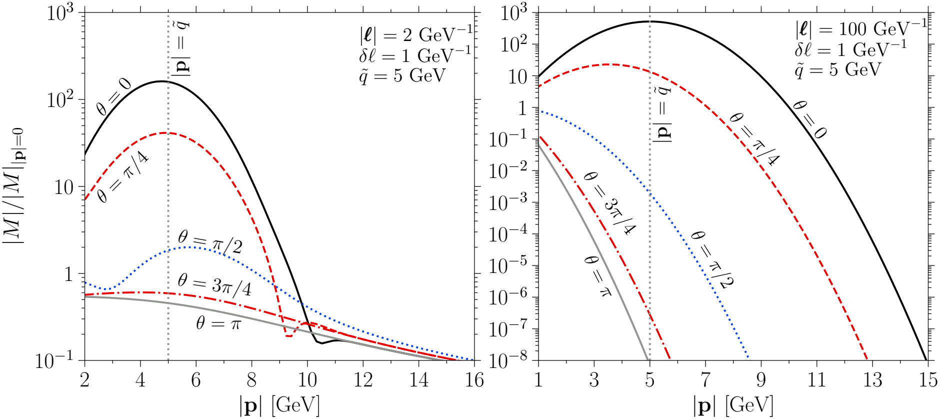
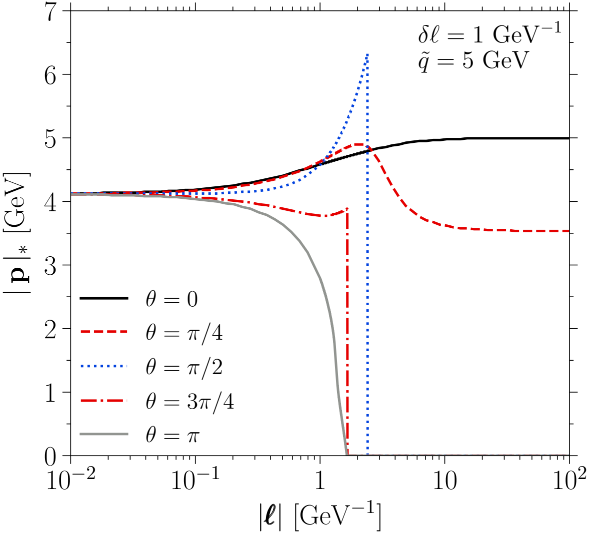
6. Summary–Future

Momentum dependence



Shift of Maximum

In general, the maxima of the amplitude do not correspond to the resonance, $|\boldsymbol{p}|_* = \tilde{q}$.



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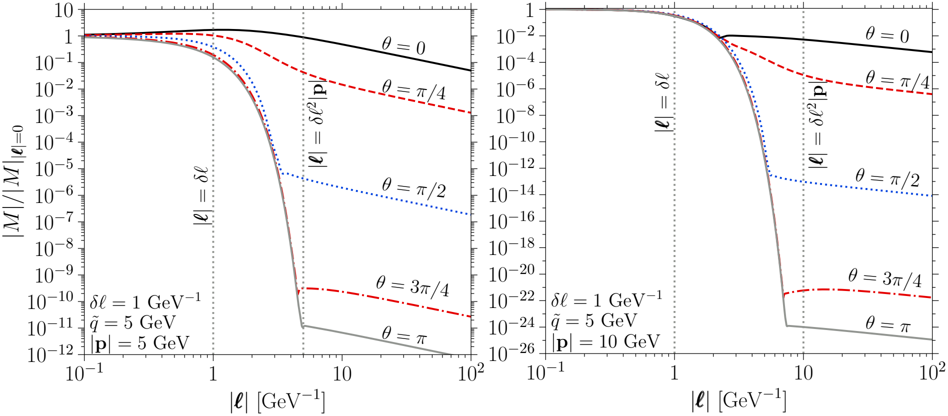

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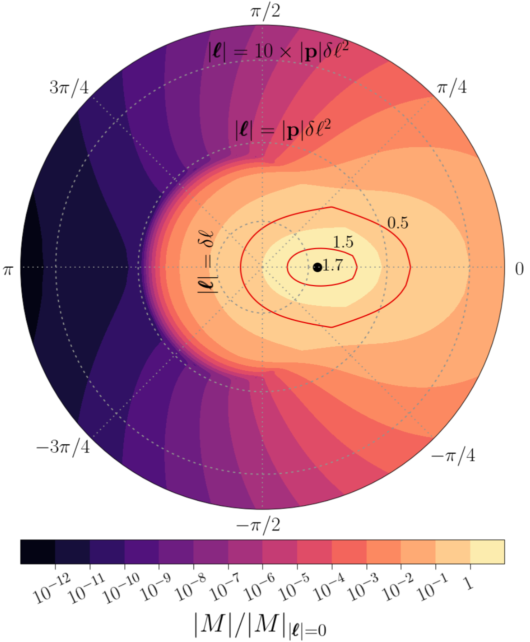

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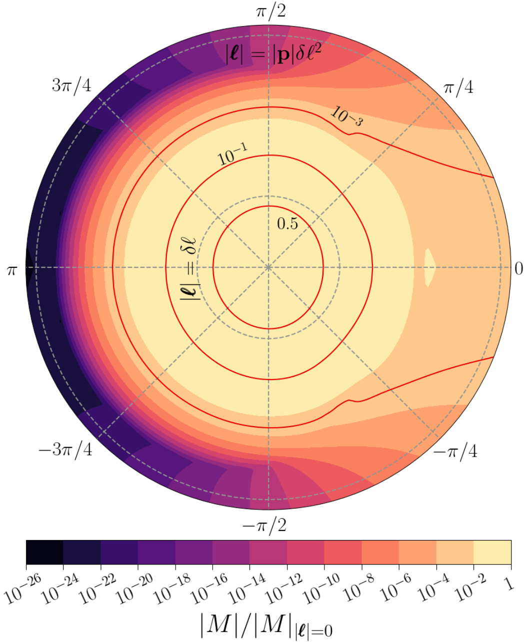

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- **Experiments**.

Thank you!