Fixed Points of Quantum Gravity and the Dimensionality of the UV Critical Surface

HECA seminar

Based on work with Daniel Litim
[2008.09181]

Yannick Kluth, 26th January 2021

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Outline

- Introduction to Asymptotic Safety and Quantum Gravity
- Quantum Gravity from the Functional Renormalisation Group
- Asymptotic Safety with Riemann tensors
- Eigenvectors in Asymptotic Safety
- Summary and Conclusions

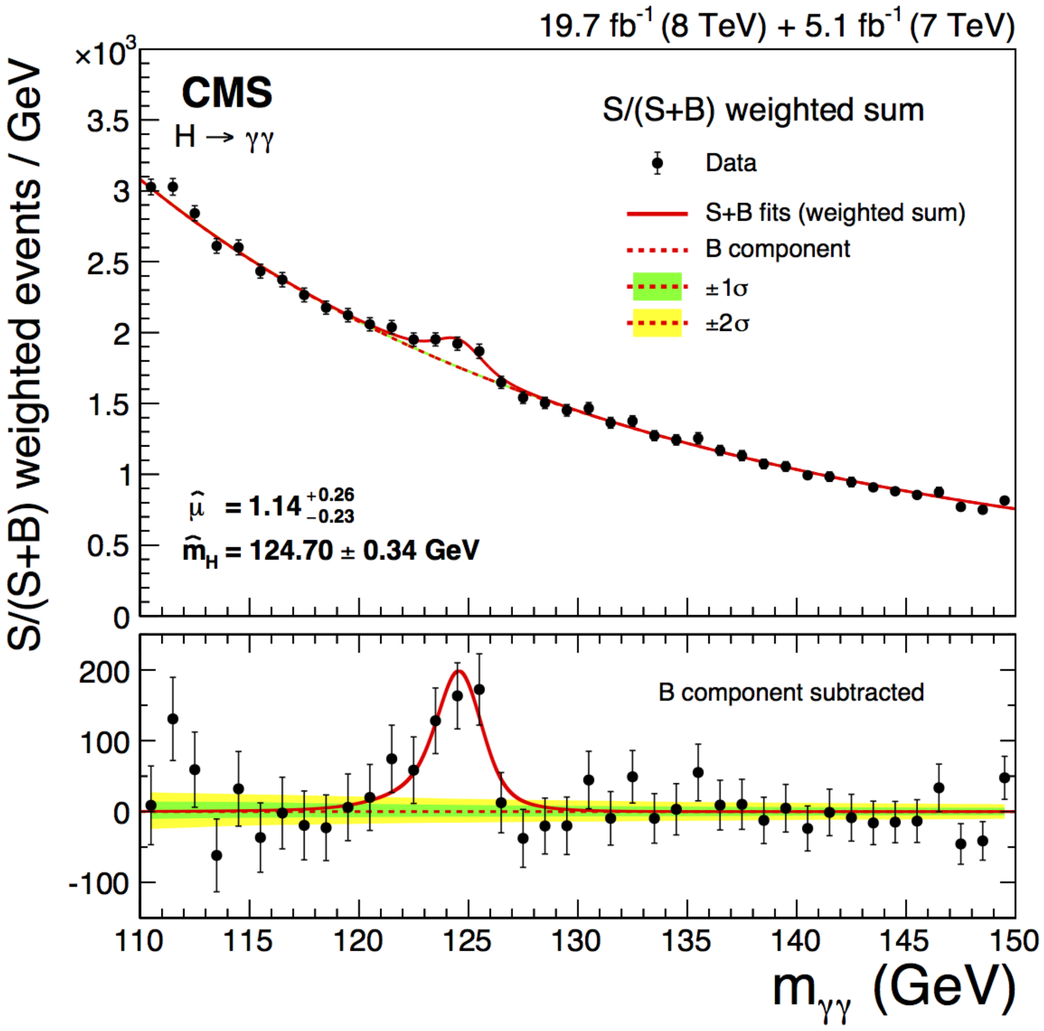
Asymptotically Safe Quantum Gravity

Gravity

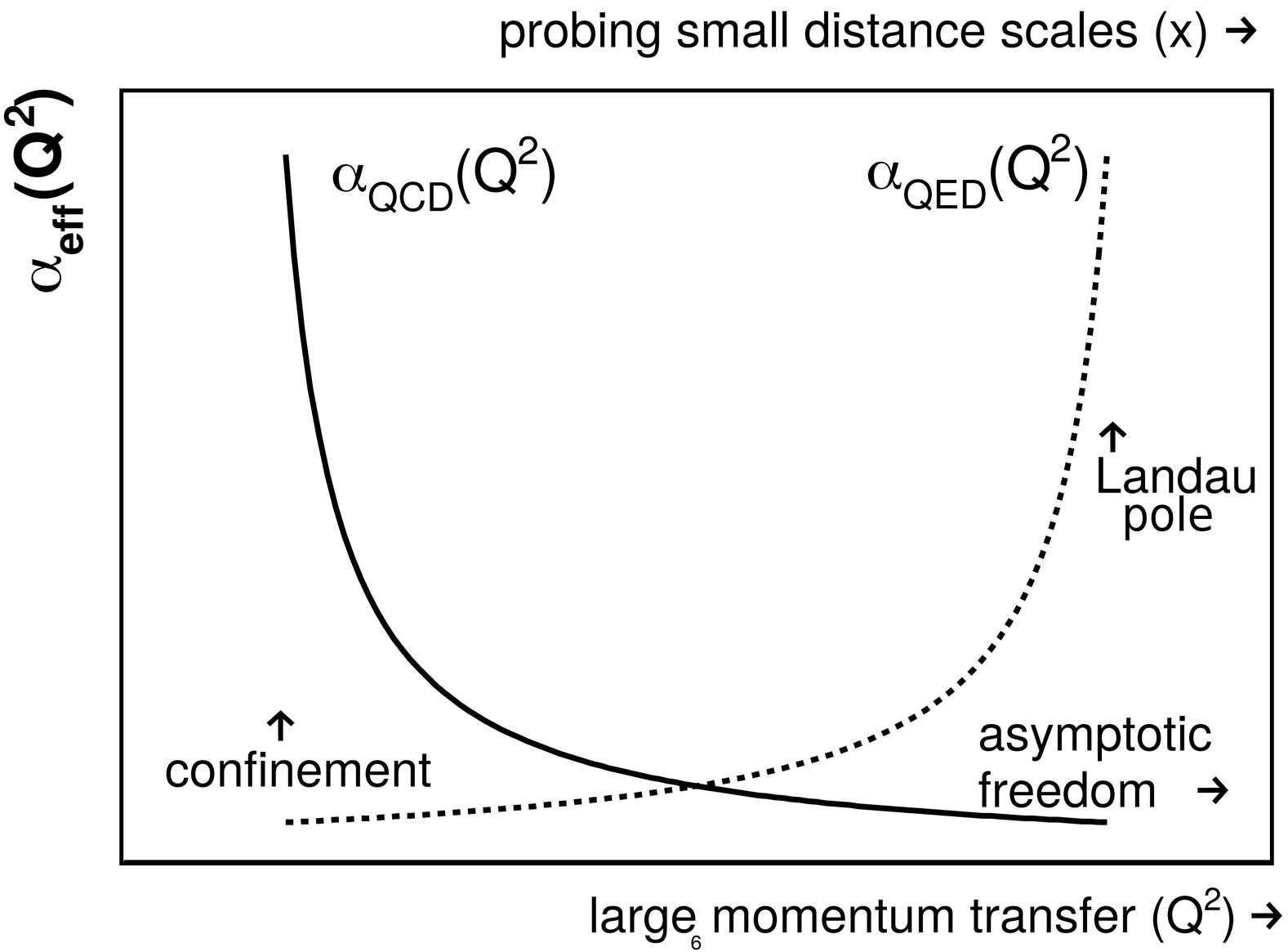
- Gravity is oldest known force of nature
  - Classically very well explained by general relativity
    - Perihelion precession of Mercury
    - Gravitational Waves
    - Black Holes
- Quantum gravity is an open problem for more than 70 years…
  - Black hole information paradox

Quantum Field Theory

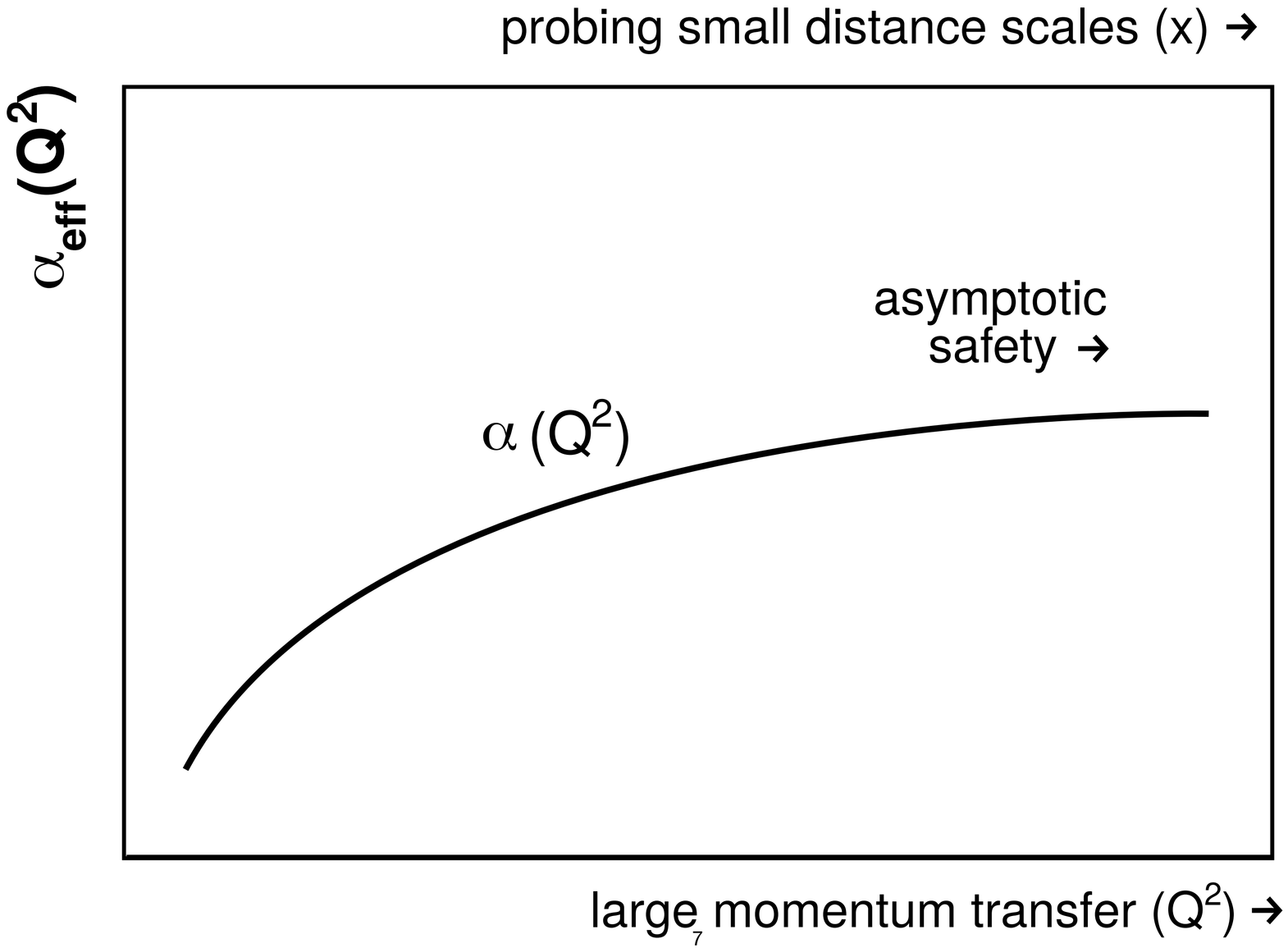
- Quantum Field Theory well established in particle physics
  - Standard Model
  - Measurement of Higgs particle
- Which QFTs can be fundamental theories?



QED vs QCD


probing small distance scales (x) →
$\alpha_{eff}(Q^2)$
$\alpha_{QCD}(Q^2)$
$\alpha_{QED}(Q^2)$
↑ Landau pole
↑ confinement
asymptotic freedom →
large momentum transfer (Q^2) →

Asymptotic Safety



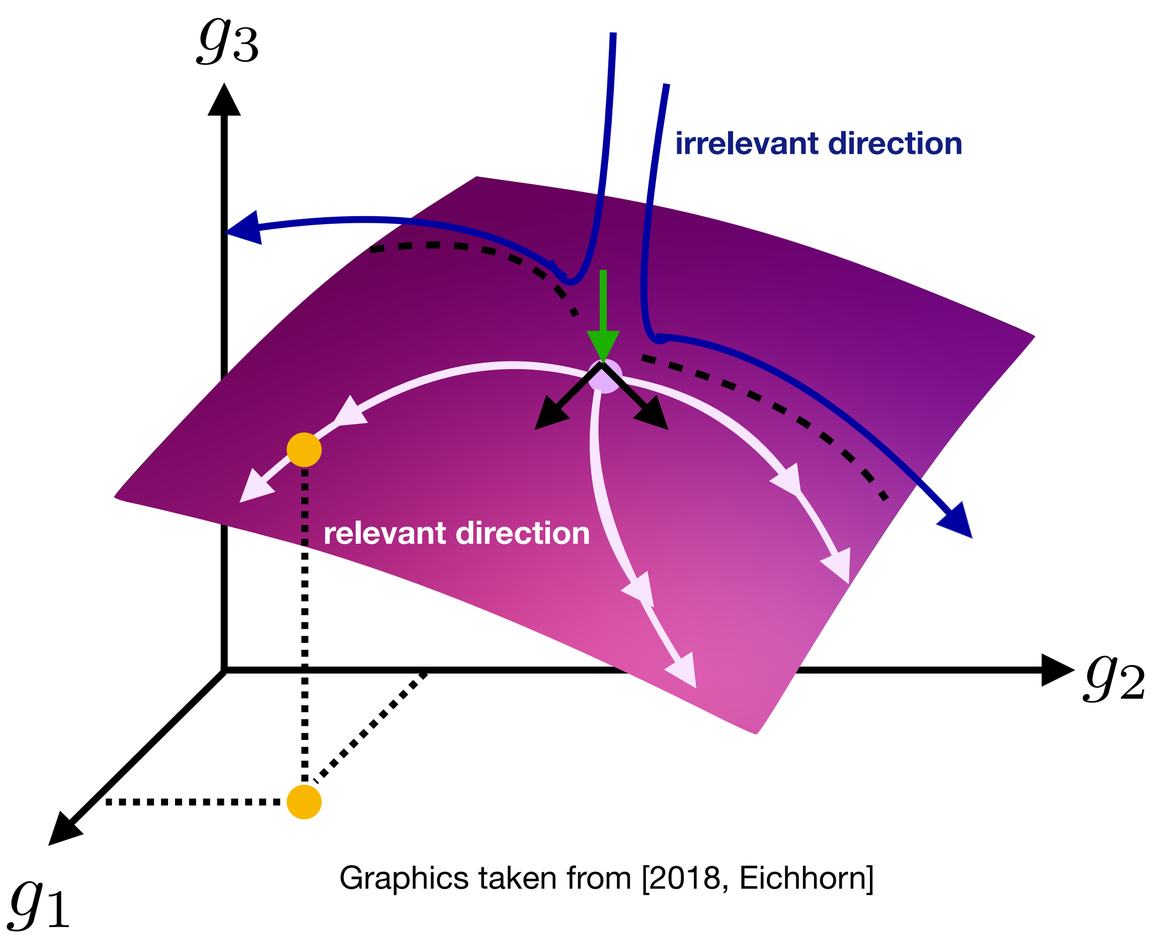
QFT and Gravity

- Can Gravity be formulated as a QFT?
  - As an effective field theory - Yes!
- But, gravity is perturbatively non-renormalisable [1985, Goroff and Sagnotti]

$$\mathcal{S} = \int \mathrm{d}^d x \sqrt{g} \left[\lambda_0 + \lambda_1 R + \lambda_{2,1} R^2 + \lambda_{2,2} R_{\mu\nu} R^{\mu\nu} + \lambda_{2,3} R_{\rho\sigma\mu\nu} R^{\rho\sigma\mu\nu} + \ldots \right]$$

  - Asymptotic Safety?
  - Infinite Number of Couplings?

The UV Critical Surface



Graphics taken from [2018, Eichhorn]

- Couplings required to flow into fixed point
- All trajectories flowing into UV fixed point form UV critical surface (purple)
  - Asymptotically safe trajectory has to stay on UV critical surface
  - Only the relevant directions are free parameters

RG Flow Close to a Fixed Point

- Expansion of beta functions around fixed point λ^*

$$\beta_i = \mathscr{M}_{ij}(\lambda_j - \lambda_j^*) + \mathscr{O}\left(\lambda_j - \lambda_j^*\right)^2$$

- Flow solved using eigenspectrum

$$\mathscr{M}_{ij} v_j^{(\ell)} = \theta_\ell v_i^{(\ell)}, \qquad \lambda_i - \lambda_j^* = \sum_\ell \mathscr{C}_\ell v_i^{(\ell)} e^{t\theta_\ell}$$

  - $\theta_\ell > 0$: Eigenmode diverges in UV (Irrelevant Direction)
  - $\theta_\ell < 0$: Eigenmode vanishes in UV (Relevant Direction)

- **Dimensionality of UV critical surface given by the number of relevant directions**

FRG in Quantum Gravity

Functional Renormalisation Group

- Γ_k is scale dependent Legendre effective action (effective average action)
  - $\Gamma_k \overset{k\to\infty}{\to} S$
  - $\Gamma_k \overset{k\to 0}{\to} \Gamma$
- RG-scale dependence introduced by IR regulator (background field method) in path integral

$$\mathscr{I}[\overline{g}] = \int \mathscr{D}h_{\mu\nu}\, e^{-\int \mathrm{d}^d x \sqrt{\overline{g}} h_{\mu\nu} \mathscr{R}_k^{\mu\nu\rho\sigma}[\overline{g}] h_{\rho\sigma} - \mathcal{S}[\overline{g}+h]}$$

- Functional Renormalisation Group

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left\{ \frac{\partial_t \mathscr{R}_k}{\Gamma_k^{(2)} + \mathscr{R}_k} \right\}$$

[1993, Wetterich]

Quantum Gravity from FRG

- Einstein-Hilbert truncation [1996, Reuter] [1999, Souma]

- EH + R^2 [2002, Lauscher and Reuter]
- EH + $C^{\rho\sigma}{}_{\mu\nu} C^{\mu\nu}{}_{\alpha\beta} C^{\alpha\beta}{}_{\rho\sigma}$ [2016, Gies et al]

} Large relevant (irrelevant) eigenvalues, 3 (2) dimensional UV critical surface

- EH + $R^2 + C_{\rho\sigma\mu\nu} C^{\rho\sigma\mu\nu}$ [2009, Benedetti and Machado and Saueressig]
- Full fourth order gravity on general background [2020, Falls et al]

} 3 dimensional UV critical surface, No effects of higher curvature invariants

Quantum Gravity from FRG

- We can investigate whole functions of curvature invariants:
  - $f(R)$ gravity [2007, Codello and Percacci and Rahmede] [2014, Falls et al]
  - $f(R, \mathrm{Ric}^2)$ gravity [2017, Falls et al]
  - Higher curvature invariants important to stabilise fixed point in $f(R)$ gravity
  - 3 dimensional UV critical surface

Which Operators?

	Curvature Invariants
dim-0	$\mathbb{1}$
dim-2	R
dim-4	R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}$, $\Box R$
dim-6	R^3, $RR_{\mu\nu}R^{\mu\nu}$, $RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}$, $R_{\mu\nu}R^{\nu\rho}R^{\mu}_{\rho}$, $R_{\mu\rho}R_{\nu\sigma}R^{\mu\nu\rho\sigma}$, $R^{\mu}_{\nu}R_{\nu\alpha\beta\gamma}R^{\mu\alpha\beta\gamma}$, $R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}R^{\nu\alpha\sigma\beta}$, $\nabla_{\mu}R\nabla^{\mu}R$, $\nabla_{\rho}R_{\mu\nu}\nabla^{\rho}R^{\mu\nu}$
	$\vdots$

Asymptotic Safety with Riemann Tensors

A Family of Gravitational Actions

- Consider Gravitational action of the form

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g} \; \left[F_k(X) + R Z_k(X) \right]$$

with

$$X = aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}$$

- We perform polynomial expansion up to order $N-1$ in curvature

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g} \; \left[\lambda_0 + \lambda_1 R + \lambda_2 X + \lambda_3 RX + \lambda_4 X^2 + \lambda_5 RX^2 + \ldots \right]$$

- Always includes Einstein Hilbert, special cases are $f(R)$ and $f(R, R_{\mu\nu}R^{\mu\nu})$

- Further special case: $\{a, b, c\} = \{0,0,1\}$, i.e. $f(R, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu})$

Which Operators?

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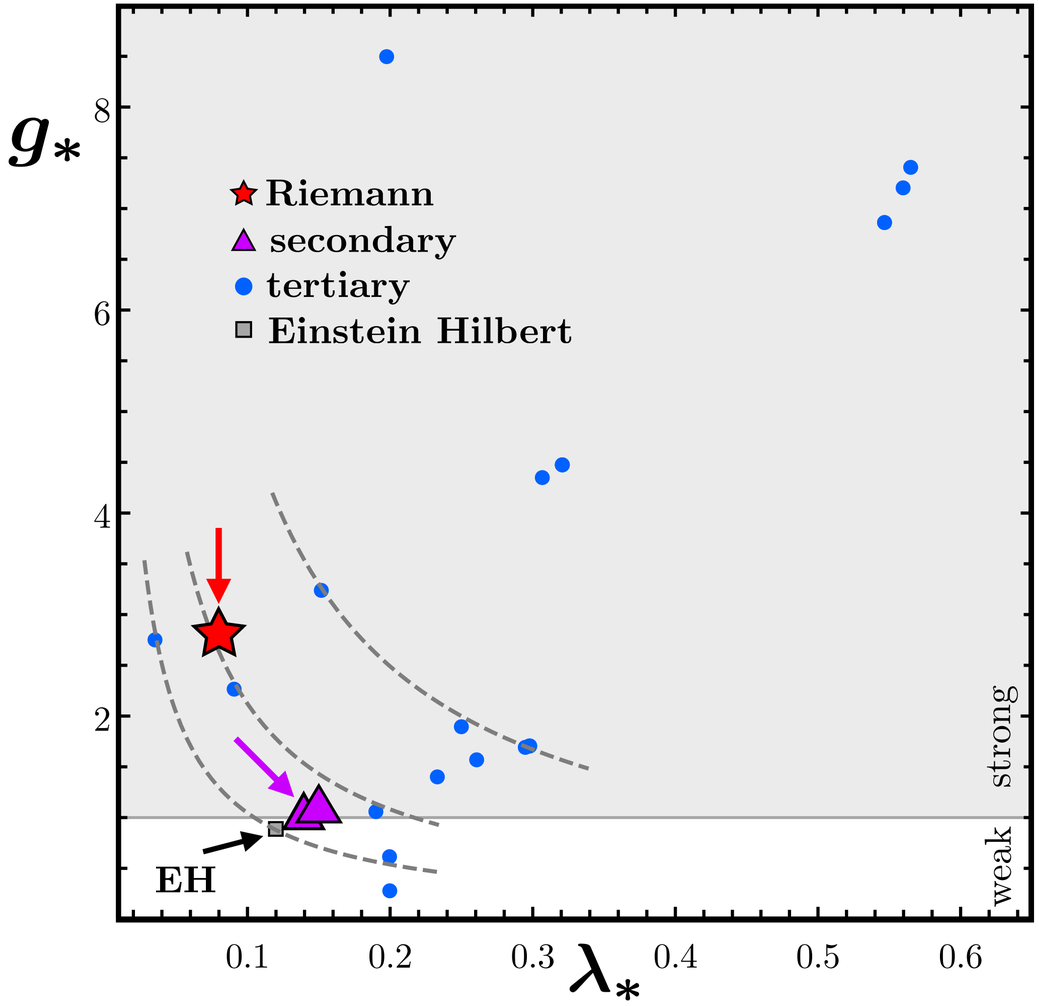
	$\vdots$

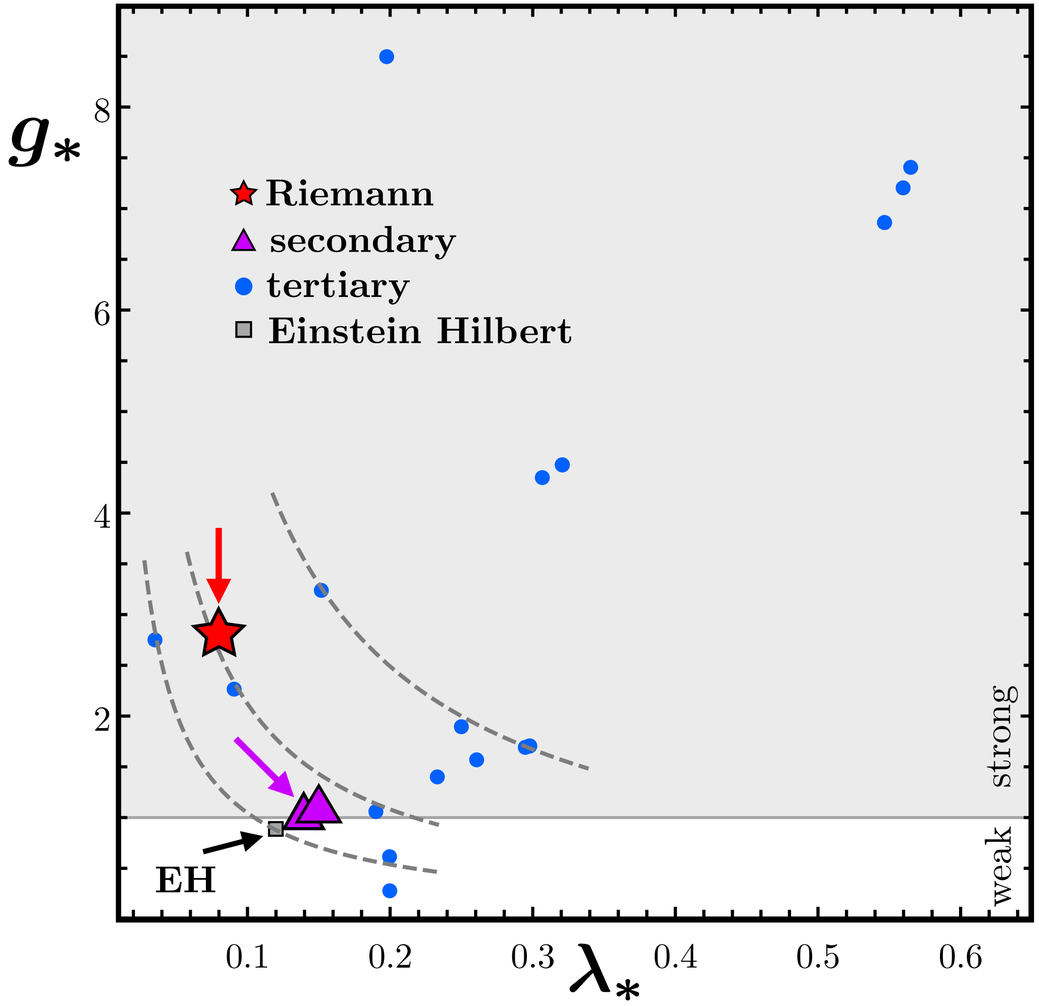
1. Is there a Fixed Point(s)?
2. UV critical surface?

Fixed Point Search



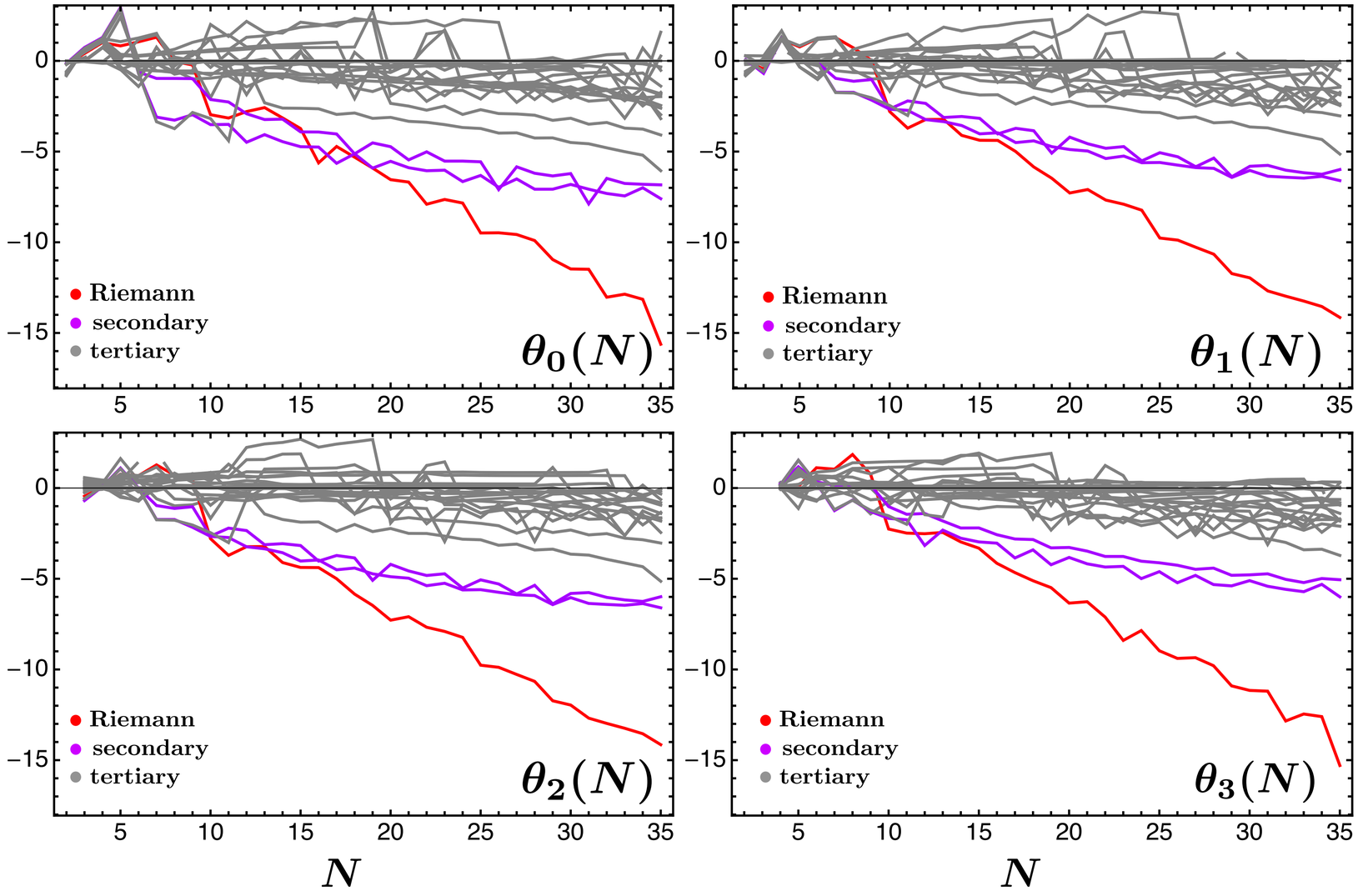
- Find fixed points at high orders by searching close to fixed points of previous orders
  - Bias: Most likely fixed points close to EH are found
  - In addition to usual method add random values to fixed point couplings at different orders

Fixed Point Search

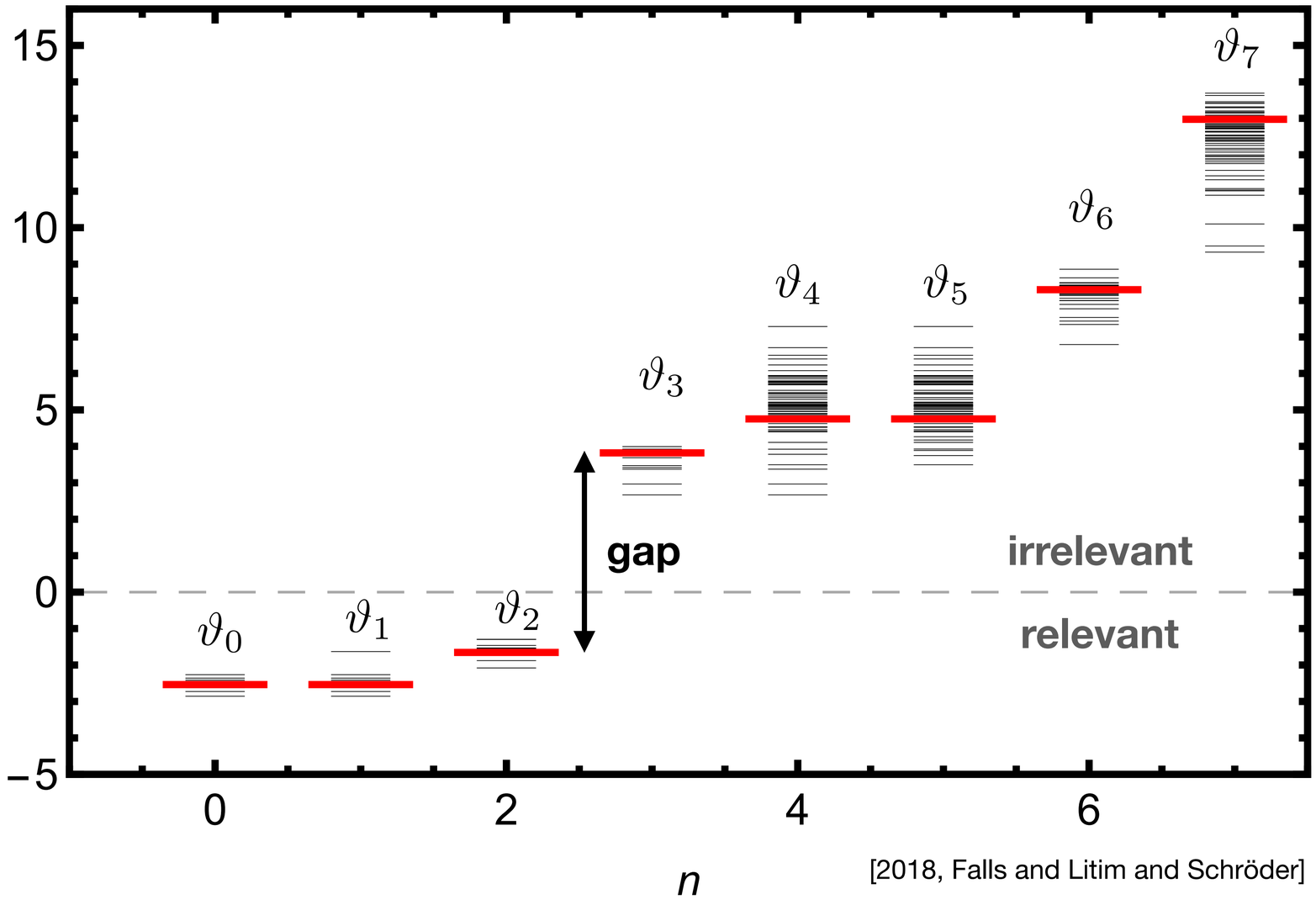


- Result of fixed point scan in $f(R, \text{Riem}^2)$ gravity at $N = 36$
- Discarding unstable candidates, three stable fixed points are found

Convergence of Eigenvalues



UV Critical Surface in $f(R)$ Gravity

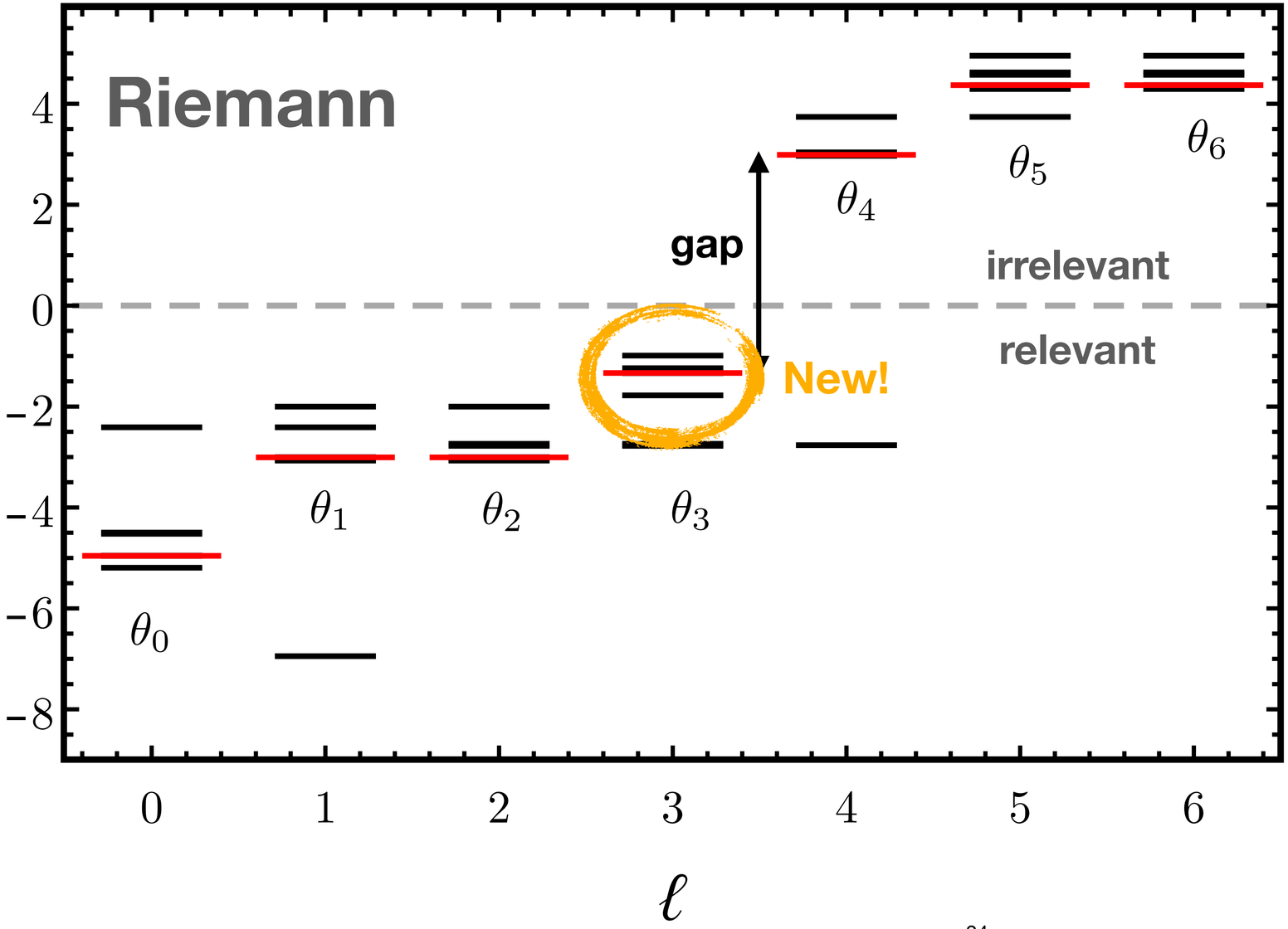


[2018, Falls and Litim and Schröder]

- Relevant Operators:

$$\Lambda, R, R^2$$

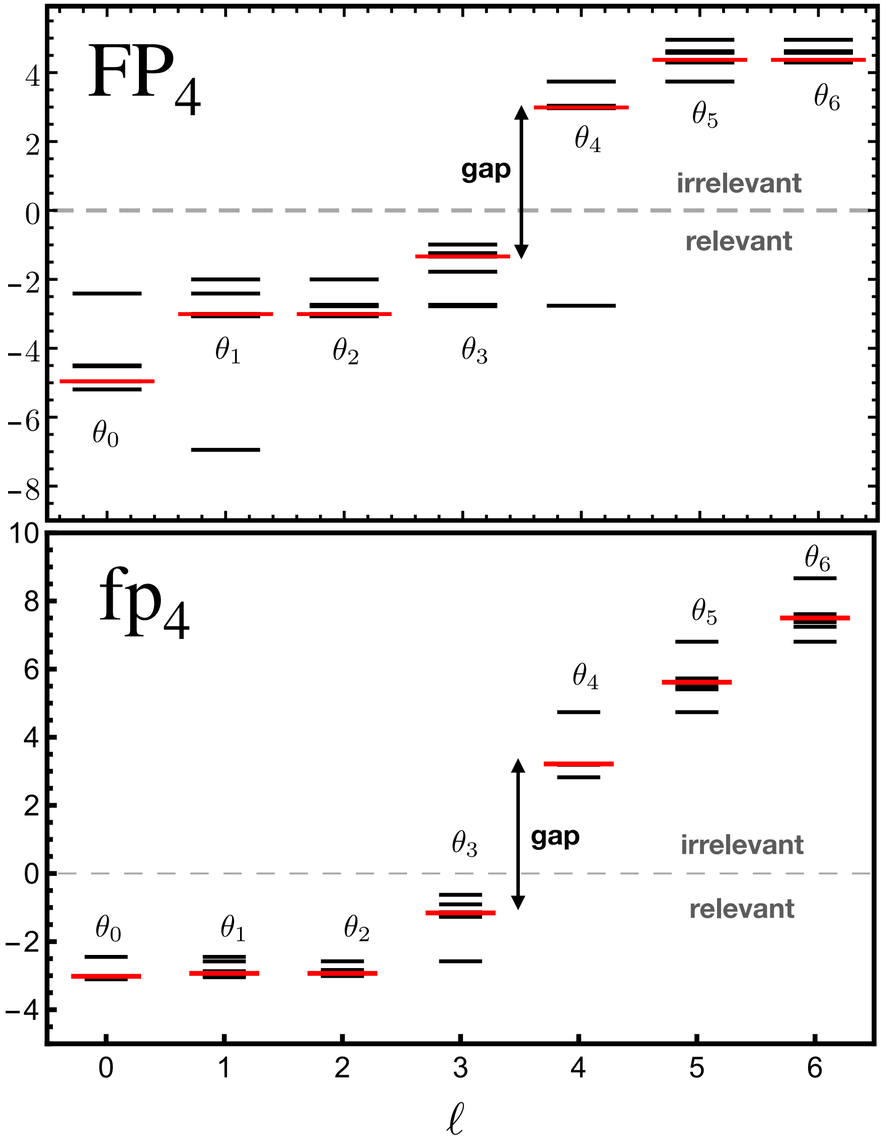
UV Critical Surface in $f(R, \text{Riem}^2)$ Gravity

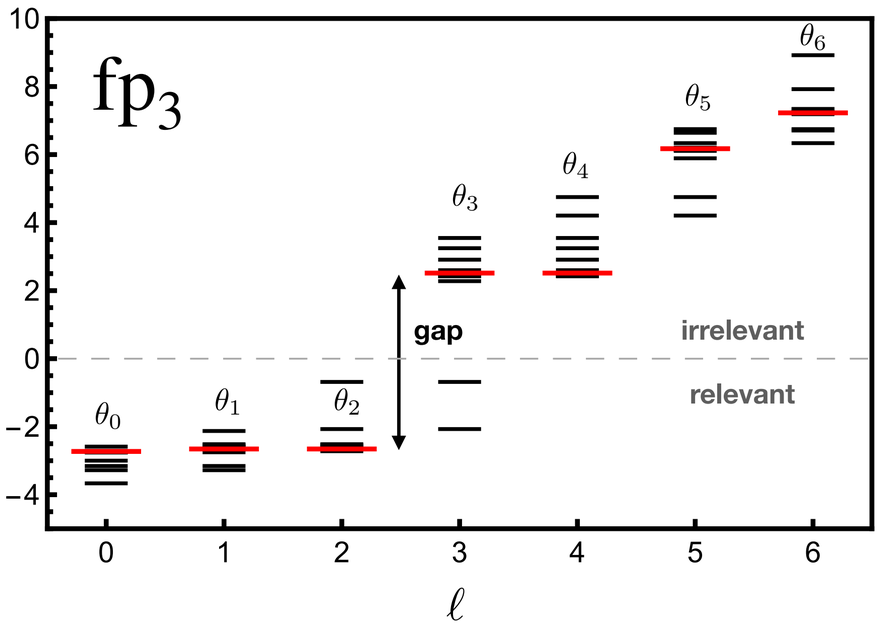


- Relevant Operators:

$$\Lambda, R, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}$$

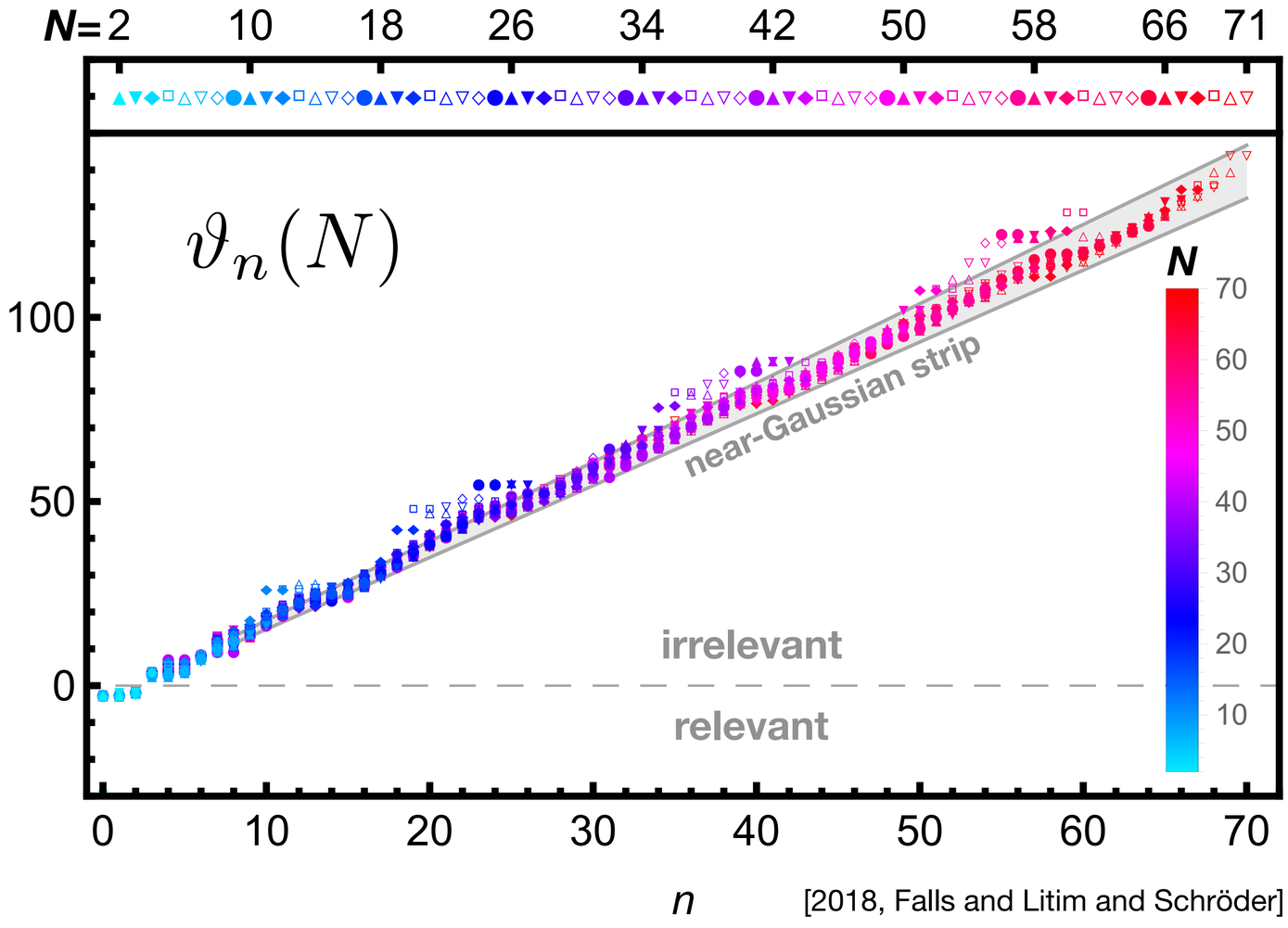
Dimensionality of the UV critical surface





- Quantum shifts can render canonically irrelevant eigenvalues relevant
  - New open parameter of AS Quantum Gravity?

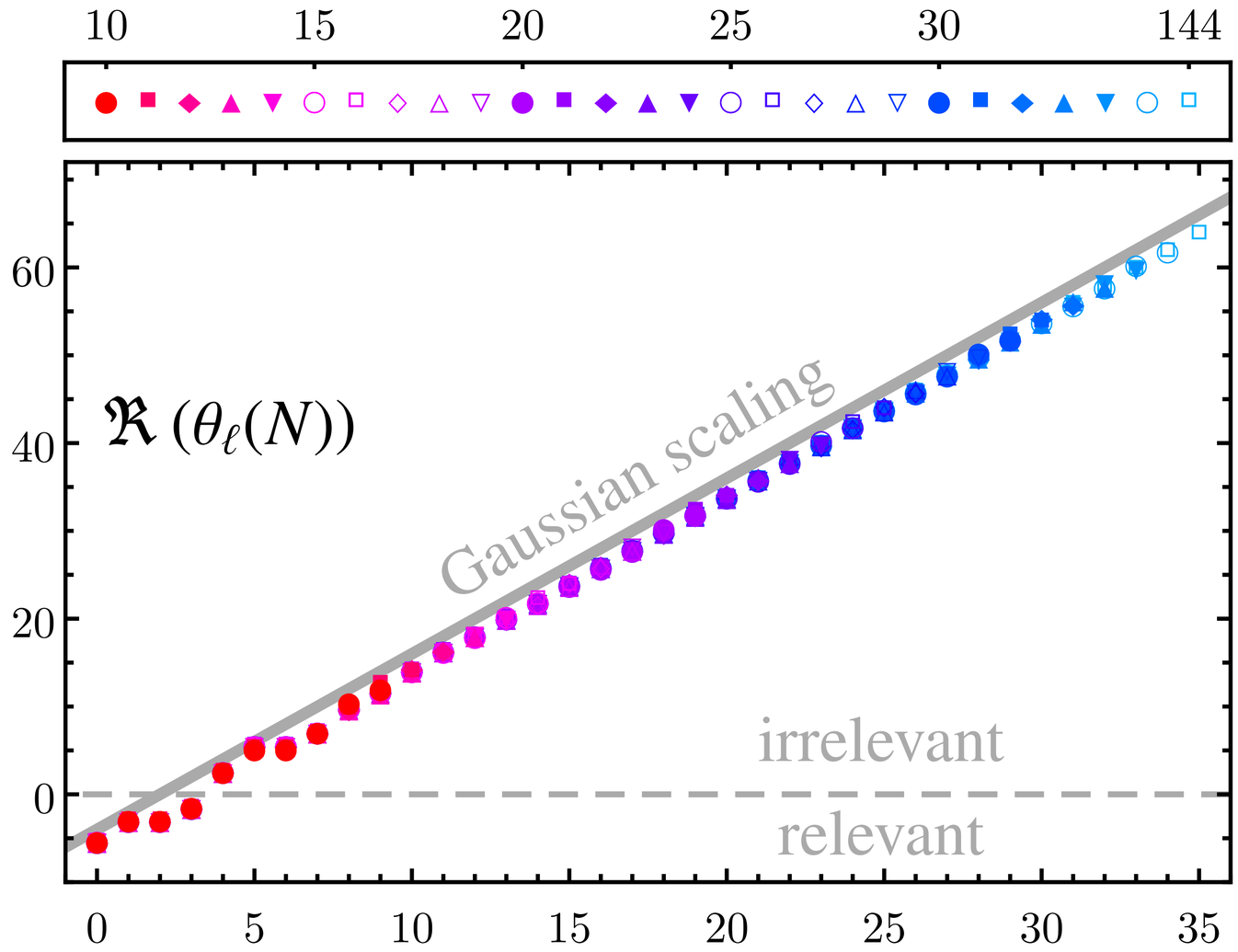
Near-Gaussian Scaling in $f(R)$ Gravity



[2018, Falls and Litim and Schröder]

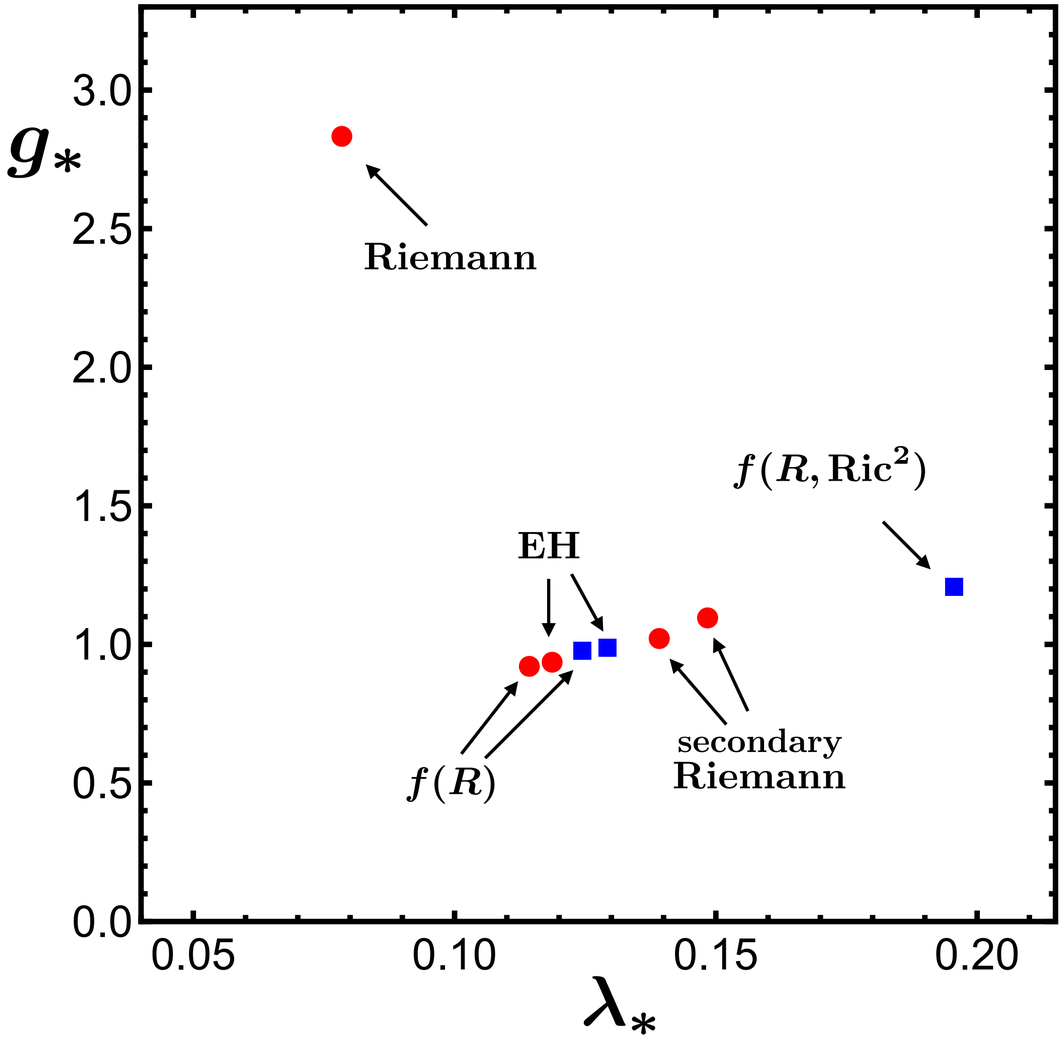
- Eigenvalues of $f(R)$ gravity follow near Gaussian scaling with small quantum induced shifts

Near-Gaussian Scaling of $f(R, \text{Riem}^2)$ Gravity



- Compared to $f(R)$ eigenvalues are shifted towards relevant directions
- Near Gaussian scaling seems to be a universal property for different fixed points

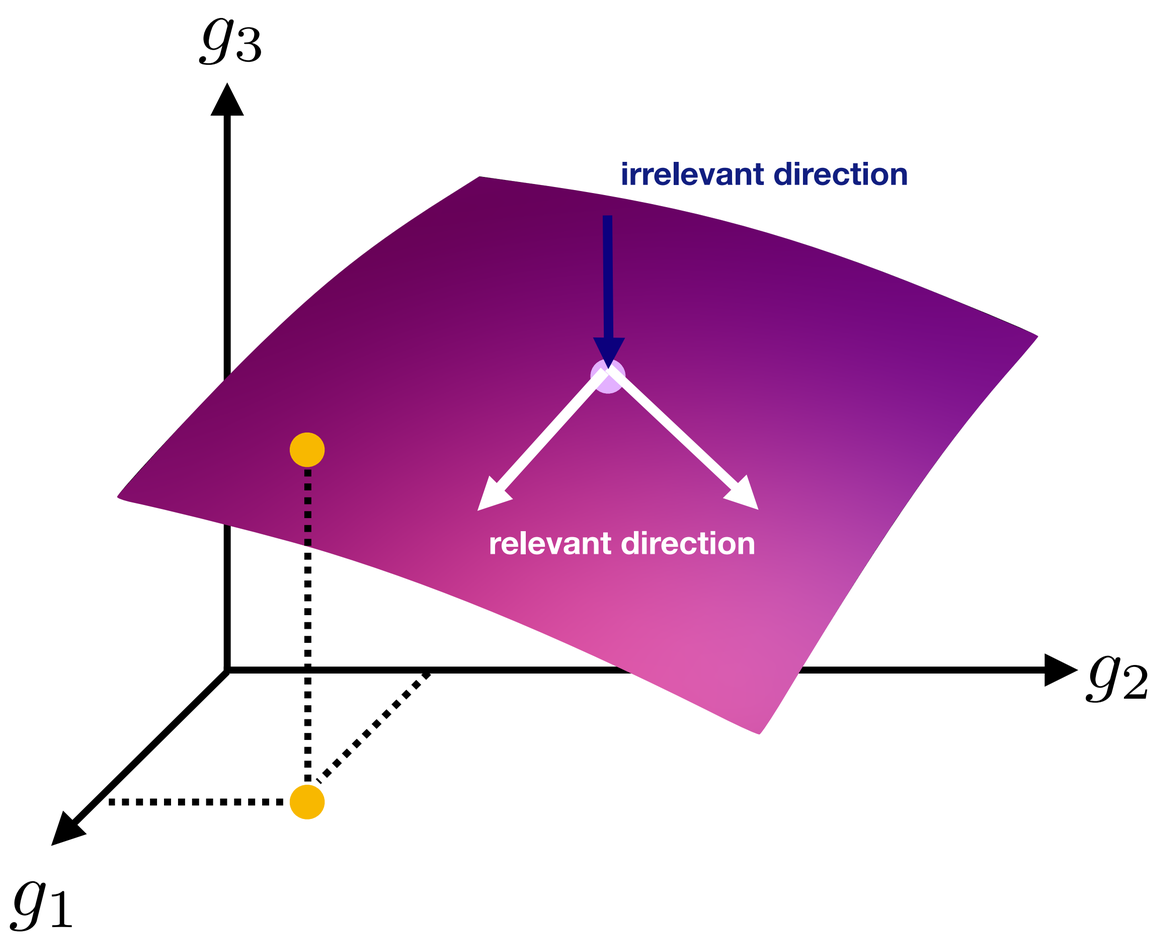
Comparison



- Shown are all higher order extensions which have been analysed in QG
- Inclusion of different higher curvature invariants shift fixed points away from Einstein Hilbert results
- Riemann fixed points only fixed points admitting a fourth relevant direction

Eigenvectors in Asymptotic Safety

Eigenvectors in Asymptotic Safety

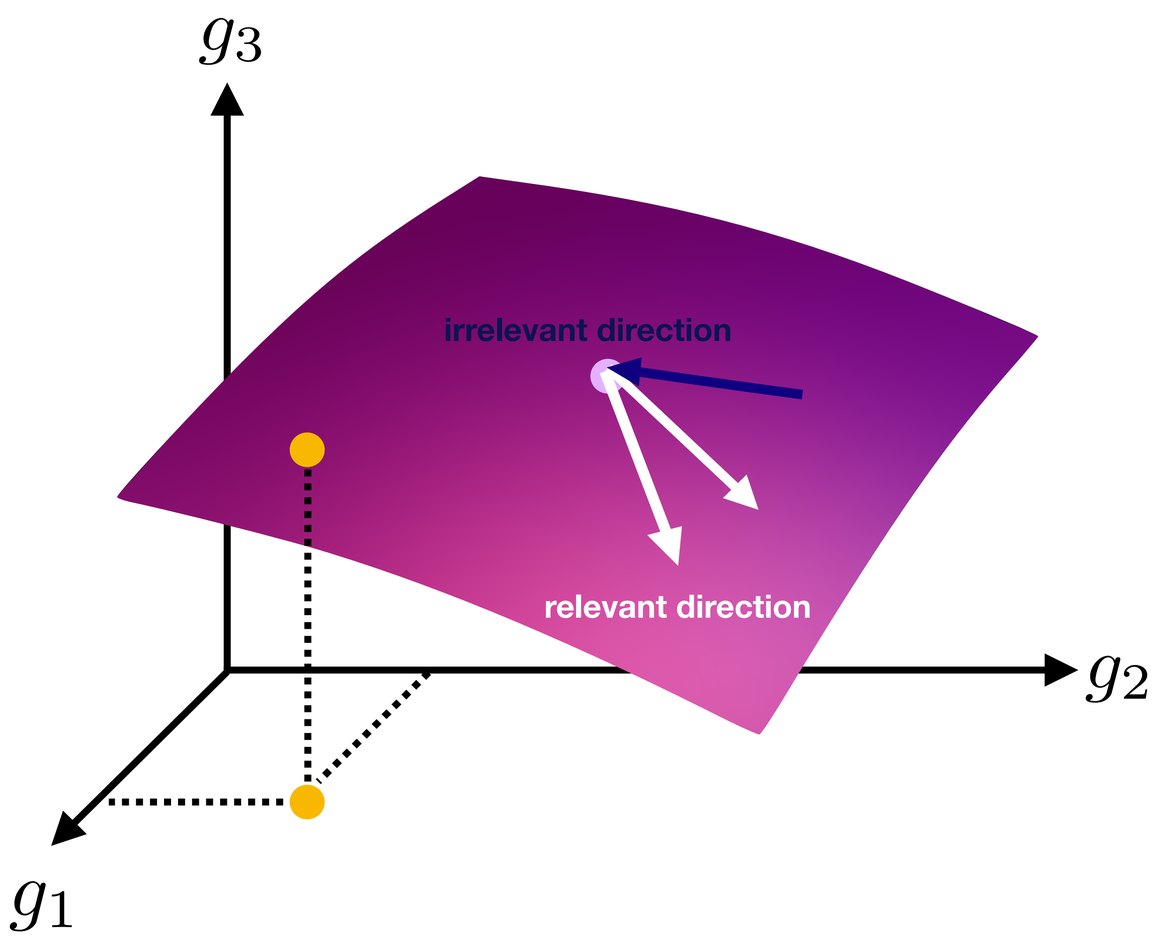


- Eigenvectors parametrise flow close to fixed point

$$\mathscr{M}_{ij} v_j^{(\ell)} = \theta_\ell v_i^{(\ell)}$$

- Which linear combination of operators lead to relevant directions?

Rescaling Ambiguity



- Eigenvectors are not invariant under rescaling of couplings
- Idea: Choose rescaling of couplings to make eigenvectors as orthogonal as possible
  - Can be implemented using equal weight condition

$$\sum_{i=0}^{N-1} \left| v_i^{(\ell)} \right|^2 = 1 \quad \forall \ell$$

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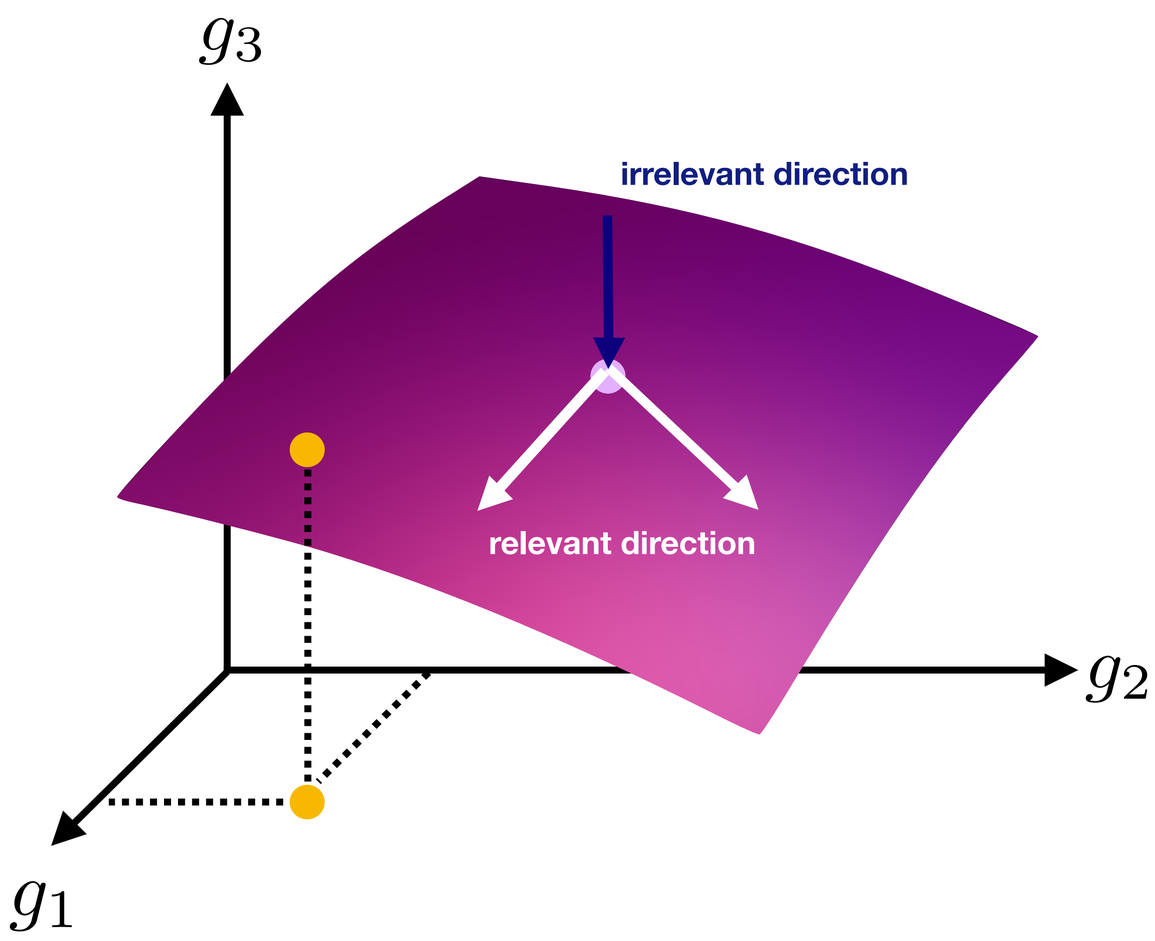
Rescaling Ambiguity



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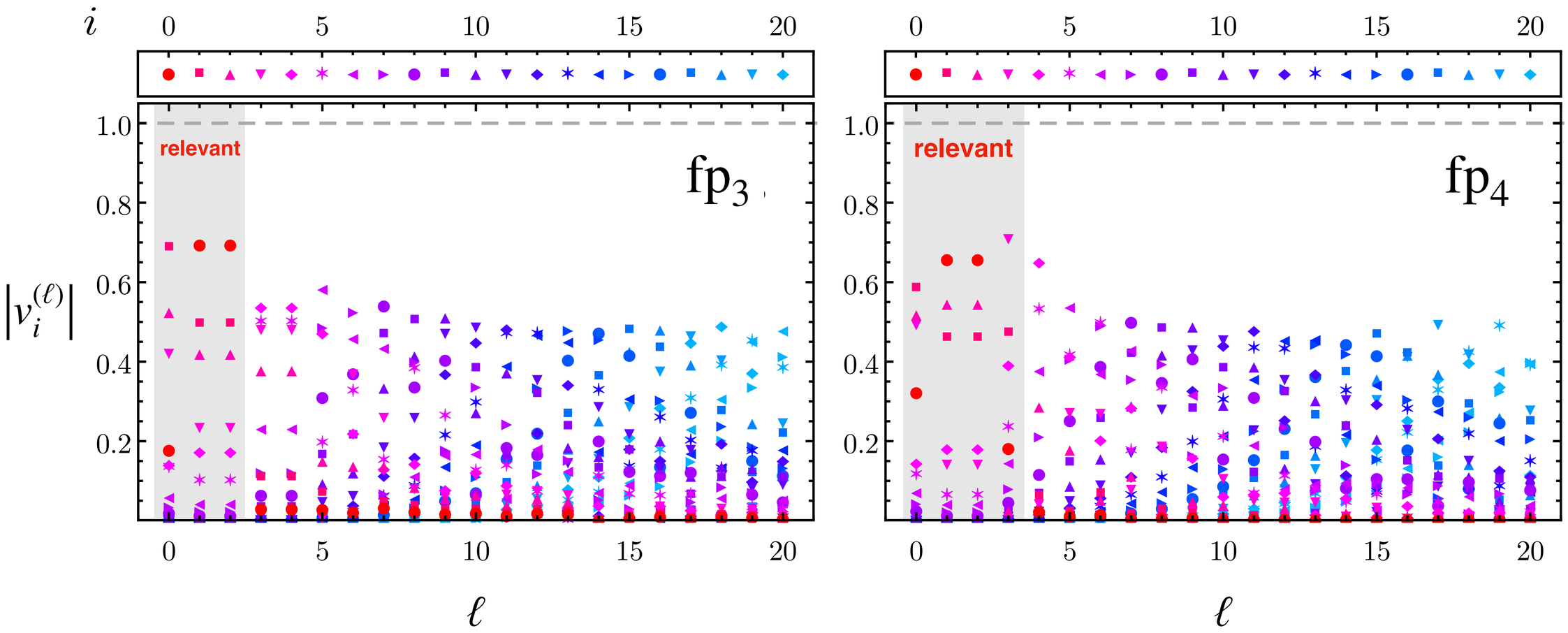
Equal Weight Eigenvectors



Summary & Outlook

Summary

- Asymptotic Safety provides a possibility to render quantum gravity predictive at all energy scales
- Flow Equation for three parameter set of gravitational actions
  - Possibility to study many further actions
  - Three fixed points in $f(R, \text{Riem}^2)$
- Rieman interactions can generate a fourth relevant direction
- **Higher curvature invariants can have non-trivial effects on fixed points in quantum gravity**
- Eigenvalues follow near-Gaussian scaling
- Eigenvectors are useful to understand how relevant/irrelevant eigenvectors are created

Outlook

- Can other dim-6 curvature invariants lead to a fourth relevant direction?
- Technical improvements (More general backgrounds, more general parametrisation, …)
- Effects of Matter?
- Behaviour of eigenvectors in different theories?

Thanks for listening!