The MESS and dualities of cosmological perturbations

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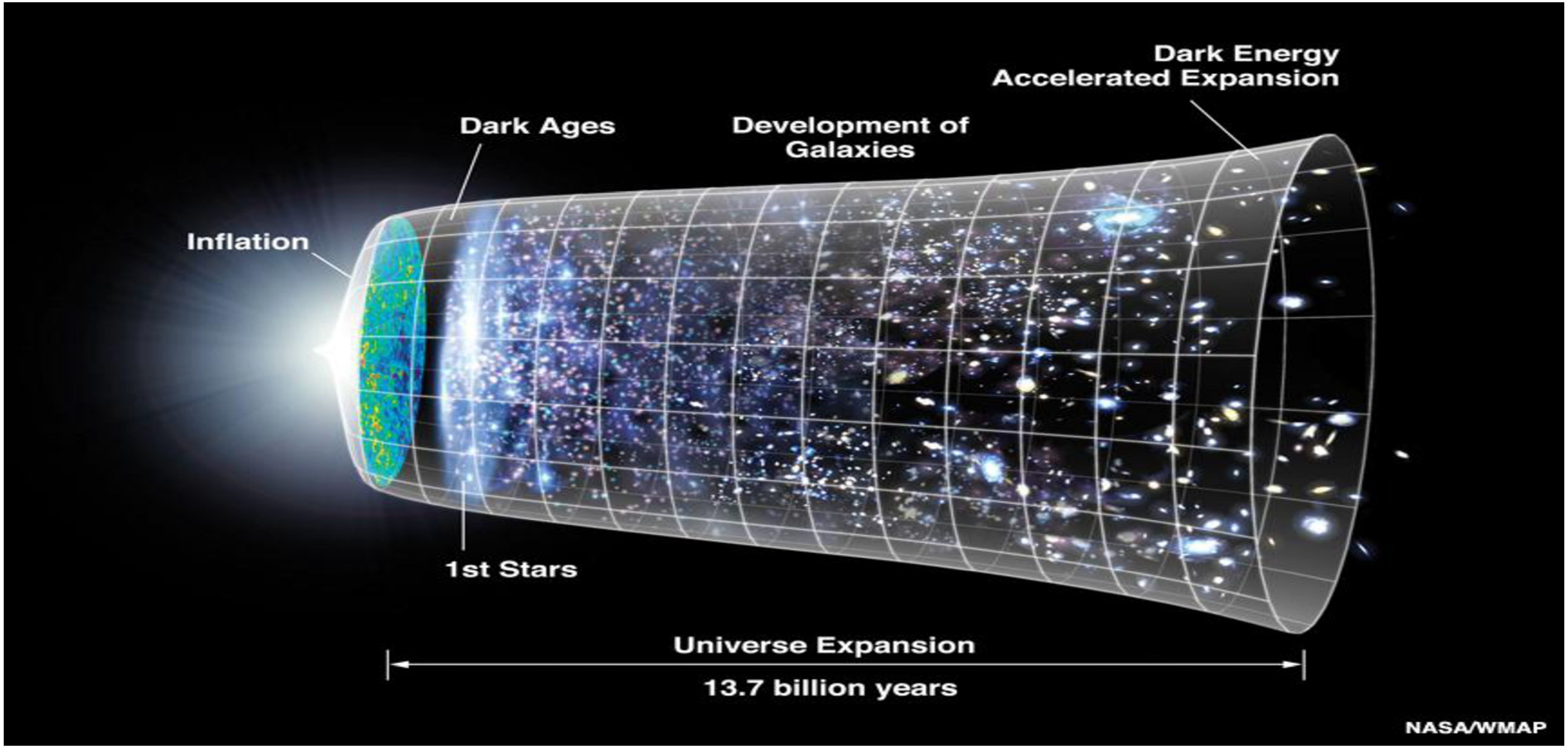
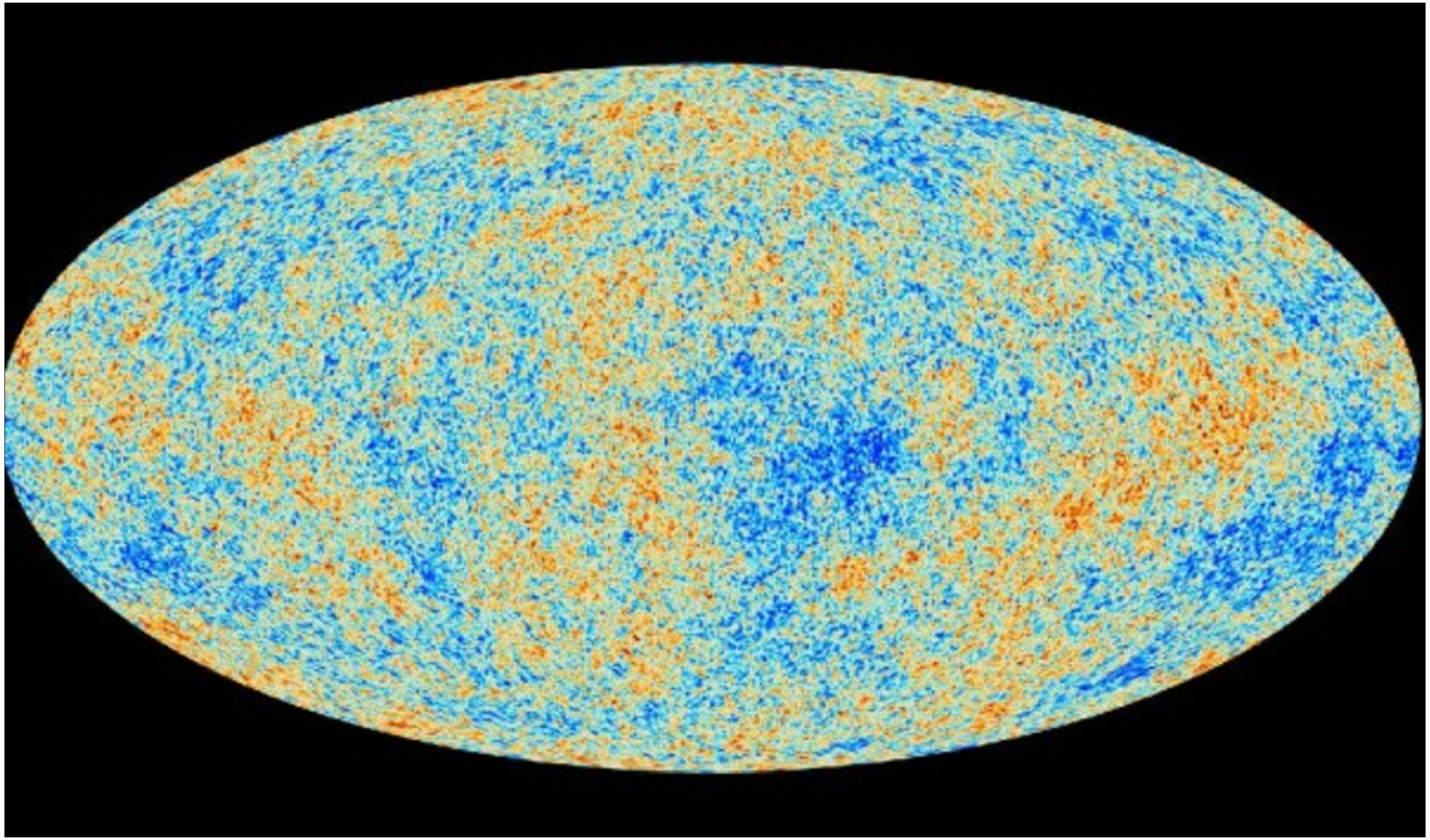
Based on work in collaboration with K . Turzynski, S. Vallejo, A. Gallego
Phys.Lett. B784
Phys.Lett. B793

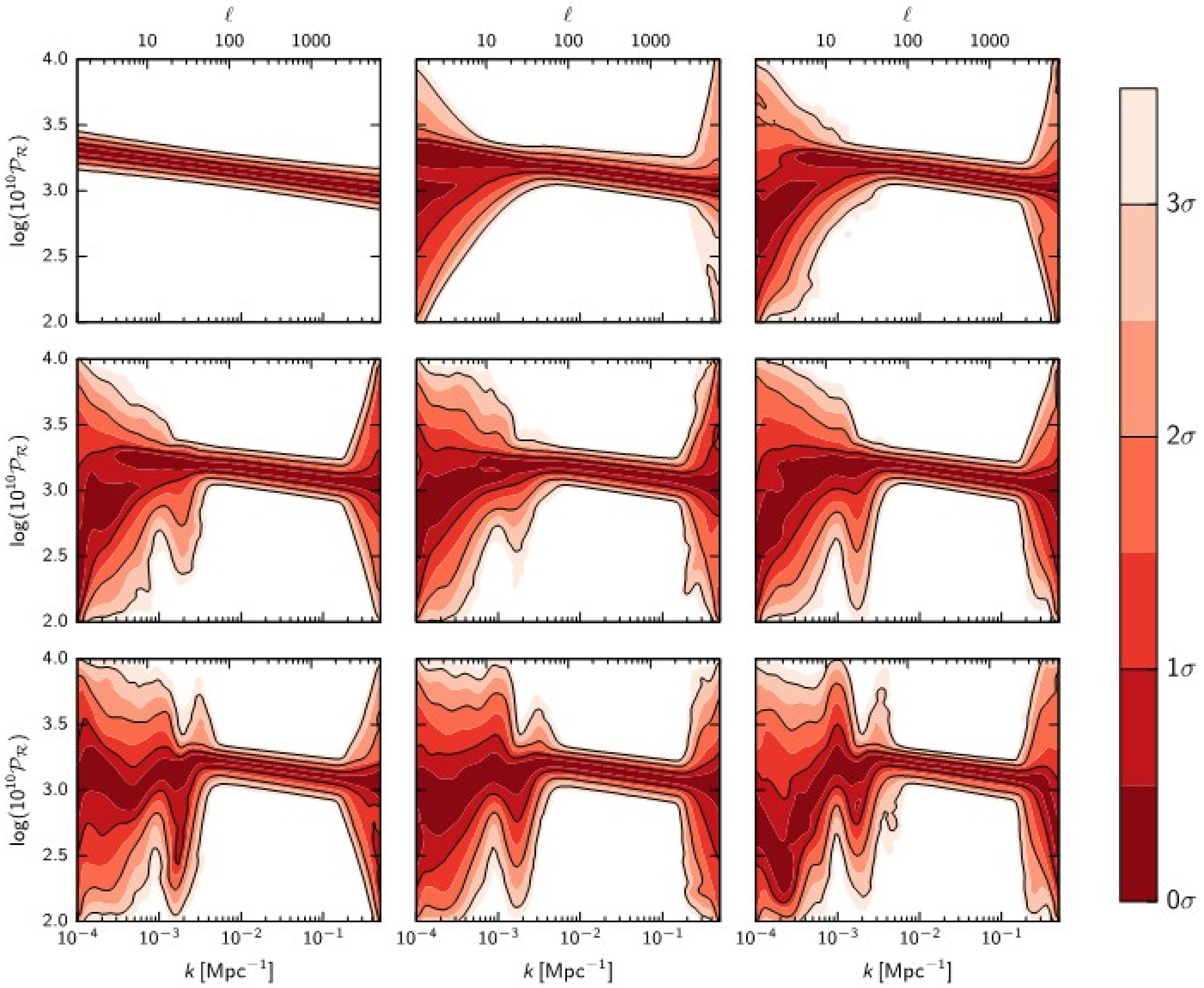
- Cosmological perturbations are fundamental to establish any predictive model of the Universe
- They provide the seeds for cosmic background radiation anisotropies and for large scale structure formation



Planck CMB temperature anisotropy map



The CMB leaves room from deviations from a power law spectrum,
Planck 2015 results. XX. Constraints on inflation



What can cause these features?

- Similar features, but on other scales, in the spectrum of primordial curvature perturbations could also cause **PBH** production which have been claimed to be within the LIGO observable range, and could also affect **LSS**.

These features can be due to several different causes such as:

- Multi-fields
- Slow-roll violation in single field
- Modification of gravity
- A combination of the above

Despite their apparent difference do all these phenomena share something?

Yes ..

SESS and MESS

We define the gauge invariant space dependent effective sound speed (SESS) of comoving curvature perturbations according to (A. Romano & S. A. Vallejo P. *PLB*, **78**, 367, 2018)

$$v_s^2(t, x^i) \equiv \frac{\delta P_c(t, x^i)}{\delta \rho_c(t, x^i)}$$

SESS reduces to the standard sound speed for single field K(X) theories.

In absence of anisotropy we can obtain a new equation, with no source term, which is a modified version of the Sasaki-Mukhanov equation

$$\ddot{\mathcal{R}} + \frac{\partial_t(Z^2)}{Z^2}\dot{\mathcal{R}} - \frac{v_s^2}{a^2} \overset{(3)}{\Delta} \mathcal{R} = 0 \,, \qquad Z^2 \equiv \frac{\epsilon a^3}{v_s^2}$$

while in the standard approach there is a source term

$$\ddot{\mathcal{R}} + \frac{\partial_t(z^2)}{z^2}\dot{\mathcal{R}} - \frac{c_s^2}{a^2} \overset{(3)}{\Delta} \mathcal{R} + \frac{1}{2z^2}\partial_t\left[\frac{a^3}{c_s^2 H}\Gamma\right] = 0 \,, \qquad z^2 \equiv \frac{a^3\epsilon}{c_s^2}$$

The equation with SESS is model independent. SESS can be treated as an effective quantity in data analysis, without assuming any model.

In presence of anisotropy the equation using SESS takes the form

$$\ddot{\mathcal{R}} + \frac{\partial_t(Z^2)}{Z^2}\dot{\mathcal{R}} - \frac{v_s^2}{a^2}\overset{(3)}{\Delta}\mathcal{R} + \frac{v_s^2}{\epsilon}\overset{(3)}{\Delta}\Pi + \frac{1}{3Z^2}\partial_t\left(\frac{Z^2}{H\epsilon}\overset{(3)}{\Delta}\Pi\right) = 0\,, \quad Z^2 \equiv \frac{\epsilon a^3}{v_s^2}$$

while using the standard approach we obtained

$$\ddot{\mathcal{R}} + \frac{\partial_t(z^2)}{z^2}\dot{\mathcal{R}} - \frac{c_s^2}{a^2}\overset{(3)}{\Delta}\mathcal{R} + \frac{c_s^2}{\epsilon}\overset{(3)}{\Delta}\Pi + \frac{1}{2z^2}\partial_t\left[\frac{a^3}{c_s^2 H}\left(\Gamma + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)\right] = 0\,,$$

$$z^2 \equiv \frac{a^3\epsilon}{c_s^2}$$

The equations we have obtained are *completely general*.

They can be applied to modified gravity theories, multi-fields systems, or any combination of these models!

The momentum dependent effective sound speed (MESS) is defined as

$$\tilde{v}_k^2(t) \equiv \frac{\delta \tilde{P}_c(t)}{\delta \tilde{\rho}_c(t)}$$

where $\delta \tilde{P}_c$ and $\delta \tilde{\rho}_c$ are the Fourier transforms of the comoving pressure and energy density respectively.

In absence of anisotropy, using MESS and manipulating the Einstein's equations in Fourier space we get

$$\ddot{\mathcal{R}}_k + \frac{\partial_t(\tilde{Z}_k^2)}{\tilde{Z}_k^2}\dot{\mathcal{R}}_k + \frac{\tilde{v}_k^2}{a^2}k^2\mathcal{R}_k = 0\,, \quad \tilde{Z}_k^2 \equiv \frac{\epsilon a^3}{\tilde{v}_k^2}$$

while in the standard approach there is a source term

$$\ddot{\mathcal{R}}_k + \frac{\partial_t(z^2)}{z^2}\dot{\mathcal{R}}_k + \frac{c_s^2}{a^2}k^2\mathcal{R}_k + \frac{1}{2z^2}\partial_t\left[\frac{a^3}{c_s^2 H}\Gamma_k\right] = 0\,, \quad z^2 \equiv \frac{a^3\epsilon}{c_s^2}$$

Important note:

The MESS $\tilde{v}_k$ is ***not*** simply the Fourier transform of the SESS v_s!

In presence of anisotropy, using MESS and manipulating the Einstein's equations in Fourier space we get

$$\ddot{\mathcal{R}}_k + \frac{\partial_t(\tilde{Z}_k^2)}{\tilde{Z}_k^2}\dot{\mathcal{R}}_k + \frac{\tilde{v}_k^2}{a^2}k^2\mathcal{R}_k - \frac{\tilde{v}_k^2}{\epsilon}k^2\Pi_k - \frac{1}{3\tilde{Z}_k^2}\partial_t\left(\frac{\tilde{Z}_k^2}{H\epsilon}k^2\Pi_k\right) = 0\,, \quad \tilde{Z}_k^2 \equiv \frac{\epsilon a^3}{\tilde{v}_k^2}$$

The equation with MESS is model independent.

MESS can be treated as an effective quantity in data analysis, without assuming any model.

Using the standard approach we get

$$\ddot{\mathcal{R}}_k + \frac{\partial_t(z^2)}{z^2}\dot{\mathcal{R}}_k + \frac{c_s^2}{a^2}k^2\mathcal{R}_k - \frac{c_s^2}{\epsilon}k^2\Pi_k + \frac{1}{2z^2}\partial_t\left[\frac{a^3}{c_s^2 H}\left(\Gamma_k - \frac{2}{3}k^2\Pi_k\right)\right] = 0\,,$$

$$z^2 \equiv \frac{a^3\epsilon}{c_s^2}$$

The equations we have obtained are *completely general*.

They can be applied to modified gravity theories, multi-fields systems, or any combination of these models!

Some notation for scalar perturbations : No gauge fixing

$$ds^2 = -(1+2A)dt^2 + 2a\partial_i B dx^i dt + $$
$$+ a^2 \{\delta_{ij}(1+2C) + 2\partial_i\partial_j E\} dx^i dx^j \ ,$$

TOTAL energy momentum tensor
Includes any matter, multi-fields,
Vector, scalar fields,
Modified gravity

$$T^0{}_0 = -(\rho + \delta\rho) \quad , \quad T^0{}_i = (\rho + P)\partial_i(v + B)$$

$$T^i{}_j = (P + \delta P)\delta^i{}_j + \delta^{ik}\partial_k\partial_j\Pi - \frac{1}{3}\delta^i{}_j \overset{(3)}{\Delta}\Pi \, .$$

Comoving slices gauge :

$$(T^0{}_i)_c = 0 \longrightarrow \alpha = \delta P_c, \beta = \delta\rho_c, \gamma = A_c, \mu = B_c, \zeta = C_c, \nu = E_c$$

$$ds^2 = -(1+2\gamma)dt^2 + 2a\partial_i\mu \, dx^i dt + $$
$$+ a^2 \{\delta_{ij}(1+2\zeta) + 2\partial_i\partial_j\nu\} dx^i dx^j$$

$$(T^0{}_0)_c = -(\rho + \beta) \quad , \quad (T^i{}_j)_c = (P + \alpha)\delta^i{}_j$$

Standard definitions of entropy in the comoving gauge and uniform density gauge

$$\delta P_u = c_w(t)^2 \delta\rho + \delta P^{nad}$$

$c_w^2 = P'/\rho'$ Adiabatic sound speed

$$\delta P_c = c_s(t)^2 \delta\rho_c + \delta P_c^{nad}$$

Comoving curvature pertubation sound speed

$$\alpha(t,x^i) = c_s(t)^2 \beta(t,x^i) + \Gamma(t,x^i)$$

But ….the one in the comoving gauge it is not unique !

$$c_s^2 \to \tilde{c}_s(t)^2 = c_s(t)^2 + \Delta c_s(t)^2 ,$$

$$\Gamma \to \tilde{\Gamma}(t,x^i) = \Gamma(t,x^i) - \Delta c_s(t)^2 \beta(t,x^i)$$

Comparing it to the SESS we can get the relation between them

$$v_s^2(t,x^i) \equiv \frac{\alpha(t,x^i)}{\beta(t,x^i)}$$

$$\alpha(t,x^i) = c_s(t)^2 \beta(t,x^i) + \Gamma(t,x^i)$$

Relation of SESS with entropy and anisotropy

In presence of anisotropies the definitions of SESS is the same and the relation with entropy is

$$v_s^2 = c_s^2 \left(1 - \frac{\Gamma}{\alpha}\right)^{-1}$$

Using the Einstein's equations $$\dot{\zeta} = -\frac{1}{2H\epsilon}\left(\alpha + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)$$

We can make explicit the relation with anisotropy

$$v_s^2 = c_s^2 \left[1 + \frac{\Gamma}{2H\epsilon\left(\dot{\zeta} + \frac{1}{3H\epsilon}\overset{(3)}{\Delta}\Pi\right)}\right]^{-1}$$

The most general equation has two source terms related to anisotropy, but no explicit entropy

Most general equation for any system including anisotropy and entropy effects

SESS

$$\dot{\zeta} = -\frac{v_s^2}{a^2 H\epsilon} \overset{(3)}{\Delta} \Psi_B - \frac{1}{3H\epsilon} \overset{(3)}{\Delta} \Pi$$

Without SESS

$$\dot{\zeta} = -\frac{c_s^2}{a^2 H\epsilon} \overset{(3)}{\Delta} \Phi_B - \frac{\Gamma}{2H\epsilon} - \frac{1}{3H\epsilon} \overset{(3)}{\Delta} \Pi$$

$$\ddot{\zeta} + \frac{\partial_t (Z^2)}{Z^2}\dot{\zeta} - \frac{v_s^2}{a^2} \overset{(3)}{\Delta} \zeta + \frac{v_s^2}{\epsilon} \overset{(3)}{\Delta} \Pi + \frac{1}{3Z^2}\partial_t \left(\frac{Z^2}{H\epsilon} \overset{(3)}{\Delta} \Pi \right) = 0$$

$$\ddot{\zeta} + \frac{\partial_t z^2}{z^2}\dot{\zeta} - \frac{c_s^2}{a^2} \overset{(3)}{\Delta} \zeta + \frac{c_s^2}{\epsilon} \overset{(3)}{\Delta} \Pi + \frac{1}{z^2}\partial_t \left[\frac{a^3}{c_s^2 H} \left(\Gamma + \frac{2}{3} \overset{(3)}{\Delta} \Pi \right) \right] = 0$$

The first and second order equations are obtained using the following important relations, obtained from Manipulating the Einstein's equations in the comoving gauge. The Poisson eq. Is more used in the modified gravity theories literature,

$$\frac{1}{a^2} \overset{(3)}{\Delta} \Psi_B = \frac{1}{2}\beta \qquad \zeta = -\Psi_B + \frac{H^2}{\dot{H}}\left(\Phi_B + H^{-1}\dot{\Psi}_B\right) \qquad \dot{\zeta} = -\frac{1}{2H\epsilon}\left(\alpha + \frac{2}{3} \overset{(3)}{\Delta} \Pi\right)$$

The difference between the uniform density field and the comoving gauge

The uniform density field (aka “unitary”) is in general different from the comoving gauge
They coincide for K(X) – inflation, but not for Horndesky theory or multi-fields systems

$$v + B \to v + B - \delta t \quad \longrightarrow \quad \delta t_c = v + B$$

We can now define explicitly gauge invariant quantities:
comoving pressure perturbation α
comoving density perturbation β
comoving curvature perturbation ζ

$$\begin{aligned}
\alpha &= \delta P + \dot{P}\delta t_c \quad , \quad \beta = \delta\rho + \dot{\rho}\delta t_c \, , \\
\gamma &= A + \dot{\delta t_c} \quad , \quad \mu = B - a^{-1}\delta t_c \, , \\
\sigma &= a\dot{E} - B + a^{-1}\delta t_c = a\dot{\nu} - \mu \, , \\
\zeta &= C - H\delta t_c \, .
\end{aligned}$$

Einstein's equations in the comoving gauge

$$\frac{1}{a^2}\overset{(3)}{\Delta}\left[-\zeta + aH\sigma\right] = \frac{\beta}{2} \, ,$$

$$\gamma = \frac{\dot{\zeta}}{H} \, ,$$

$$-\ddot{\zeta} - 3H\dot{\zeta} + H\dot{\gamma} + (2\dot{H} + 3H^2)\gamma = \frac{\alpha}{2} \, ,$$

$$\dot{\sigma} + 2H\sigma - \frac{\gamma + \zeta}{a} = 0 \, ,$$

How general is this equation?

- SESS reduces to the standard definition of sound speed for single field K(X) theories
- It is a space dependent quantity which effectively reproduces the effects of the source terms in the EOM which in the standard formulation are associated to entropy perturbations
- Given the generality of the assumptions this formulation is valid for any system for which an energy momentum tensor can be defined, including multi-fields systems or modified gravity theories (MGT)
- It is also valid for MGT, after writing the MGT field equations as Einstein's equations with an appropriate definition of an effective energy momentum tensor

How useful SESS and MESS are?

- These equations are completely general and can be applied to any physical system for example:
- Multi-fields , scalar or vector fields (scalar part)
- Modified gravity, e.g. Horndesky theory, in terms of an effective EM tensor : $G_{\mu\nu}=T_{\mu\nu}^{eff}$
- Non-Gaussianity can be studied in terms of MESS and SESS
- The anisotropy stress term is easy to add and does not modify the definition of MESS and SESS
- Mess is not simply the Fourier transform of SESS !

Example: 2 minimally coupled scalar fields

$$L = \sum_{n}^{N} X_n + 2V(\Phi_n) \qquad X_n = g^{\mu\nu}\partial_\mu \Phi_n \partial_\nu \Phi_n \qquad \Phi_n(x^\mu) = \phi_n(t) + \delta\phi_n(x^\mu)$$

$$\delta T^0{}_0 = -\dot\phi\delta\dot\phi - \dot\psi\delta\dot\psi + A(\dot\phi^2 + \dot\psi^2) - V_\phi\delta\phi - V_\psi\delta\psi\,,$$

$$\delta T^i{}_j = \delta^i_j\left[\dot\phi\delta\dot\phi + \dot\psi\delta\dot\psi - A(\dot\phi^2 + \dot\psi^2) - V_\phi\delta\phi - V_\psi\delta\psi\right]$$

$$\delta T^0{}_i = \partial_i\left(-\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{a}\right)$$

Comoving gauge

$$\dot\phi\widetilde{\delta\phi} + \dot\psi\widetilde{\delta\psi} = 0 \;\longrightarrow\; \widetilde{\delta\phi} = \delta\phi + \dot\phi\delta t \;,\; \widetilde{\delta\psi} = \delta\psi + \dot\psi\delta t \;\longrightarrow\; \delta t_c = -\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}$$

Comoving field perturbations

Comoving pressure and enegy density

$$U_\phi = \delta\phi - \dot\phi\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}\,,\quad U_\psi = \delta\psi - \dot\psi\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}$$

$$\alpha = \delta P_c = \dot\phi\dot U_\phi + \dot\psi\dot U_\psi - \gamma(\dot\phi^2 + \dot\psi^2) + \\ + (\ddot\phi + 3H\dot\phi)U_\phi + (\ddot\psi + 3H\dot\psi)U_\psi\,,$$

$$\beta = \delta\rho_c = \dot\phi\dot U_\phi + \dot\psi\dot U_\psi - \gamma(\dot\phi^2 + \dot\psi^2) + \\ - (\ddot\phi + 3H\dot\phi)U_\phi - (\ddot\psi + 3H\dot\psi)U_\psi\,.$$

We can substitute the gauge invariant comoving fields in the comoving pressure and density perturbations

$$\beta = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H} - \frac{\Theta(\dot{\phi}^2+\dot{\psi}^2)}{2}, \quad \alpha = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H}$$

$$\Theta = \left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right)\frac{\partial}{\partial t}\left(\frac{\dot{\phi}^2-\dot{\psi}^2}{\dot{\phi}^2+\dot{\psi}^2}\right)$$

Note that this quantity is gauge invariant, as expected

$$\left(\frac{\delta\phi_{c_s}}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right) = \left(\frac{Q_\phi}{\dot{\phi}} - \frac{Q_\psi}{\dot{\psi}}\right) = \left(\frac{U_\phi}{\dot{\phi}} - \frac{U_\psi}{\dot{\psi}}\right)$$

Assuming a classical trajectory of the form $\psi(\phi)$ →

$$\Theta = 4\dot{\phi}\frac{\partial\psi}{\partial\phi}\frac{\partial^2\psi}{\partial\phi^2}\left[\left(\frac{\partial\psi}{\partial\phi}\right)^2+1\right]^{-2}\left(\frac{U_\psi}{\dot{\psi}} - \frac{U_\phi}{\dot{\phi}}\right)$$

The SESS is different from cs only when there is a turn in field space

$$v_s^2 = \left(1 + \frac{H\Theta}{2\dot{\zeta}}\right)^{-1}$$

After substituting SESS we get the "standard " source term

$$\dot{\zeta} = \frac{H}{a^2\dot{H}} \overset{(3)}{\Delta} \Phi_B - \frac{1}{2}H\Theta\,,$$

$$\ddot{\zeta} + \frac{\partial_t(z^2)}{z^2}\dot{\zeta} - \frac{1}{a^2}\overset{(3)}{\Delta}\zeta + \frac{1}{z^2}\partial_t\left(\frac{z^2 H\Theta}{2}\right) = 0$$

Generalization to multi-fields

$$\theta_{ij} = \left(\frac{\delta\phi_i}{\dot{\phi}_i} - \frac{\delta\phi_j}{\dot{\phi}_j}\right)\frac{\partial}{\partial t}\left(\frac{\dot{\phi}_i^{\,2} - \dot{\phi}_j^{\,2}}{\sum_i^n \dot{\phi}_i^{\,2}}\right)\,,\ \Theta = \chi_N \sum_{i>j}^{N} \theta_{ij}$$

Single field KGB : intrinsic entropy

$$L_{KG}(\Phi, X) = K(\Phi, X) + G(\Phi, X)\,\Box\Phi$$

$$\alpha = c_s^2(t)\beta + \Gamma^{int}$$

$$v_{KG}^2 = c_s^2\left(1 + \frac{\Gamma^{int}}{2\epsilon H\dot{\zeta}}\right)^{-1}$$

Multi-field Horndeski : nKGB

$$\Gamma_{NKG} = \sum_{i}^{N}\Gamma_i^{int} + \chi_N \sum_{i>j}^{N}\Gamma_{ij}$$

$$v_{NKG}^2 = c_s^2\left(1 + \frac{\Gamma_{NKG}}{2\epsilon H\dot{\zeta}}\right)^{-1}$$

Applications: features in primordial curvature spectrum motivated by CMB or PBH

Considering the phenomenological ansatz of time independent MESS we get:

$$z^2 = 2a^2\epsilon$$

$$\zeta_k'' + \frac{\partial_\eta(z^2)}{z^2}\zeta_k' + \tilde{v}_k^2 k^2 \zeta_k = 0$$

$$u_k \equiv \tilde{Z}_k \zeta_k$$

$$u_k'' + \left(\tilde{v}_k^2 k^2 - \frac{z''}{z}\right) u_k = 0$$

Due to the MESS modes freeze after horizon crossing time, around $\eta_k = -\frac{1}{v_k k}$

This super-horizon evolution is the cause of the features in the spectrum

For example for a multi-fields model with standard kinetic term this super-horizon evolution is attributed to entropy perturbations while in the MESS picture it is just due the difference between the freezing time and the horizon crossing time

Effects of a local momentum variation of the MESS:

$$\tilde{v}_k = 1 + A_c \exp\left[-\left(\frac{k-k_0}{\sigma}\right)^2\right]$$

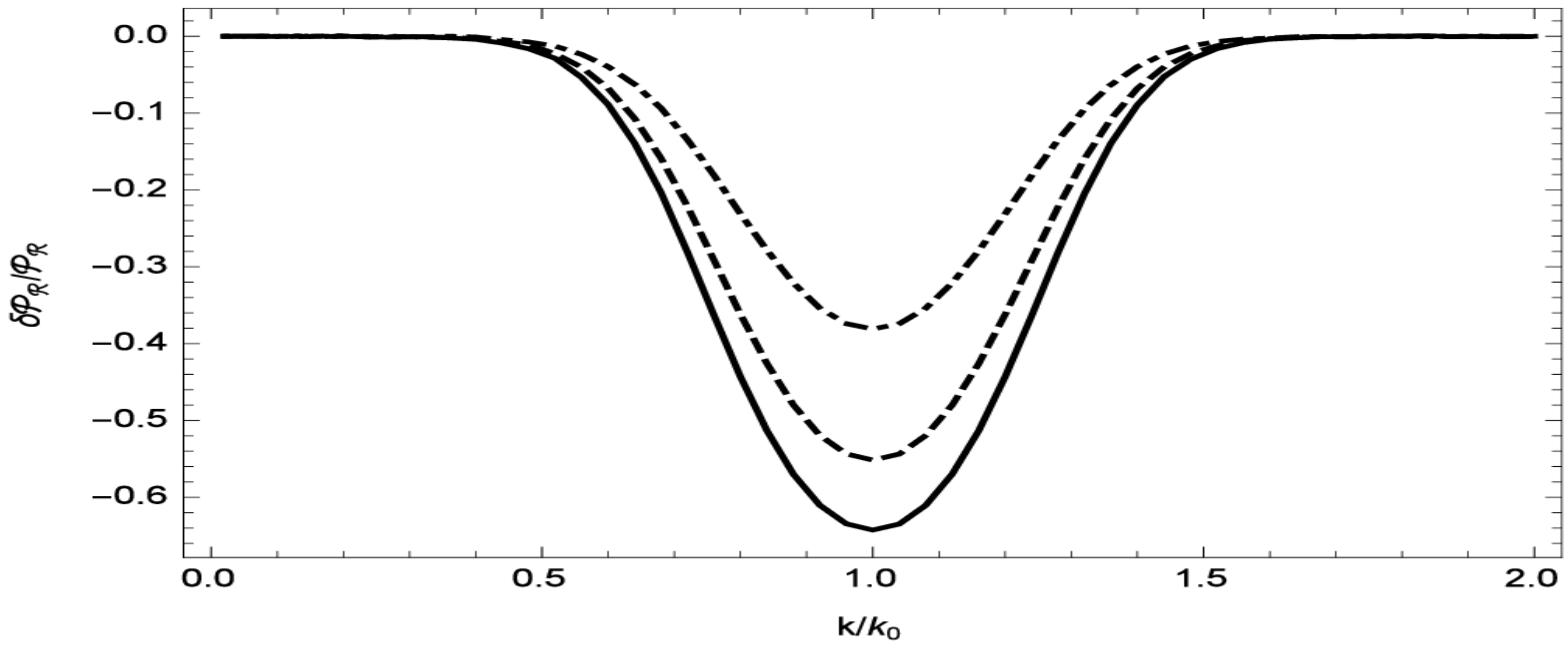
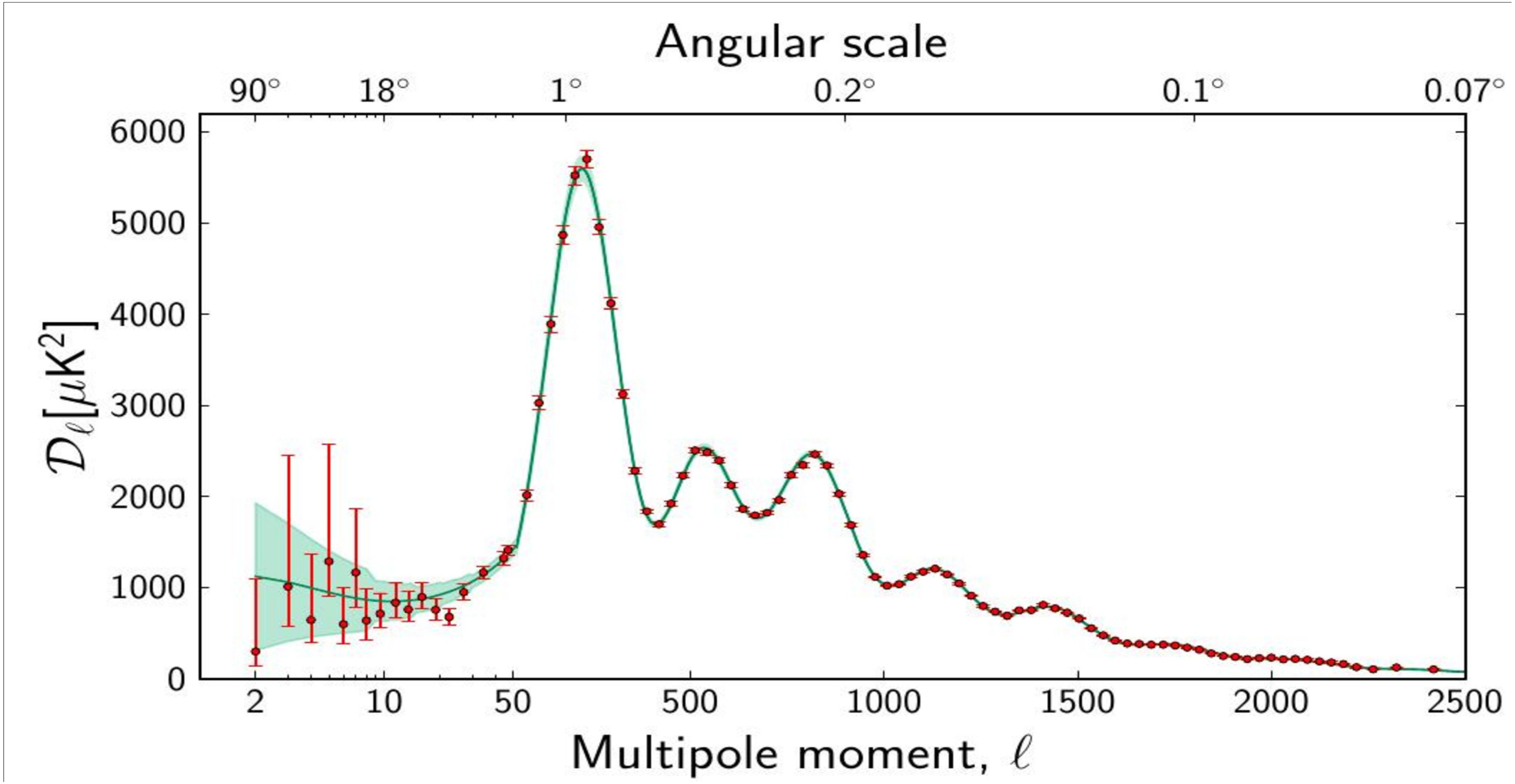


FIG. 1: The relative difference $\Delta\mathcal{P}_\zeta/\mathcal{P}_\zeta$ is plotted as a function of k/k_0. The solid, dashed and dot-dashed lines correspond $\sigma = 2.5 \times 10^{-1} k_0$ and $A_c = 4 \times 10^{-1}$, $A_c = 3 \times 10^{-1}$ and $A_c = 1.7 \times 10^{-1}$ respectively.

The scale k0 could have different origins: turning point in multi-fields modes, particle production, modification of gravity, etc.

CMB anisotropy spectrum : there exists some anomalies which could be explained by MESS



One spectrum to rule them all?

$$\mathcal{R}_c''(k) + 2\frac{z'}{z}\mathcal{R}_c'(k) + c_s^2 k^2 \mathcal{R}_c(k) = 0,$$

$$h_k'' + 2\frac{z_\gamma'}{z_\gamma}h_k' + c_\gamma^2 k^2 h_k = 0\,,$$

$$z = \frac{a\sqrt{2\epsilon}}{c_s} = \frac{1}{c_s}\sqrt{2\left(a^2 - \frac{a^3\ddot{a}}{\dot{a}^2}\right)}.$$

Freedom to choose the initial condition condition for a(t) for a given z(t) !!

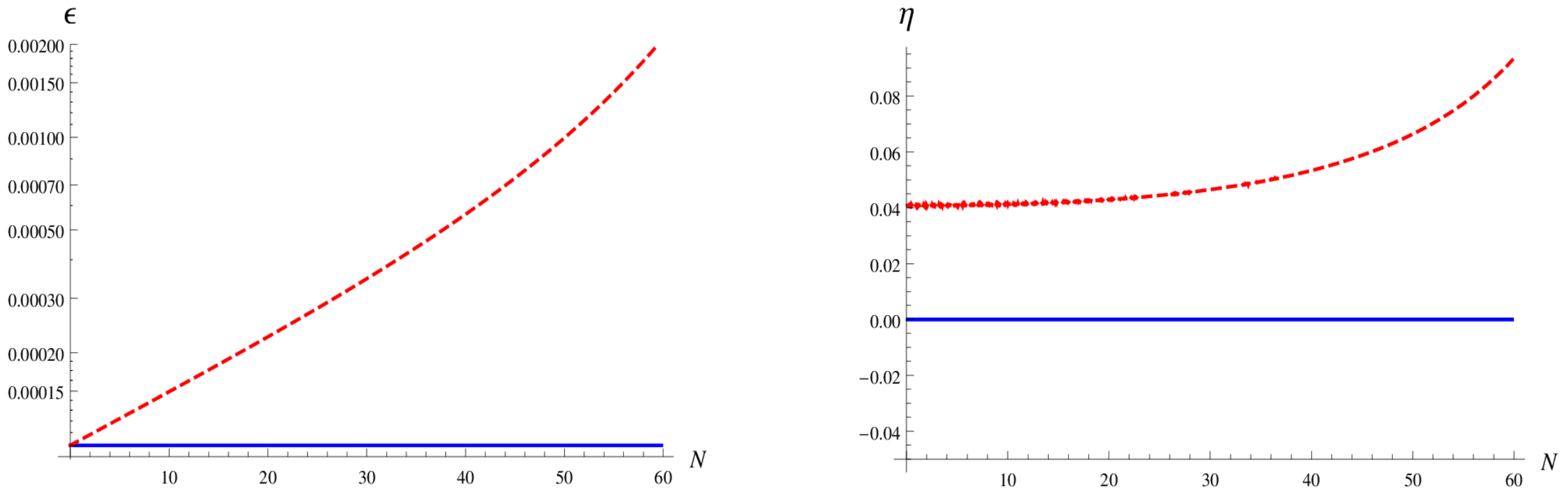
Recipe to construct dual models:

- Fix z0(t), c(t)
- Solve $z(t)=z_{dual}(t)$ with different initial H, i.e. different initial derivative a'
- The new a(t) will by construction give the same z(t) but different slow roll parameters
- The spectra will be the same
- Higher order correlation functions for scalar perturbations will be different
- Gravitational waves spectra will be different

Examples of homospectral models

$$a_{\rm ref}(t) = \left(1+\epsilon_{\rm c} H_{{\rm ref},i} t\right)^{1/\epsilon_{\rm c}} \left[1+\lambda e^{-\left(\frac{t-t_0}{\sigma}\right)^2}\right],$$

No feature



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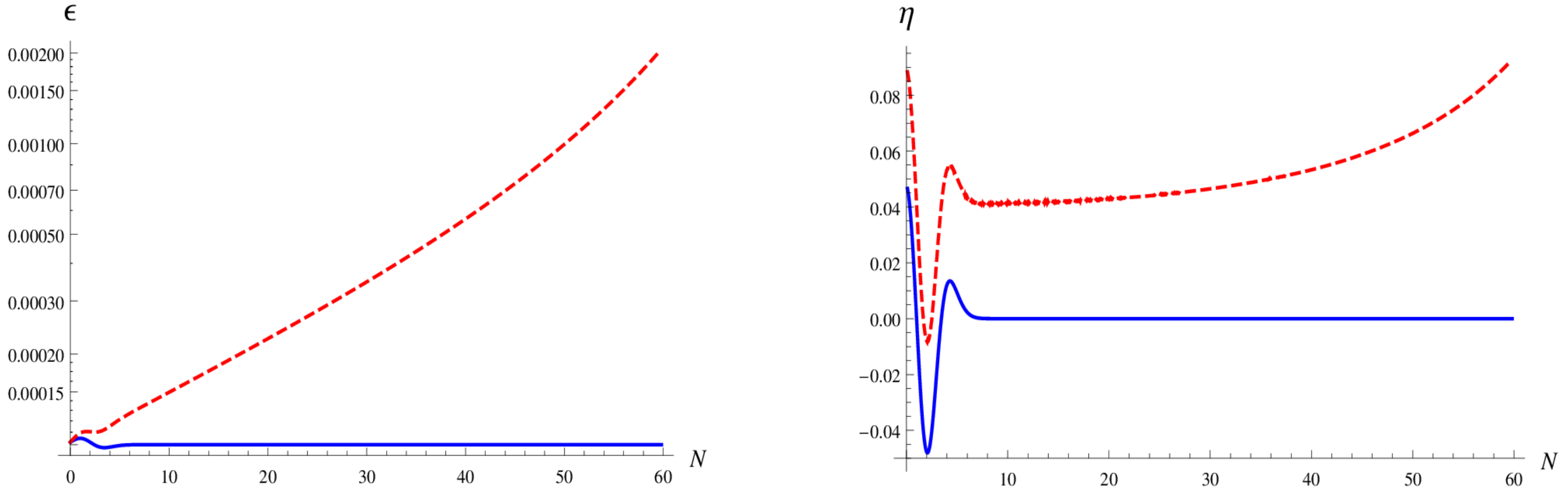
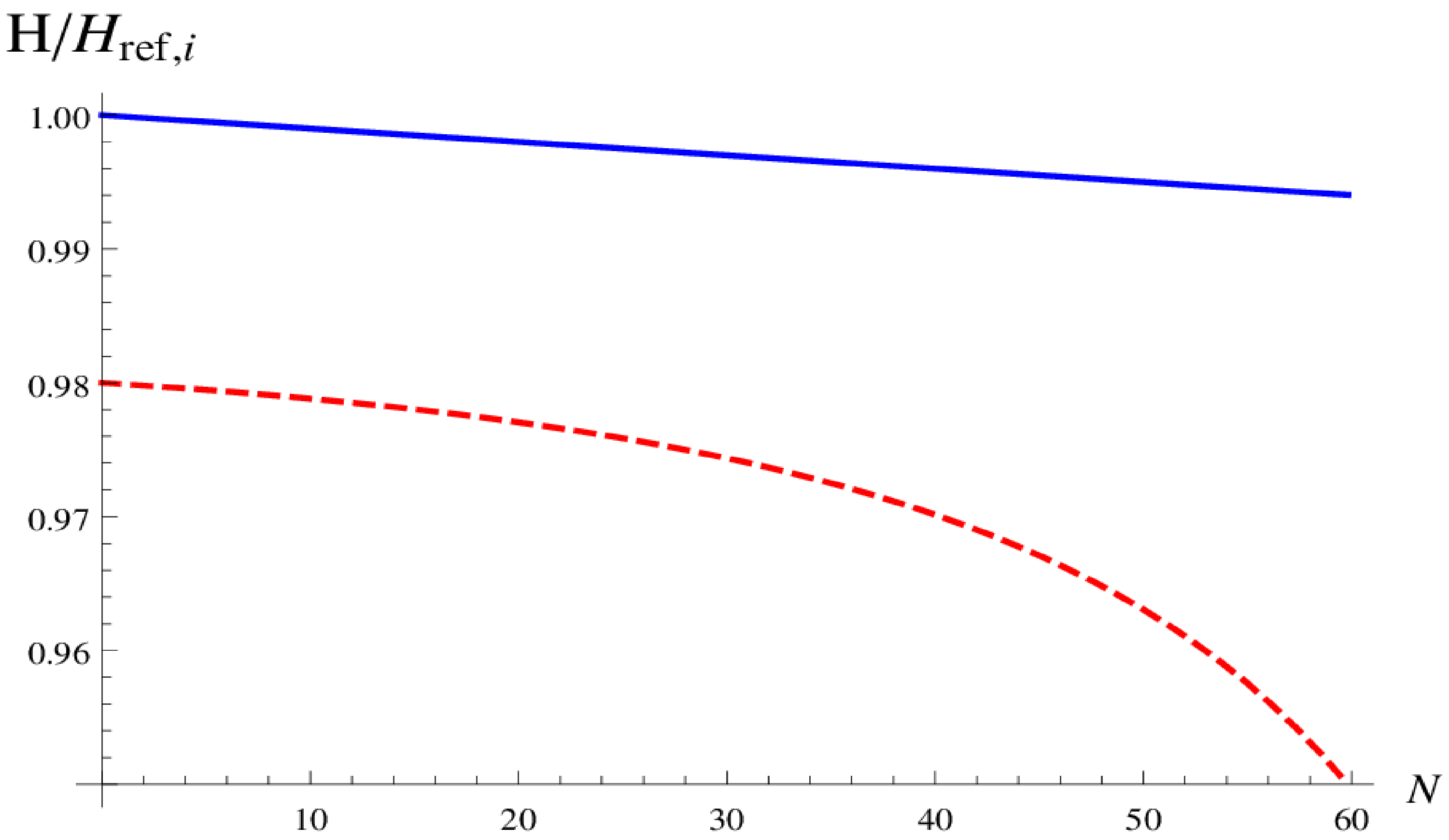
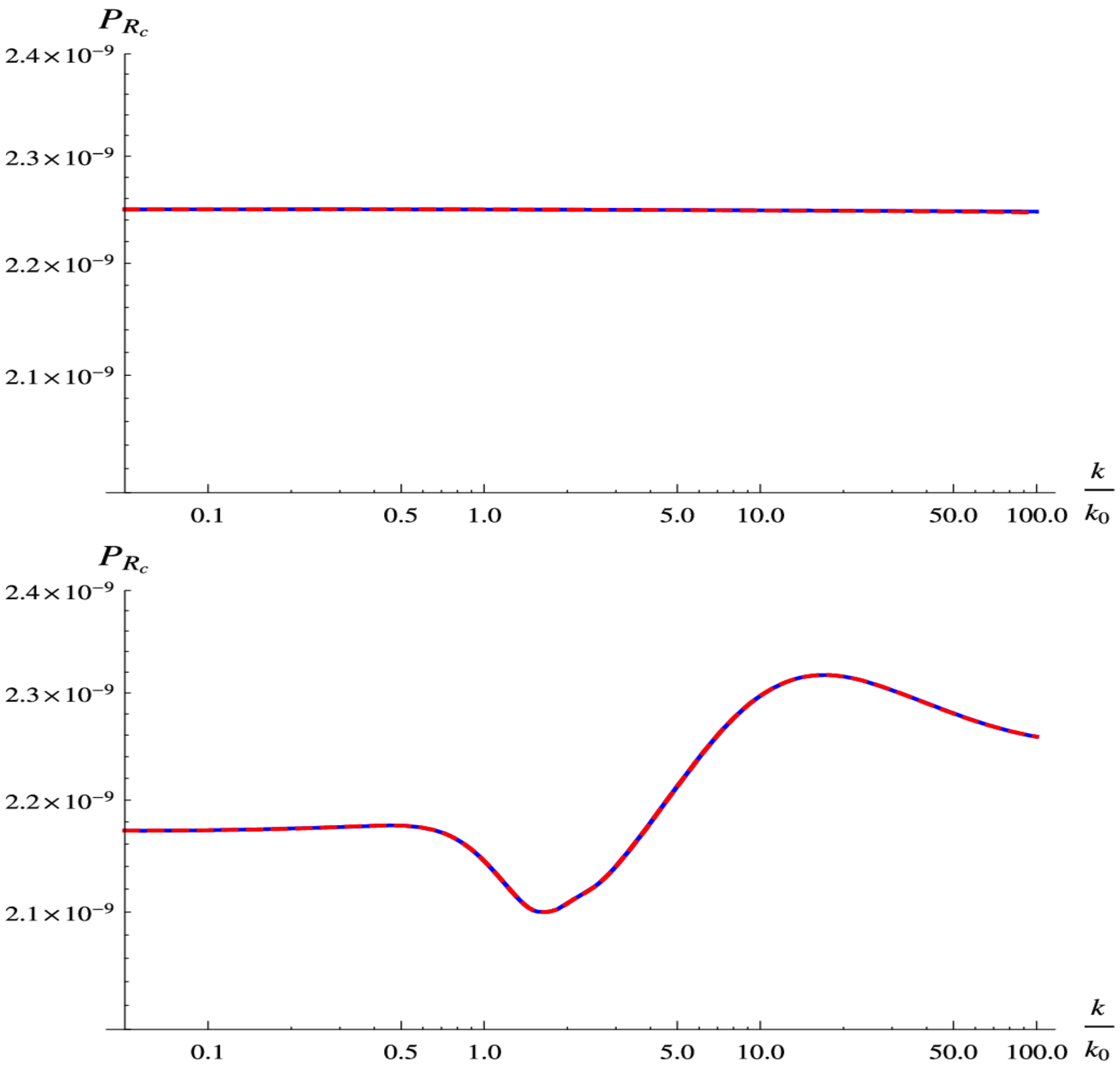


Hubble parameter for homospectral models without feature



Curvature spectra of homospectral models



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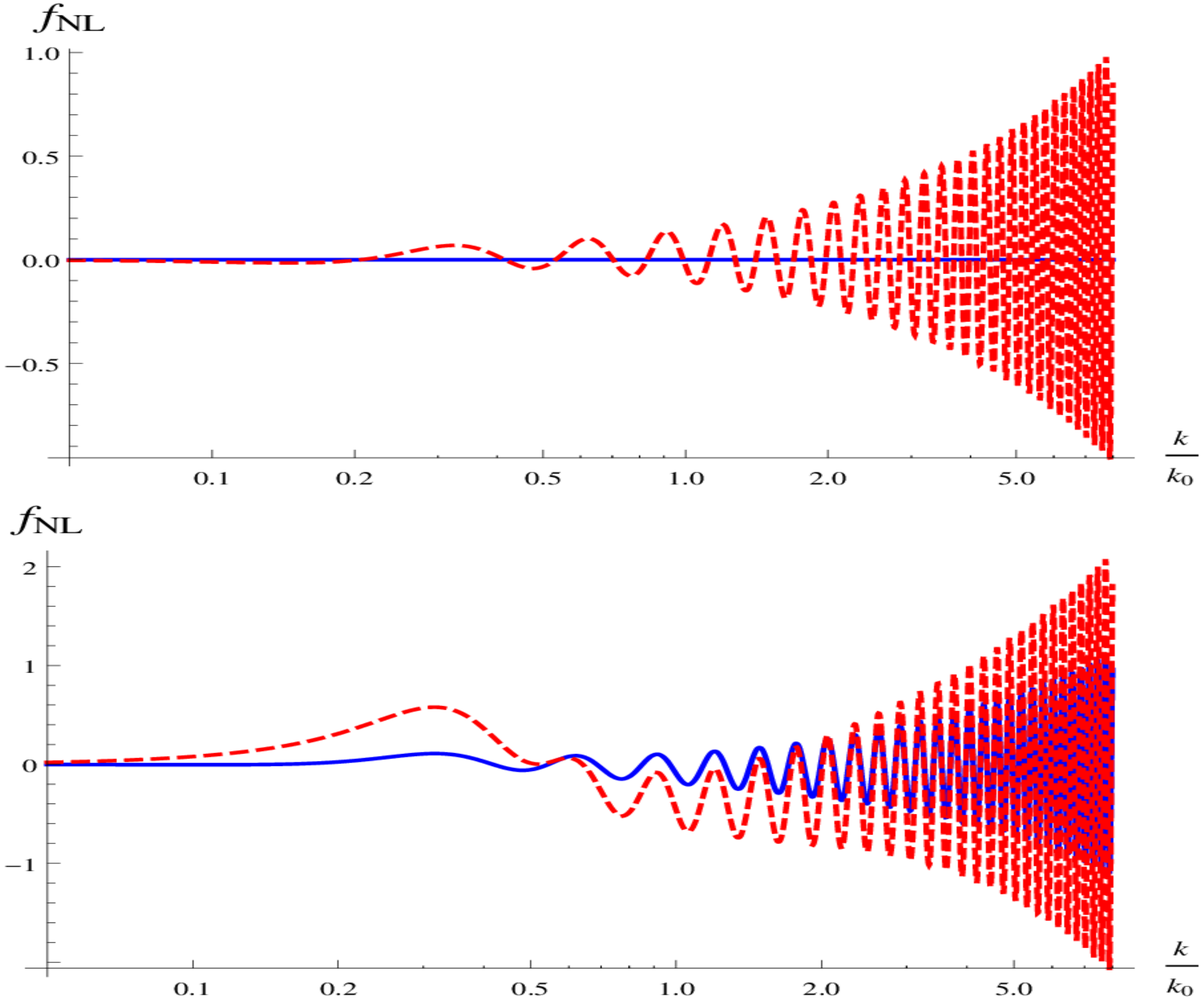
Bispectrum equilateral configuration



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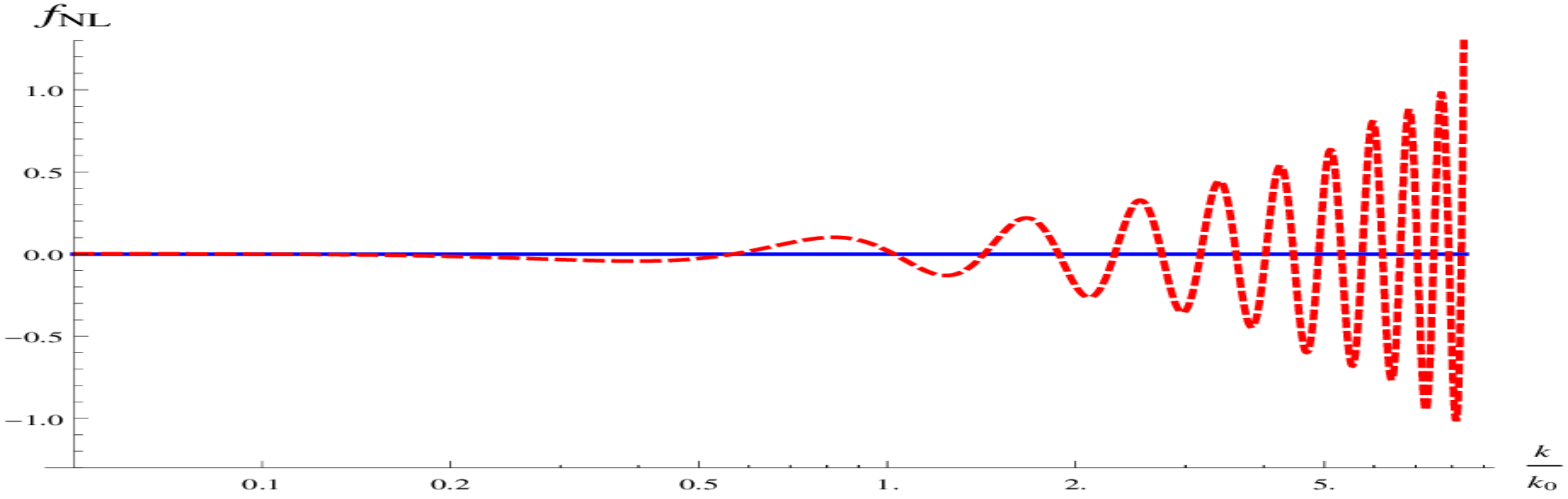
Bispectrum Squeezed configuration



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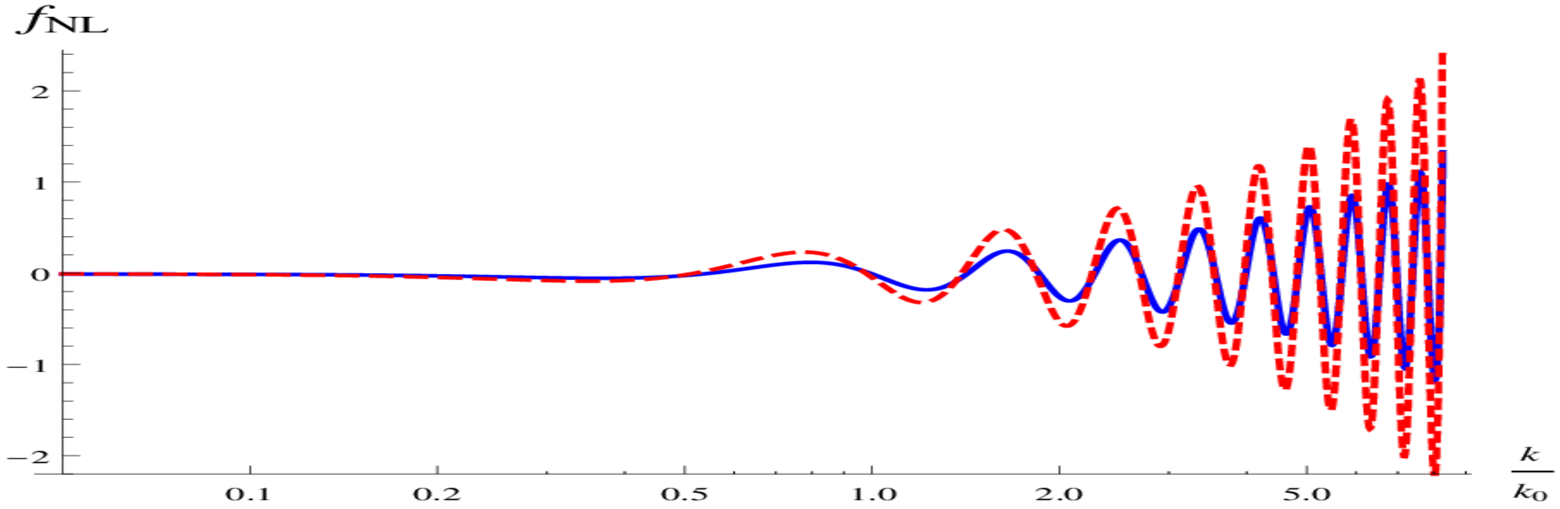

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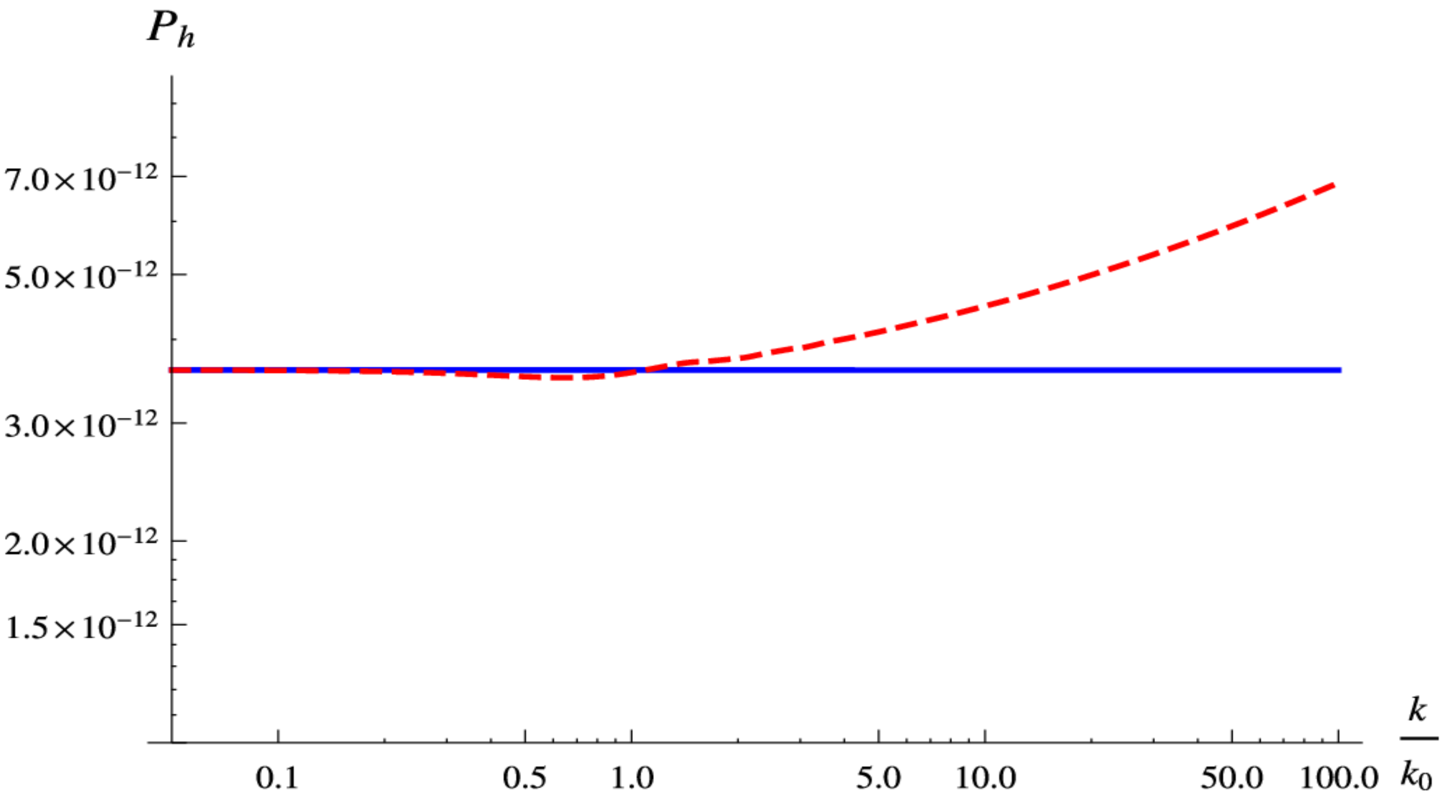
The spectrum of tensor modes is different for scalar homospectral models



Violation of “general consistency condition”(JCAP 1504 (2015), Palma), not the squeezed limit Maldacena 's

$$f_{NL} \simeq \frac{5}{12}\frac{k_1k_2k_3}{k_1^3+k_2^3+k_3^3}\left[\frac{d^2}{d\ln k^2}\frac{\Delta P_{\mathcal{R}_c}}{P^0_{\mathcal{R}_c}}(k)\right],$$

One spectrum to rule them all?

Any (not just scale invariant) spectrum of comoving curvature perturbation can be obtained

- with an infinite class of homospectral dual background histories, including contracting Universes
- different theoretical scenarios with the same MESS such as multi-fields, modified gravity, or their combination
- further degeneracy due to combination of MESS and background evolution degeneracy
- Higher order correlation functions and gravitational waves can reduce the degeneracy
- MESS is a useful model independent quantity to span the full space of theoretical scenarios

MESS of multifields system

$$\tilde{v}_k^2(t) = \left(1 + \frac{H\Theta}{2\dot{\zeta}}\right)^{-1} = \left(1 + \frac{2HV_{,s}Q_{,s}}{\dot{\zeta}\dot{\sigma}^2}\right)^{-1} = \left(1 - \frac{2H^2\eta_\perp Q_{,s}}{\dot{\zeta}\dot{\sigma}}\right)^{-1}$$

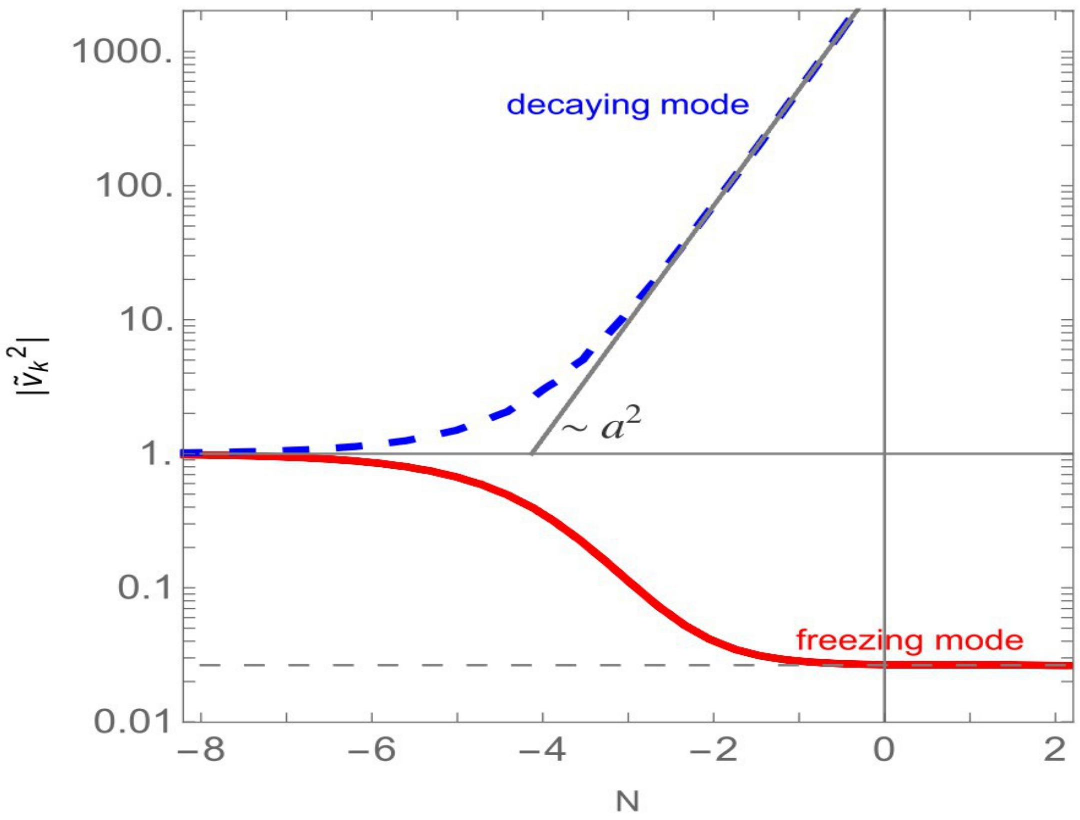
$$\Theta \equiv -\frac{4\dot{\phi}_1\dot{\phi}_2}{\dot{\sigma}^3}\sqrt{G}\left(\frac{\delta\phi_1}{\dot{\phi}_1} - \frac{\delta\phi_2}{\dot{\phi}_2}\right)V_{,s} = \frac{4}{\dot{\sigma}^2}Q_{,s}V_{,s}$$

$$G \equiv \det(G_{IJ}),\ Q_{,s} \equiv Q_{,K}e_s^K\ V_{,s} \equiv V_{,K}e_s^K;$$

$$e_s^K = \left(e_s^1, e_s^2\right) = \left(\frac{G_{21}\dot{\phi}_1 + G_{22}\dot{\phi}_2}{\dot{\sigma}\sqrt{G}}, -\frac{G_{11}\dot{\phi}_1 + G_{12}\dot{\phi}_2}{\dot{\sigma}\sqrt{G}}\right).$$

$$\eta_\perp \equiv -\frac{V_{,s}}{H\dot{\sigma}}.$$

Example


decaying mode
freezing mode
~ a^2
1000.
100.
10.
1.
0.1
0.01
−8
−6
−4
−2
0
2
N
$|\tilde{v}_k^2|$

Effects of the MESS on the spectrum

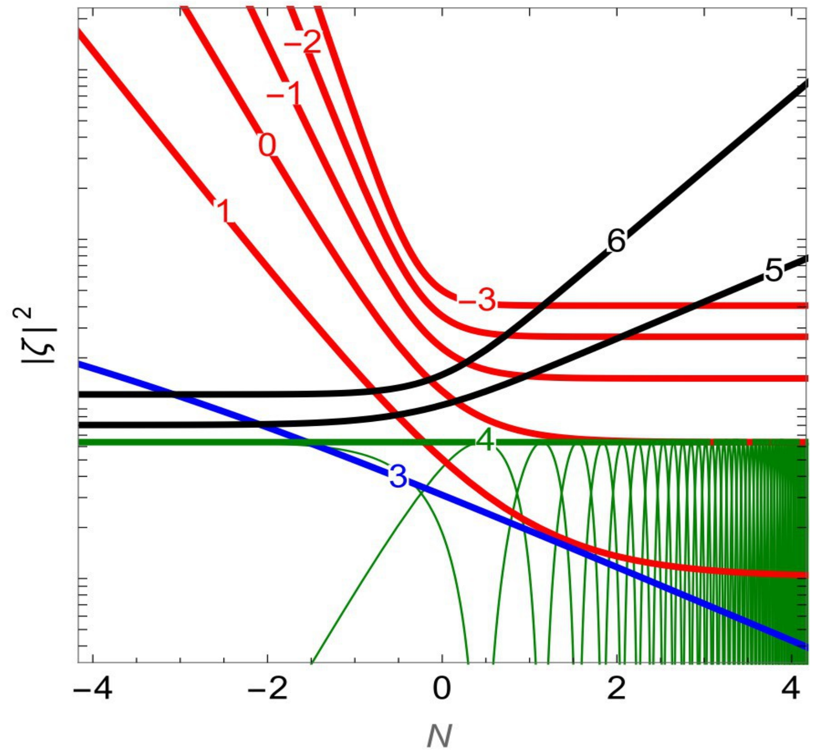


FIG. 1: Evolutions of the amplitude of curvature perturbation ζ given in eq. (8) for different sound speeds $\tilde{v}_k^2 \propto a^p$ with different values of p is shown as thick lines. Color coding corresponds to late-time behavior: freezing (red), decaying (blue) and growing (black); the special case of $p = 4$ is shown in green. Thin green lines indicate the real and imaginary part of of ζ for $p = 4$. Normalization of ζ is arbitrary.

Conclusions

Model independent analysis based on MESS or SESS can set constraints on a wide class of models/theories, comparing different categories of theoretical scenarios, not only models, within a unified phenomenological framework.

- MESS and SESS are model independent and can be applied to any physical system for example:
- Multi-fields , scalar or vector fields (scalar part)
- Modified gravity, e.g. Horndesky theory, in terms of an effective EM tensor : $G_{\mu\nu}=T^{eff}_{\mu\nu}$
- Non-Gaussianity can be studied in terms of MESS and SESS
- The anisotropy stress term can be added but does not modify the definition of MESS and SESS
- Another convenient quantity to parametrizes the effect in a model independent way is the effective Z ZEFF:

Einstein's equations in the comoving gauge and derivations of EOM in terms of MESS

$$\frac{1}{a^2}\overset{(3)}{\Delta}[-\zeta + aH\sigma] = \frac{\beta}{2}\,, \qquad (11)$$

$$\gamma = \frac{\dot{\zeta}}{H}\,, \qquad (12)$$

$$-\ddot{\zeta} - 3H\dot{\zeta} + H\dot{\gamma} + (2\dot{H} + 3H^2)\gamma = \frac{\alpha}{2}\,, \qquad (13)$$

$$\dot{\sigma} + 2H\sigma - \frac{\gamma + \zeta}{a} = 0\,, \qquad (14)$$

12 ,13 >

$$\dot{\zeta} = -\frac{1}{2H\epsilon}\alpha\,, \qquad (15)$$

$$\sigma = \frac{\Phi_B + \zeta}{aH} \quad , \quad \gamma = \Phi_B + \partial_t(a\sigma)\,. \qquad (16)$$

16,11 ->

$$\frac{1}{a^2}\overset{(3)}{\Delta}\Phi_B = \frac{1}{2}\beta\,. \qquad (17)$$

$$v_s^2(t, x^i) \equiv \frac{\alpha(t, x^i)}{\beta(t, x^i)}\,, \qquad (18)$$

15,17,18 ->

$$\dot{\zeta} = -\frac{v_s^2}{a^2 H\epsilon}\overset{(3)}{\Delta}\Phi_B\,. \qquad (19)$$

16,12->

$$\zeta = -\Phi_B + \frac{H^2}{\dot{H}}\left(\Phi_B + H^{-1}\dot{\Phi}_B\right) = \frac{H^2}{a\dot{H}}\partial_t\left(\frac{a\Phi_B}{H}\right) \qquad (20)$$

20,d/dt 19->

$$\partial_t\left(\frac{a^3\epsilon}{v_s^2}\dot{\zeta}\right) - a\epsilon\overset{(3)}{\Delta}\zeta = 0$$