KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

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Aspects of dark matter long-range interactions at finite temperature
(Part II)

Tobias Binder

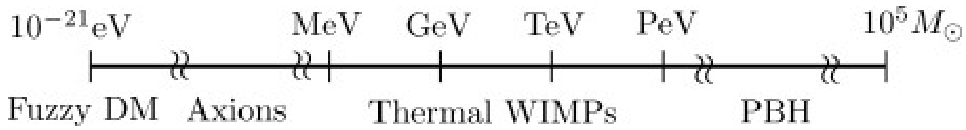
based on

1808.06472, **1910.11288**, **2002.07145,**

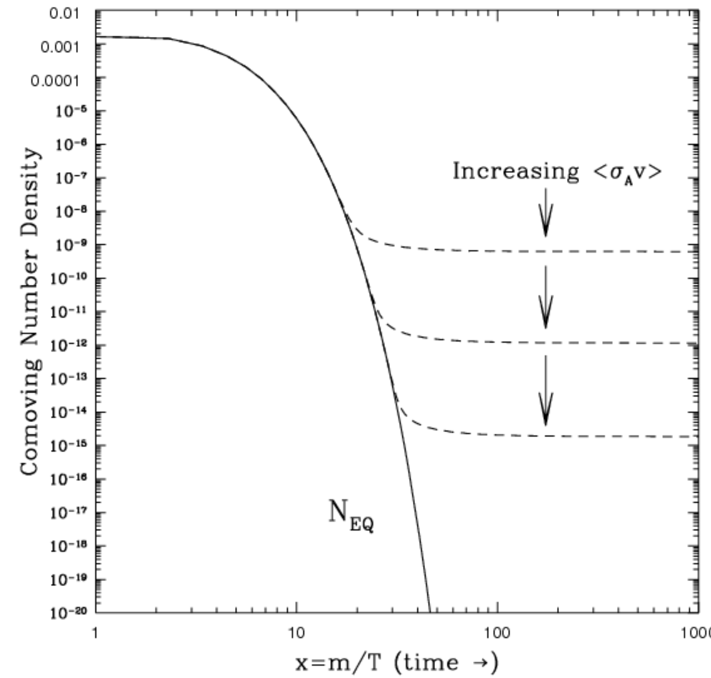
in collaboration with

Burkhard Blobel, Laura Covi, Julia Harz, Kyohei Mukaida (Part I) and **Kalliopi Petraki.**

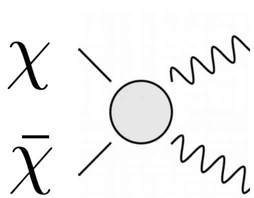
Motivation



- Strong constraints put many MeV-TeV mass realizations in thermal WIMP scenarios under tension.
- TeV-scale and above still remains attractive and much less constrained.
- Prediction of heavy thermal relics and indirect signals requires inclusion of long-range effects.

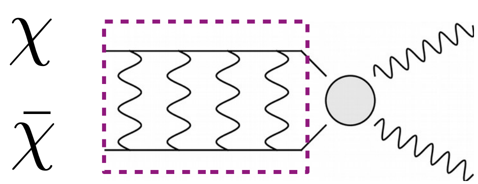


"Light WIMPs"

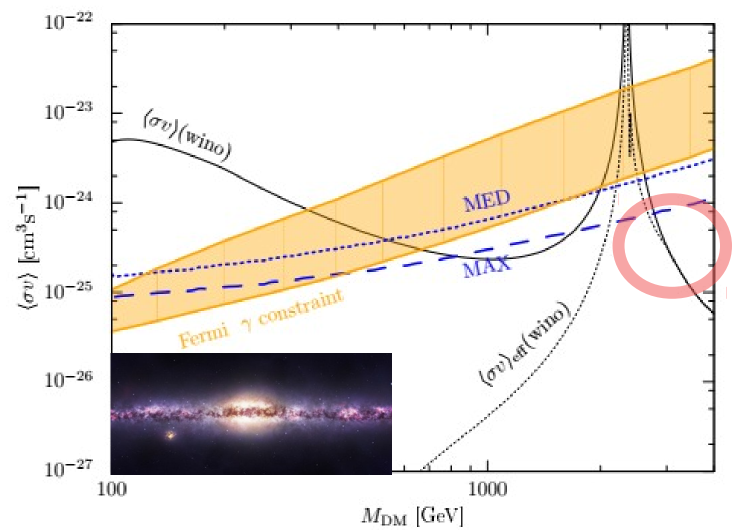


$m_{\text{mediator}} \gg \alpha m_\chi$

"Heavy WIMPs"



$m_{\text{mediator}} \lesssim \alpha m_\chi$



- Experimental probes sensitive to predicted WIMP mass
- Can finite temperature effects modify prediction?

Overview

1 Introduction: long-range force effects in vacuum

- Sommerfeld-enhanced annihilation and bound-state decay (Nonrelativistic EFT)
- Bound-state formation (Potential nonrelativistic EFT)

2 High-to-intermediate temperature regime

- Number density equation from Keldysh-Schwinger formalism
- Effective in-medium potential for SE and BS decay
- *Melting* of bound states

1808.06472

3 Intermediate-to-low temperature regime

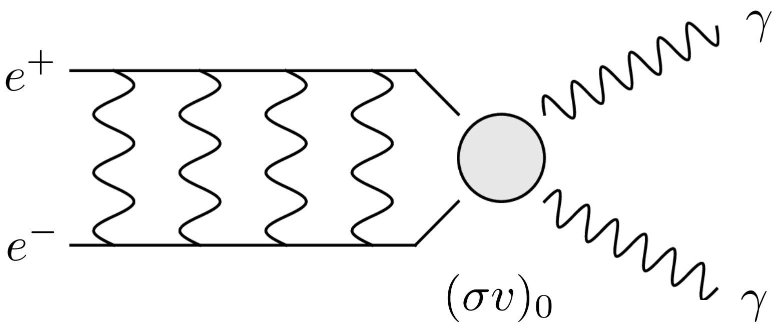
- Number density equation from density-matrix formalism
- Bound-state formation via bath-particle scattering

1910.11288, 2002.07145

4 Impact on thermal relic abundance

Quantum mechanical effects: Positronium

➢ Bound-state decay and Sommerfeld-enhanced annihilation:



Nonrelativistic description allows to factorize long- and short-wavelength parts.

(for s-wave)

$$\Gamma = (\sigma v)_0 \times |\psi_{E<0}(r=0)|^2$$

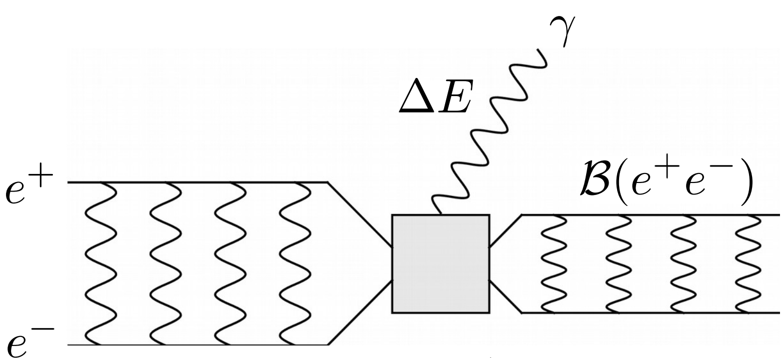
J. Pirenne 1946,
J. Wheeler 1946

$$(\sigma v) = (\sigma v)_0 \times |\psi_{E>0}(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v)\,, \text{ for } v \lesssim \alpha.$$

A. Sakharov 1948
(A. Sommerfeld 1931)

Nonrelativistic EFT: Caswell & Lepage 1986

➢ Bound-state formation:



$$\langle \psi_{E>0} | \, g\vec{r}\cdot\vec{E} \, | \psi_{E<0} \rangle$$

Electric dipole transitions

Capture into ground state nlm=100 via on-shell photon emission:

$$(\sigma v)^{\text{bsf}}_{100} \sim 3 \times (\sigma v)^{\text{ann}}, \text{ for } v \ll \alpha$$

Potential Nonrelativistic EFT:
Brambilla, Pineda, Soto & Vairo 2000

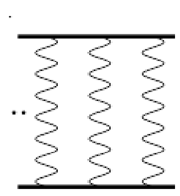
Quantum mechanical effects: dark matter

SM mediated:

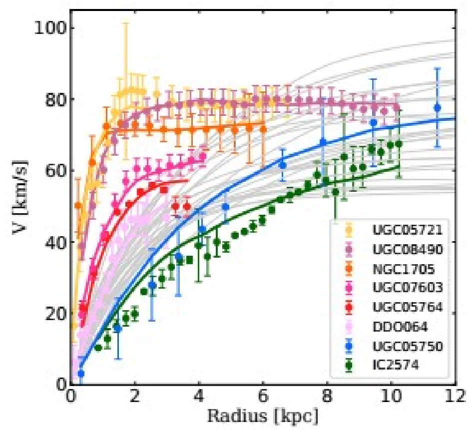
- EW charged DM, Minimal DM
  Hisano *et al.* '03, '05, '06, Cirelli *et al.* '07,..., Hryczuk *et al.* '10+, ..., Mitridate *et al.* '17,...
- Colored coannihilation
  J. Ellis *et al.* '16, Kim&Laine '17, Harz&Petraki '18, S. Biodini *et al.* '19+,...
- Higgs mediated bound states
  Harz&Petraki '18, S. Biodini '18+,...
  - ► Covers all SM force-carriers: $W^{\pm}, Z, \gamma, g, H.$

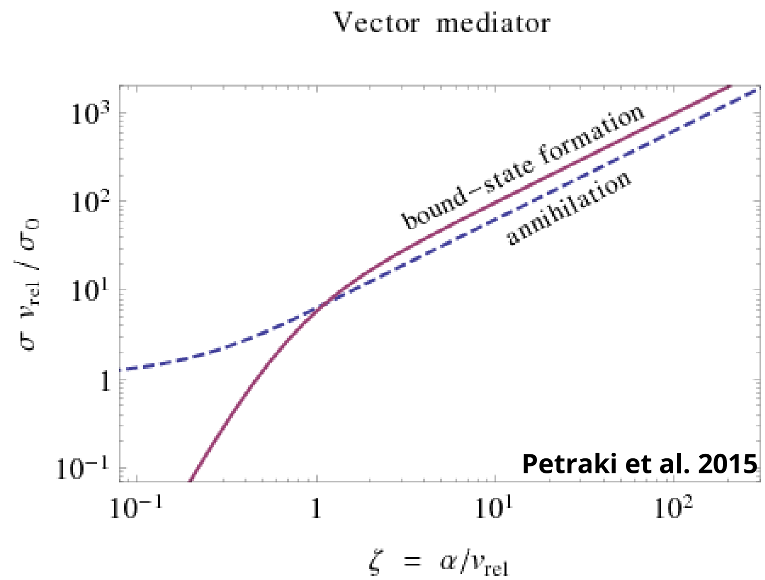
BSM mediated:

- **S**elf-**I**nteracting **DM** with new light mediators
  von Harling&Petraki *'14*, ... [many]

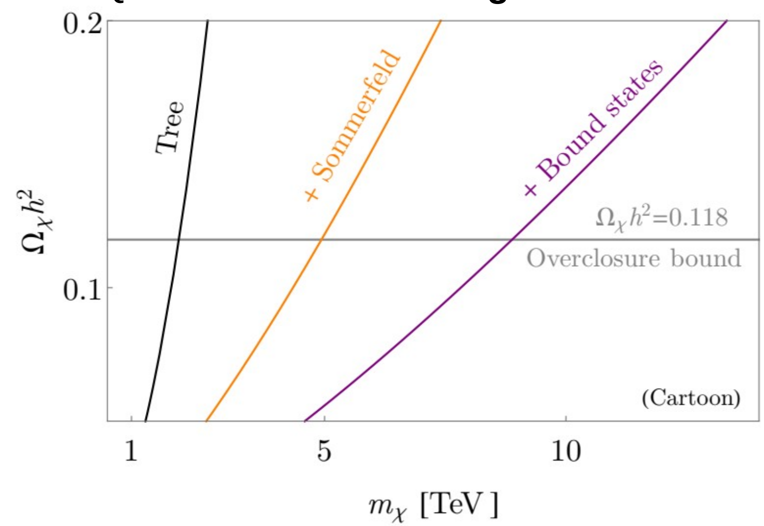


- ► **SIDM can solve Diversity problem**
  Kamada *et al.* '16,...., Kaplinghat *et al.* '19

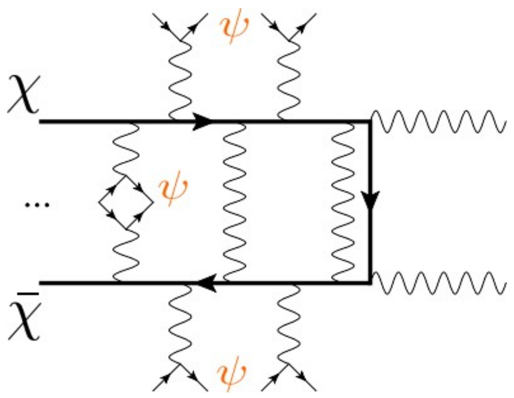




QM effects allow for **larger DM masses**:



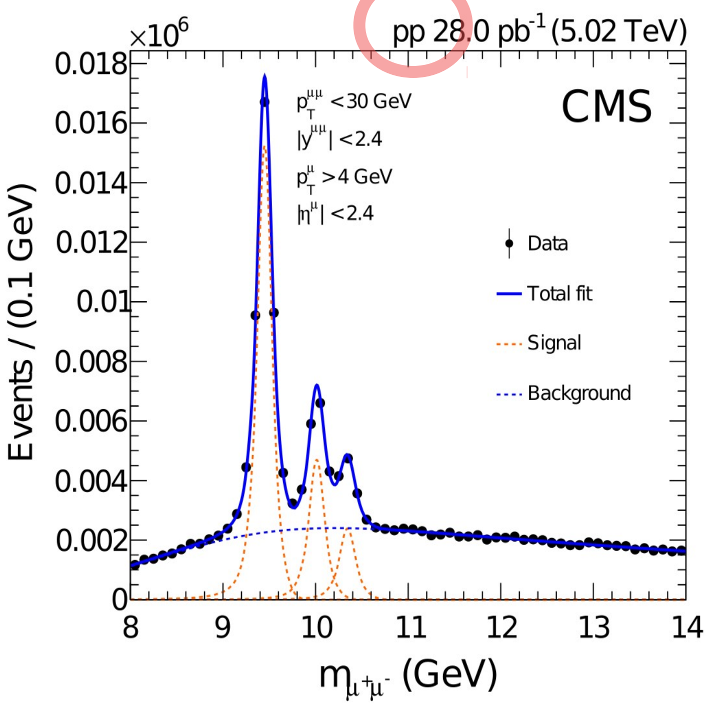
High-to-intermediate temperature regime

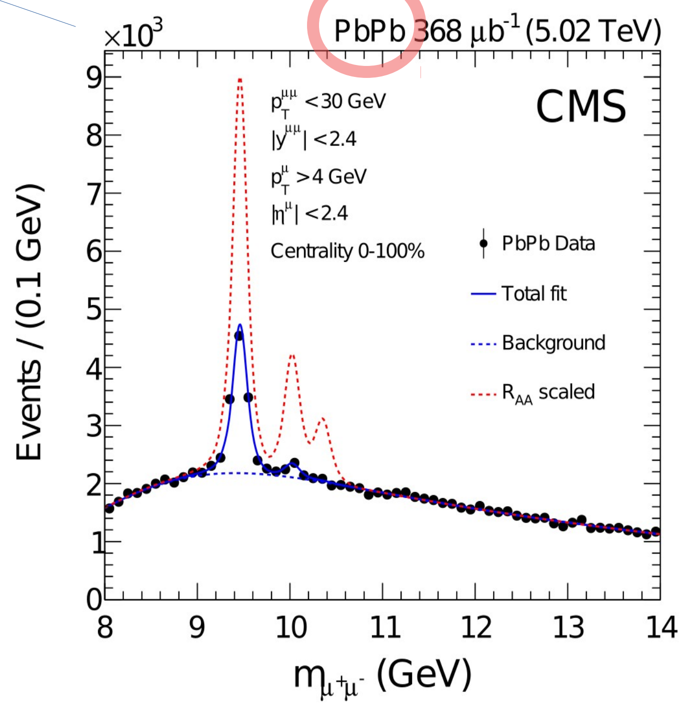


Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

What do we expect?

- Nice analogy between heavy DM in the primordial plasma and heavy quarkonia in quark-gluon plasma.





CMS collaboration,
Phys.Lett. **B790** (2019) 270-293

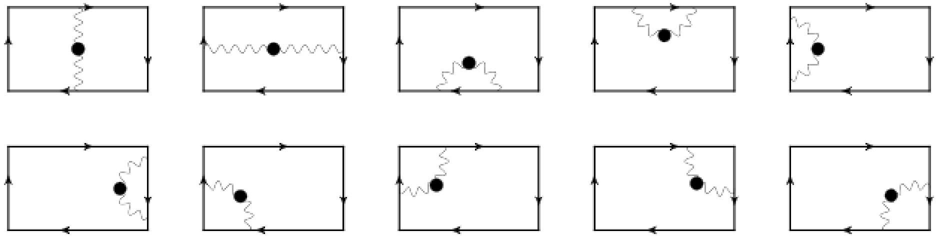
Sequential melting of b-bbar bound states inside QGP plasma observed.

Historical remarks

Heavy Quarkonia annihilation/decay in QGP plasma

- **Matsui & Satz 1986: J/Psi suppression in QGP due to screening effect**
  - In-medium potential from 2 Polyakov loop (Wilson line)
  - $$\lim_{t\to\infty} V(t,\mathbf{r}) = -\frac{\alpha}{r}e^{-m_D r}$$

- **M. Laine et al. 2007 (seminal)**
  - 4 Polyakov loop method



$$\lim_{t\to\infty} V(t,\mathbf{r}) = -\alpha m_D - \frac{\alpha}{r}e^{-m_D r} - i\alpha T\phi(m_D r)$$

Salpeter correction

Thermal width

Dark Matter Sommerfeld-enhanced annihilation/decay in primordial plasma

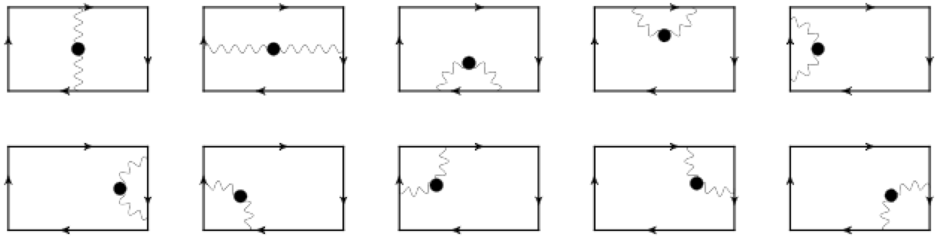
- **Cirelli et al. 2007 (Minimal DM)**
  - (only) Debye screening included
  - Wino mass lowered (?)

- **Bödeker & Laine 2012**
  - **For quarkonia, spectral information sufficient. For DM, we need dynamics!**
  - Linear response "*matched*" to Boltzmann equation in the linear regime close to chemical equilibrium
  - Formalism allows to include thermal corrections to Sommerfeld enhanced annihilation and bound state decay
  - See also follow-ups by Biondini, Kim, Laine

Previous literature in more detail

1) **Effective in-medium potential** **[M. Laine *et al.* ‘07]**

extraced from Euclidian 4-Polyakov loop (Wilson lines) method



$$\Rightarrow \lim_{t\to\infty} V(t,\mathbf{r}) \Rightarrow \rho(E)$$

2) **Number density equation** **[Bödeker&Laine ‘12]**

$$\dot{\delta n} = -\Gamma_{\rm chem}\delta n(t) + \xi(t) \ \ \text{(Langevin equation)}$$

matching $\Gamma_{\rm chem}$

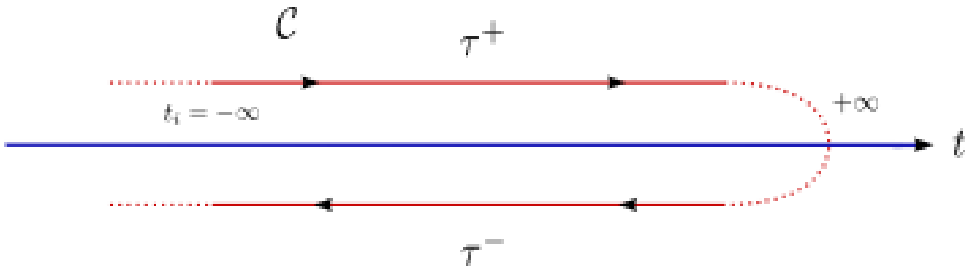
$$\dot{n} + 3Hn = -\Gamma_{\rm chem}(n - n_{\rm eq}) + \mathcal{O}([n - n_{\rm eq}]^2) \ \ \text{(linearized BE)}$$

$$\dot{n} + 3Hn = -\frac{\Gamma_{\rm chem}}{2n_{\rm eq}}(n^2 - n_{\rm eq}^2) \ \ \text{(quadratic BE)}$$

CTP-formalism approach

Non-equilibrium QFT ⇒ In-medium potential + dynamics in one formalism

$$G_O(x,y) \equiv \langle T_{\mathcal{C}} O(x) O^\dagger(y) \rangle$$
$$\langle ... \rangle = \mathrm{Tr}[\rho ...]$$



Scheme overview:

I Non-relativistic effective action

II Number density equation from EoM of 2-point correlation functions

III Resummation scheme of 4-point correlator and effective potential

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Scheme

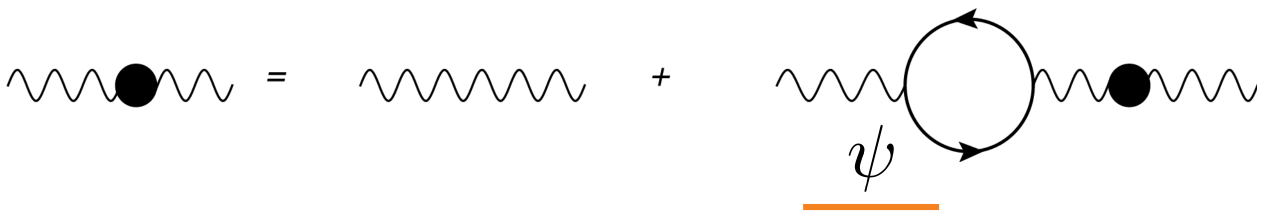
$$\mathcal{L} \supset g\bar{\chi}\gamma^{\mu}\chi A_{\mu} + \underline{g\bar{\psi}\gamma^{\mu}\psi A_{\mu}}$$

I Non-relativistic effective action on CTP-contour

Essentially the same as in vacuum

$$S_{\rm NR}[\eta,\xi] = \int_{x\in\mathcal{C}} \eta^{\dagger}\left[i\partial_t + \frac{\Delta}{2M}\right]\eta + \xi^{\dagger}\left[i\partial_t - \frac{\Delta}{2M}\right]\xi + \int_{x,y\in\mathcal{C}} i\frac{g^2}{2}\underbrace{J(x)D(x,y)J(y)}_{\text{“potential”}} + i\underbrace{O^{\dagger}(x)\Gamma(x,y)O(y)}_{\text{“annihilation”}},$$

- Separation of short- and long-range contributions, $J \equiv \eta^{\dagger}\eta + \xi^{\dagger}\xi, O \equiv \xi^{\dagger}\eta$
- Correlator $D(x,y) \equiv \langle T_{\mathcal{C}} A_0(x) A_0(y)\rangle$ contains information of the bath



Scheme

II Number density equation from EoM of two-point function

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 \left[G^{++--}_{\eta\xi}(x,x,x,x) - G^{++--}_{\eta\xi}(x,x,x,x)\Big|_{\rm eq} \right].$$

$$n_\eta(x) \equiv \langle \eta^\dagger(x)\eta(x)\rangle,\ G_{\eta\xi}(x,y,z,w) \equiv \langle T_{\mathcal{C}}\eta(x)\xi^\dagger(y)\xi(w)\eta^\dagger(z)\rangle$$

Free limit: $2G^{++--}_{\eta\xi}(x,x,x,x) \simeq n_\eta n_\xi \longrightarrow$

$$\dot{n}_\eta + 3Hn_\eta = -(\sigma v)_0 \left[n_\eta n_\xi - n_\eta^{\rm eq} n_\xi^{\rm eq} \right]$$

Lee-Weinberg equation ✓

- Treat annihilation as perturbation
- Assume grand canonical state $\rho \propto e^{-\beta(\hat{H}-\mu_\eta \hat{N}_\eta - \mu_\xi \hat{N}_\xi)}$ and use KMS relation:

$$G^{++--}_{\eta\xi}(x,x,x,x) = e^{-2\beta(M-\mu)} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^\rho_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

Compute spectral function from retarded correlator: $G^\rho_{\eta\xi} = 2\Im[iG^R_{\eta\xi}]$

Scheme

After truncation:

$$G_{\eta\xi} = G_\eta G_\xi + g_\chi^2 \int_{\mathcal{C}} G_\eta G_\xi D G_{\eta\xi}$$

"Bethe-Salpeter equation"

3-time problem, KMS ?, no thermal width appears in static limit.

Instead, resummation of **ALL** contributions to order g_χ^2

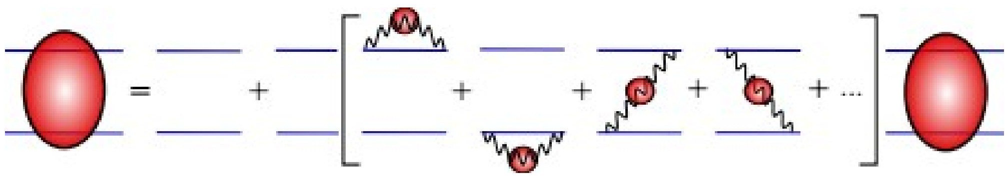
$$G_\eta = G_\eta^0 + g_\chi^2 \int_{\mathcal{C}} G_\eta^0 \Sigma_\eta G_\eta^0$$

$$G_{\eta\xi} = G_\eta^0 G_\xi^0 + g_\chi^2 \int_{\mathcal{C}} G_\eta^0 G_\xi^0 \left[D G_\eta^0 G_\xi^0 + \Sigma_\eta G_\eta^0 + \Sigma_\xi G_\xi^0 \right]$$

No 3-time problem, resummation scheme respects
KMS condition, reproduces 4-Polyakov result...

Scheme

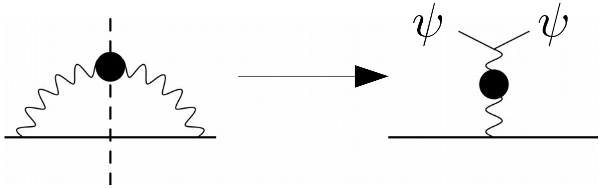
III Resummation and effective potential



Retarded component is of special interest:

$$\left[\frac{\nabla_{\mathbf{r}}^2}{M} + E + i\epsilon - V_{\rm eff}(\mathbf{r},T)\right] G^R_{\eta\xi}(\mathbf{r},\mathbf{r}';E) = 2i\delta(\mathbf{r}-\mathbf{r}')$$

$$G^\rho_{\eta\xi} = 2\Im[iG^R_{\eta\xi}]$$



Soft bath particle scattering aka “Landau damping” leads to thermal width.

In static limit and Hard-Thermal-Loop approximation:

$$\boxed{V_{\rm eff}(\mathbf{r},T) \equiv -ig_\chi^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3}(1-e^{i\mathbf{pr}})D^{++}(0,\mathbf{p}) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r}e^{-m_D r} - \underline{i\alpha_\chi T\phi(m_D r)}}$$

consistent with **[M. Laine *et al.* ‘07]**

Independent derivation.

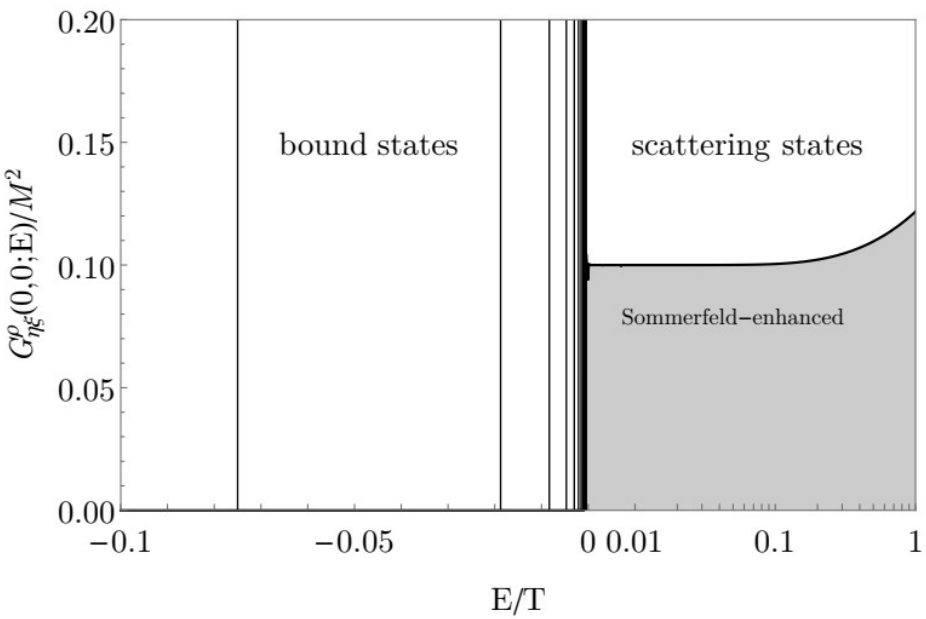
Phenomenology: vacuum limit

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\text{eq}}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\text{eq}} = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$\lim_{T\to 0} V_{\text{eff}}(r,T)$$

Coulomb potential, M=5TeV, α_χ=0.1, T=M/30



$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_{\mathcal{B}} \Gamma_{\mathcal{B}} \frac{n^{\text{eq}}_{\mathcal{B}}}{n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}}}\right)\left[(\alpha n)^2 - n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}}\right]$$

BEs in (Saha) ionization equilibrium ✓

$$\boxed{\frac{n_\chi n_{\bar{\chi}}}{n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}}} = \frac{n_{\mathcal{B}}}{n^{\text{eq}}_{\mathcal{B}}}}$$

That's one of the limitation!

$$(\sigma v)_0 G^{\rho}_{\eta\xi}(E)|_{E>0} \propto E^{1/2}(\sigma v)_0 |\psi(0)|^2$$

$$(\sigma v)_0 G^{\rho}_{\eta\xi}(E)|_{E<0} \propto \sum_n \delta(E - E_n)\Gamma_n$$

Phenomenology: finite temperature

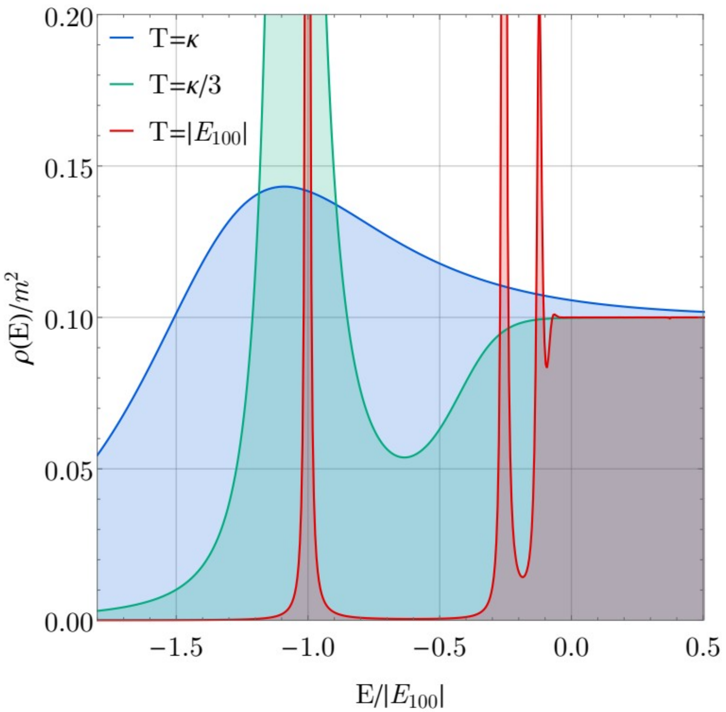
$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{d^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty}\frac{dE}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$\text{full } V_{\rm eff}(r,T) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r} e^{-m_D r} - i\alpha_\chi T\phi(m_D r)$$

1 **high** α **intermediate** α^2 **low** T/m_χ

Finite temperature



DM bound states melted around freeze-out T

Advantage

- Everything included
- Finite temperature corrections

Limitation

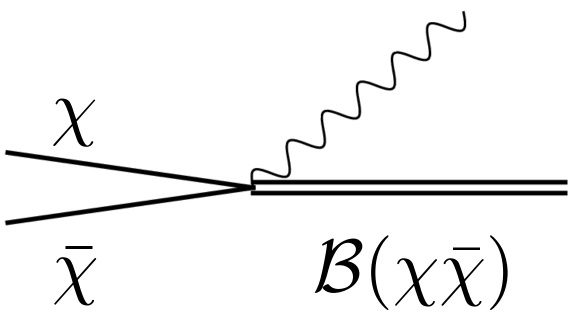
- HTL resummation
- Ionization equilibrium
- **When does the system depart from ionization equilibrium?**

Relevance

- Compared to SE only, corrections are relevant
- Compared to SE+bound states in vacuum, corrections might be less relevant. ("Probability conservation" ?)

Intermediate-to-low temperature regime

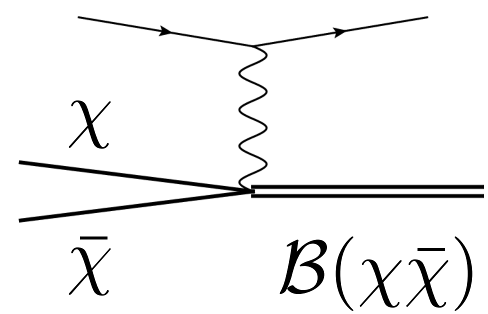
BSF via on-shell mediator emission



(previous literature)

vs.

BSF via bath-particle scattering



(similar diagram leads to bound-state melting)

3.) Which process dominates?

2.) Cancellation of collinear divergences?

1.) How can we systematically compute higher order BSF processes?

Number density equation

$$\mathcal{L} \supset g\bar{\chi}\gamma^{\mu}\chi A_{\mu} + \mathcal{L}_{\text{environment}}$$

Dipole Operator (pNREFT):

$$H_{\text{dip}}(t) = -\sum_{\text{Spin}} \int \mathrm{d}^3x \mathrm{d}^3r \, O^{\dagger}_{sr}(\mathbf{x},\mathbf{r},t) \left[g \, \mathbf{r} \cdot \mathbf{E}(\mathbf{x},t) \right] O_{sr}(\mathbf{x},\mathbf{r},t), \text{ where}$$

$$O_{sr}(\mathbf{x},\mathbf{r},t) = \int \frac{\mathrm{d}^3K}{(2\pi)^3} \left\{ \sum_{\mathcal{B}} e^{-i(\mathcal{E}_{\mathcal{B}}t - \mathbf{K}\cdot\mathbf{x})} \psi_{\mathcal{B}}(\mathbf{r}) \hat{c}^{sr}_{\mathcal{B},\mathbf{K}} + \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{-i(\mathcal{E}_{\mathbf{k}}t - \mathbf{K}\cdot\mathbf{x})} \psi_{\mathbf{k}}(\mathbf{r}) \hat{a}^{s}_{\mathbf{K}/2+\mathbf{k}} \hat{b}^{r}_{\mathbf{K}/2-\mathbf{k}} \right\}.$$

Density matrix formalism:

$$\begin{aligned} \dot{n}_{\chi} &= \frac{1}{\mathrm{vol}(\mathbb{R}^3)} \int \frac{\mathrm{d}^3k_{\chi}}{(2\pi)^3} \mathrm{Tr}\left[\dot{\hat{\rho}}_I \hat{n}_{\mathbf{k}_{\chi}} \right] \\ &\simeq -\frac{1}{\mathrm{vol}(\mathbb{R}^3)} \int \frac{\mathrm{d}^3k_{\chi}}{(2\pi)^3} \lim_{t\to\infty} \int_0^t \mathrm{d}t' \mathrm{Tr}\left\{ \left[\left[\hat{n}_{\mathbf{k}_{\chi}}, \hat{H}_I(t) \right], \hat{H}_I(t') \right] \hat{\rho}(t=0) \right\}. \end{aligned}$$

Computation of double commutator for dipole Hamiltonian gives:

$$\dot{n}_{\chi} + 3Hn_{\chi} = -\sum_{\mathcal{B}} \langle \sigma^{\text{BSF}}_{\mathcal{B}} v_{\text{rel}} \rangle \left[n_{\chi} n_{\bar{\chi}} - n_{\mathcal{B}} n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}} / n^{\text{eq}}_{\mathcal{B}} \right] + \text{annihilation}$$

$$\dot{n}_{\mathcal{B}} + 3Hn_{\mathcal{B}} = +\langle \sigma^{\text{BSF}}_{\mathcal{B}} v_{\text{rel}} \rangle \left[n_{\chi} n_{\bar{\chi}} - n_{\mathcal{B}} n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}} / n^{\text{eq}}_{\mathcal{B}} \right] + \text{decay}$$

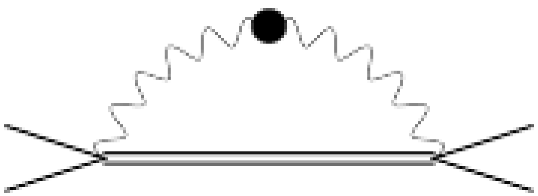
$$\boxed{\sigma^{\text{BSF}}_{nlm} v_{\text{rel}} = \int \frac{\mathrm{d}^3p}{(2\pi)^3} D^{-+}_{\mu\nu}(\Delta E, \mathbf{p}) \mathcal{T}^{\mu}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}) \mathcal{T}^{\nu\star}_{\mathbf{k},nlm}(\Delta E, \mathbf{p}).}$$

BSF efficient:

$$\boxed{\Rightarrow \frac{n_{\chi} n_{\bar{\chi}}}{n^{\text{eq}}_{\chi} n^{\text{eq}}_{\bar{\chi}}} = \frac{n_{\mathcal{B}}}{n^{\text{eq}}_{\mathcal{B}}}}$$

Ionization equilibrium

Generalized bound-state formation cross section



$$\sigma_{nlm}^{\mathrm{BSF}} v_{\mathrm{rel}} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\mu}(\Delta E, \mathbf{p}) \mathcal{T}_{\mathbf{k},nlm}^{\nu\star}(\Delta E, \mathbf{p}).$$

Mediator two-point correlation fct.:

$$D_{\mu\nu}^{-+}(x,y) \equiv \langle A_\mu(x) A_\nu(y) \rangle$$

$$\langle ... \rangle = \mathrm{Tr}[e^{-H_{\mathrm{env}}/T} ...]$$

Kubo-Martin-Schwinger relation:

$$D_{\mu\nu}^{-+}(\Delta E, \mathbf{p}) = [1 + f_B^{\mathrm{eq}}(\Delta E)]\, D_{\mu\nu}^{\rho}(\Delta E, \mathbf{p})$$

$$D_{\mu\nu}^{\rho} = 2\Im\left[i D_{\mu\nu}^{R} \right]$$

$$D_{\mu\nu}^{R} = D_{\mu\nu}^{R,0} + D_{\mu\alpha}^{R,0} \Pi_R^{\alpha\beta} D_{\beta\nu}^{R,0} + ...$$

First contact with the plasma

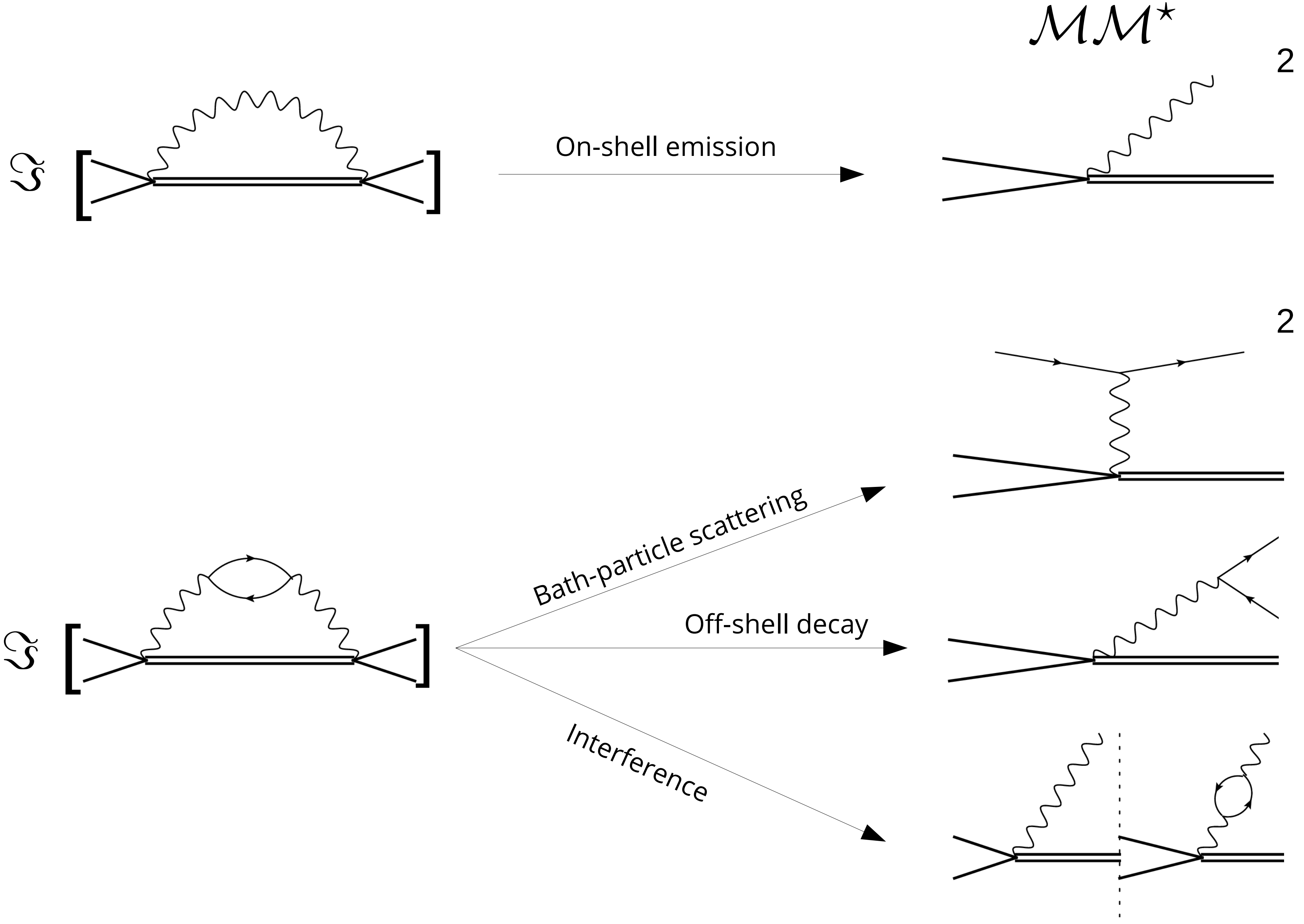
Dipole transition matrix elements:

$$\mathcal{T}_{\mathbf{k},nlm}^{0}(\Delta E, \mathbf{p}) \propto \mathbf{p} \langle \psi_{nlm} | \, \mathbf{r} \, | \psi_{\mathbf{k}} \rangle$$

$$P_\mu \mathcal{T}_{\mathbf{k},nlm}^{\mu}(P) = 0 \text{ (components are related)}$$

Previous literature provides many other types of transition ME as well.

Leading and next-to-leading order

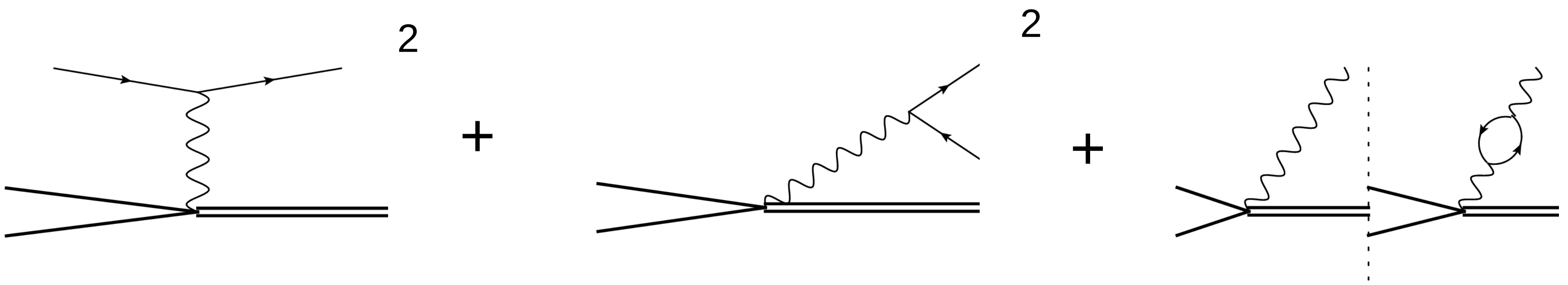


Next-to-leading order in more detail

UV finite,
collinear divergent.

UV finite,
collinear divergent.

Vacuum part UV
divergent,
collinear divergent.



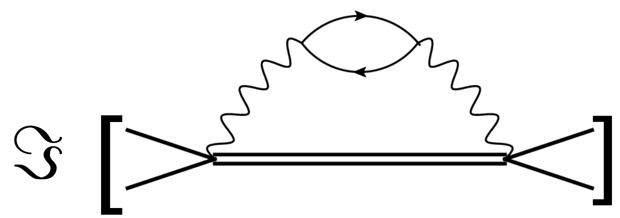
= Finite in collinear direction, and UV finite after vacuum renormalization.

- Provide mathematical proof for cancellation of collinear divergences.
- Holds even for arbitrary phase-space distribution of bath particles,

  i.e. bath particles do not have to be in thermal equilibrium in order

  to guarantee finiteness in the forward scattering direction.
- Bloch-Nordsieck or Kinoshita–Lee–Nauenberg theorem does not help here

Sketch of proof



$$\Im[D^R_{\mu\alpha}(P)\Pi^{\alpha\beta}_R(P)D^R_{\beta\nu}(P)]$$

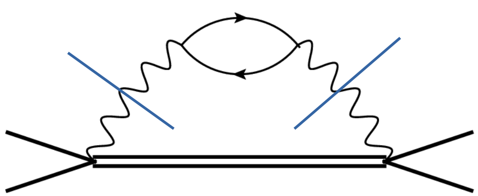
$$G(z) \equiv \frac{H(z)}{(z-z_2)^2(z-z_1)}$$

$$\text{Collinear limit: } z_1 \to z_2$$

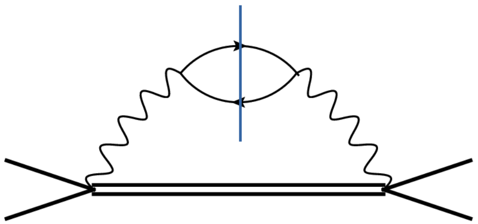
double pole cut

single pole cut

$$=$$



$$+$$



$$\mathrm{Res}(G, z_2) = -\cancel{\frac{H(z_2)}{(z_1-z_2)^2}} - \cancel{\frac{H'(z_2)}{z_1-z_2}}$$

$$\mathrm{Res}(G, z_1) = \frac{H(z_1)}{(z_1-z_2)^2}$$

$$\simeq \cancel{\frac{H(z_2)}{(z_1-z_2)^2}} + \cancel{\frac{H'(z_2)}{z_1-z_2}} + \boxed{\frac{1}{2}H''(z_2)}$$

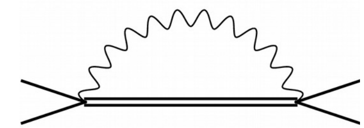
Collinear divergences cancel in the sum!

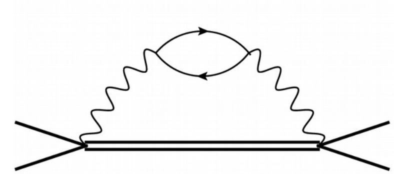
Bound-state formation at NLO: massless case

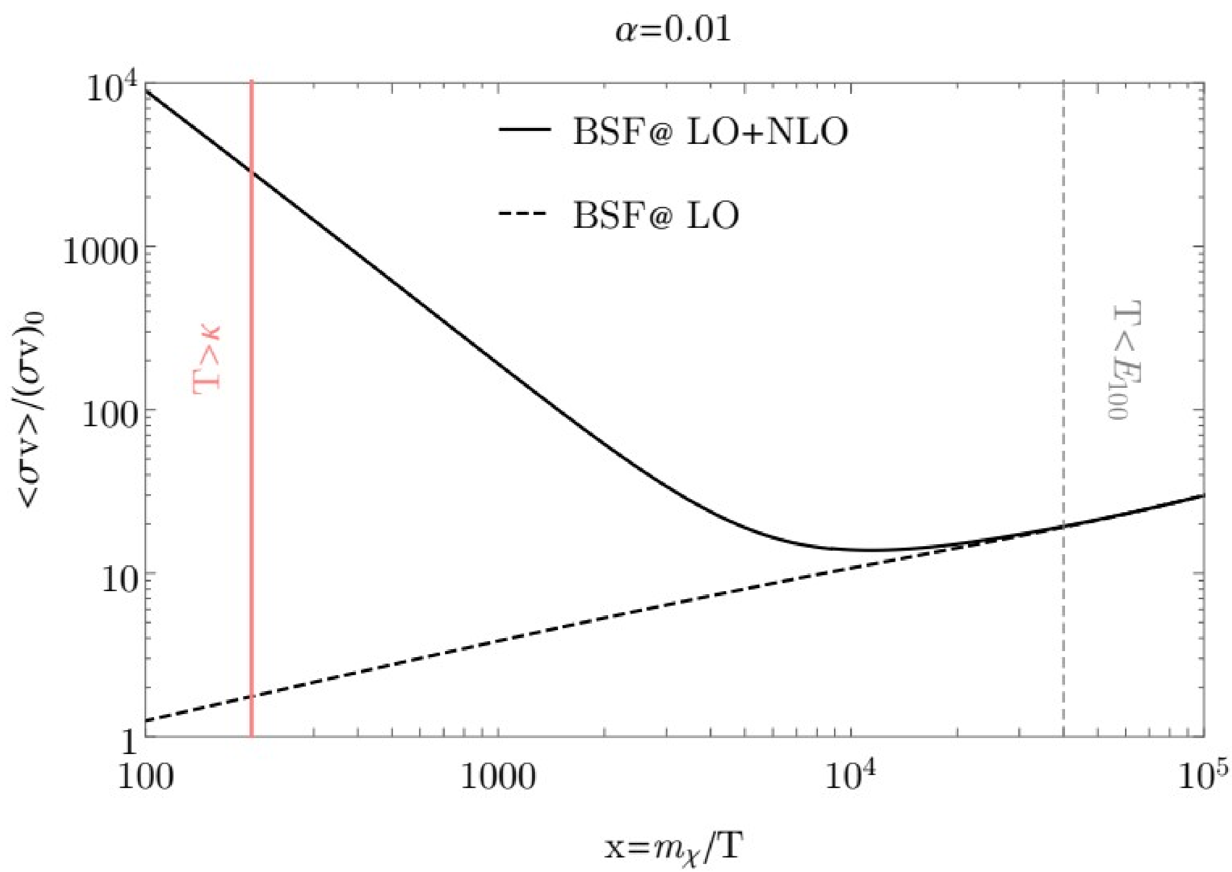
- Interference terms **cancel collinear divergences,** resulting in a finite cross section.

- For T> binding energy **BSF via bath-particle scattering dominates over on-shell mediator emission.**

- Variation of renormalization scale between DM mass and binding energy does not affect plot visually, hence Log-contributions are under control.

$(\sigma v)^{\mathrm{LO}}_{\mathrm{BSF}} \equiv \Im$ []

$(\sigma v)^{\mathrm{NLO}}_{\mathrm{BSF}} \equiv \Im$ []



Impact on thermal relic abundance

BSF via bath-particle scattering: massive case

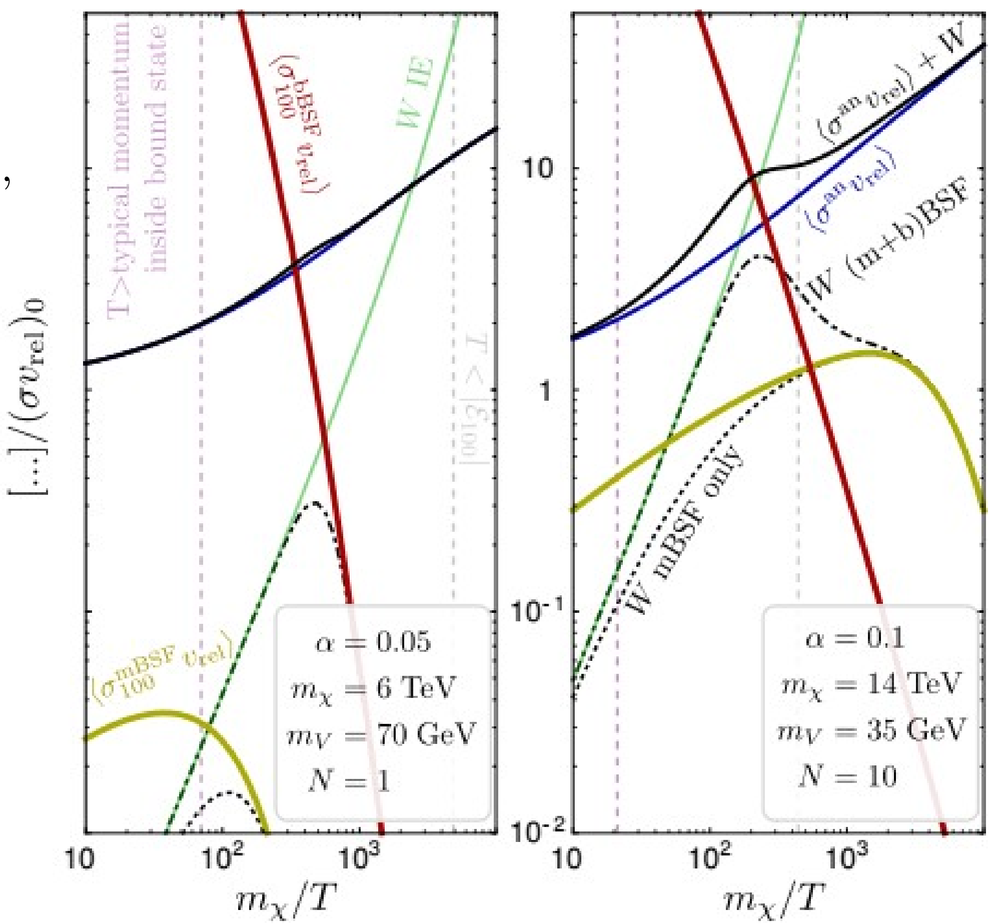
$$\dot{n}_\chi + 3Hn_\chi = -\left[\langle\sigma^{\rm an} v_{\rm rel}\rangle + W(T)\right]\left[n_\chi^2 - (n_\chi^{\rm eq})^2\right],$$

$$W \equiv \langle\sigma_{100}^{\rm BSF} v_{\rm rel}\rangle \frac{(1/4)\,\Gamma^{\rm dec}_{100,S}}{\Gamma^{\rm dec}_{100,S} + \Gamma^{\rm dis}_{100}} + \frac{(3/4)\,\Gamma^{\rm dec}_{100,T}}{\Gamma^{\rm dec}_{100,T} + \Gamma^{\rm dis}_{100}}.$$

$$\Gamma^{\rm dis}_{100} = \langle\sigma^{\rm BSF}_{100} v_{\rm rel}\rangle (n_\chi^{\rm eq})^2 / n^{\rm eq}_{100}$$

Enhanced BSF via bath-particle scattering leads to longer period of **ionization equilibrium,** where **effective depletion cross section W takes maximum value** for fixed T.

no kinematical block

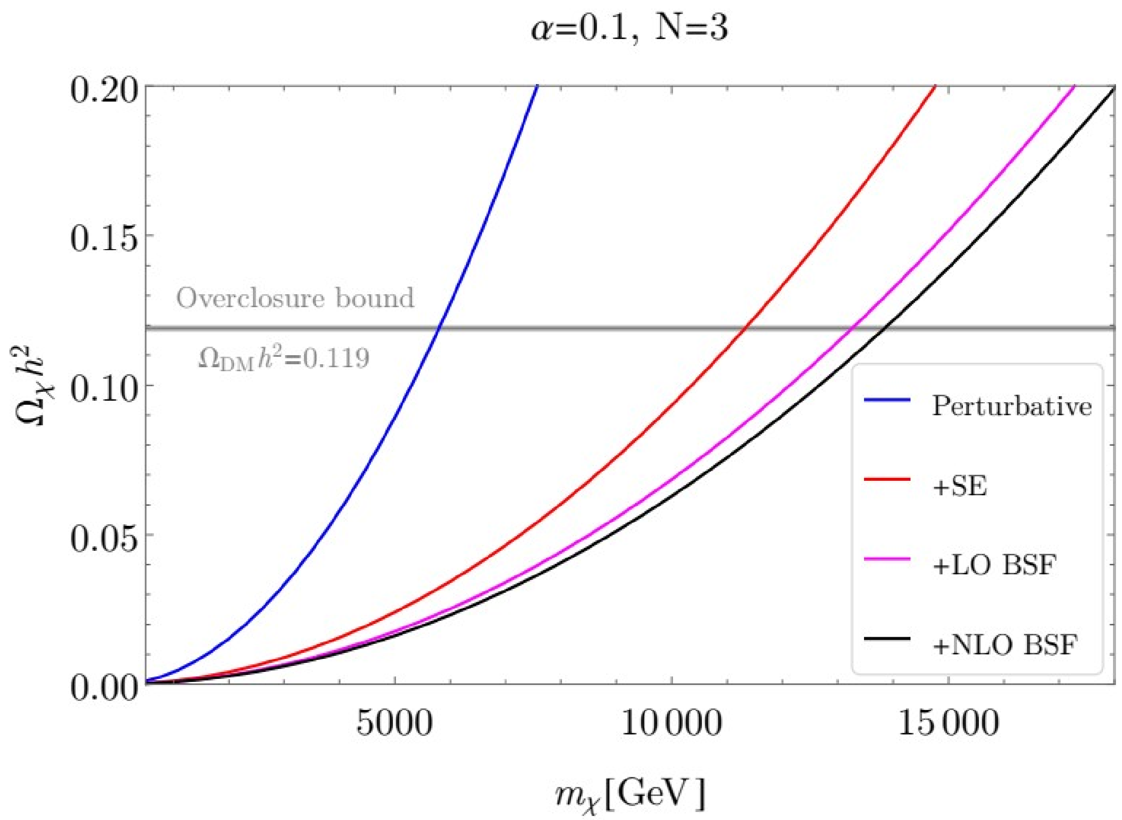


$\sim 2\%$ $\sim 20\%$

additional depletion

Relic abundance: massless vector mediator

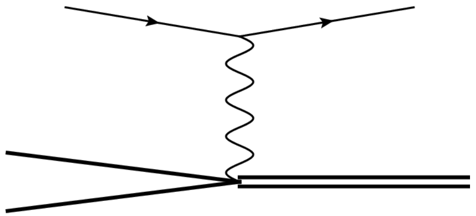
“One quark” scenario:
(more conservative)



Remarks:

- For larger N, effect increases.
- Boltzmann formalism would have failed in massless limit.
- Insertion of screening mass by hand would have overestimated the effect.

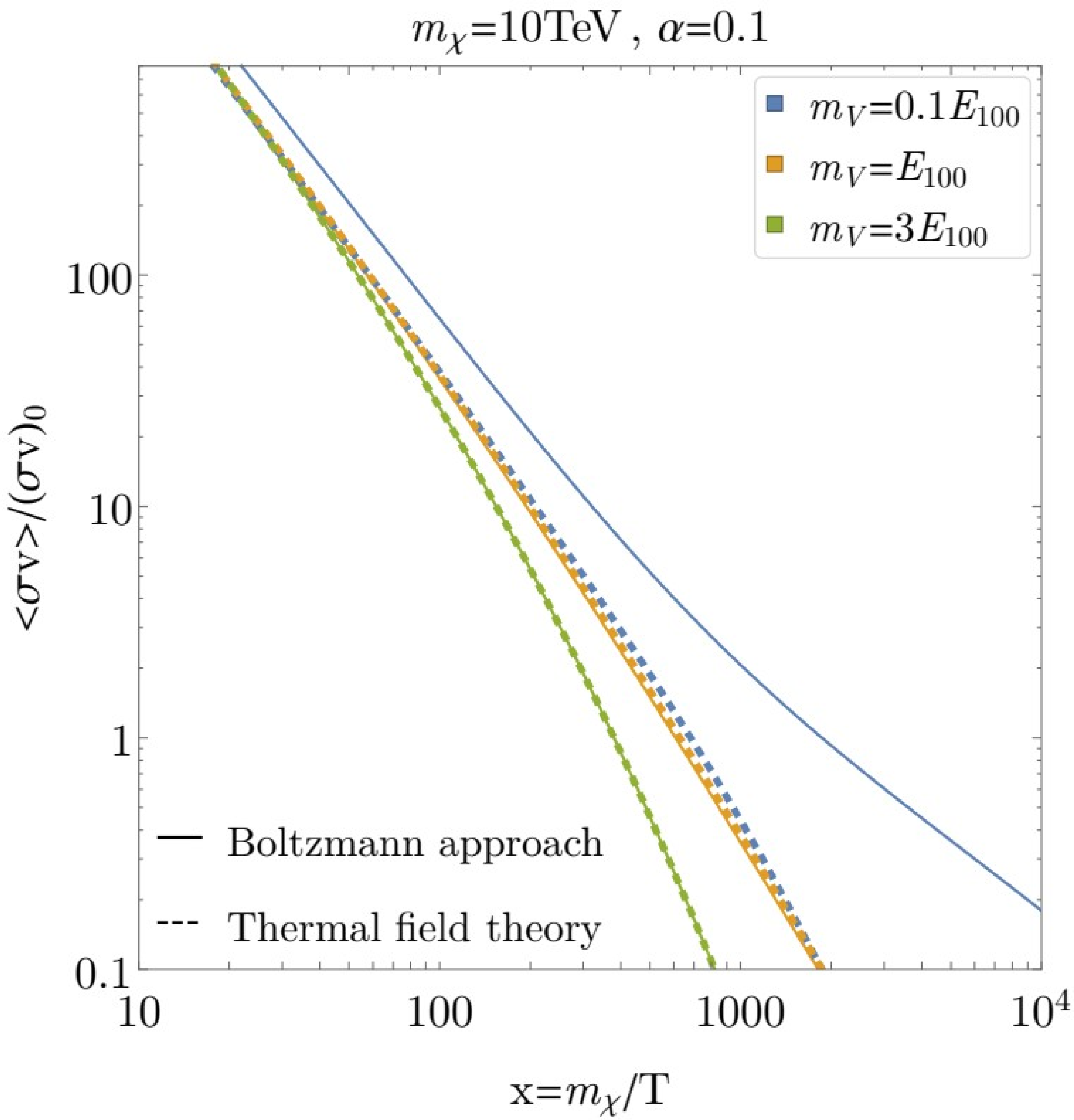
BSF via bath-particle scattering: massive case



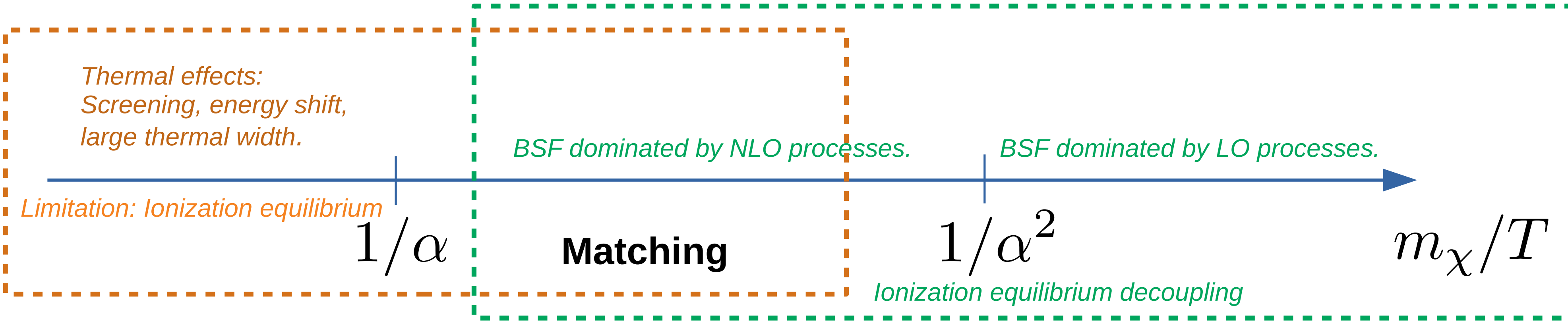
+ Interference

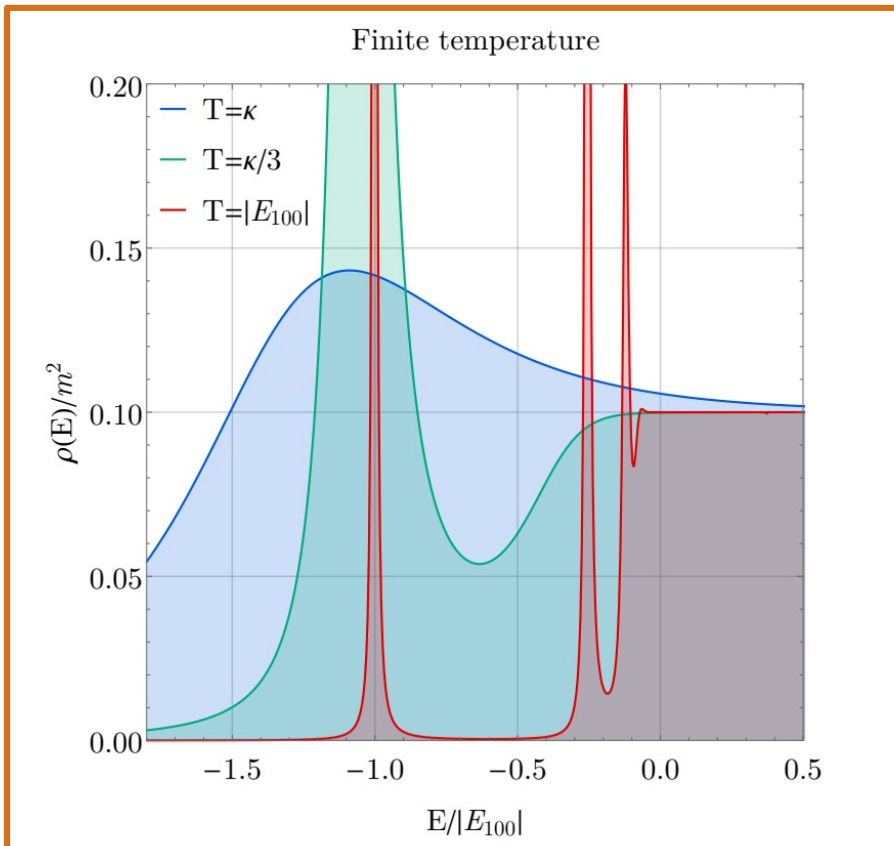
- Interference terms negligible for mediator masses much larger than binding energy: Boltzmann computation ok.

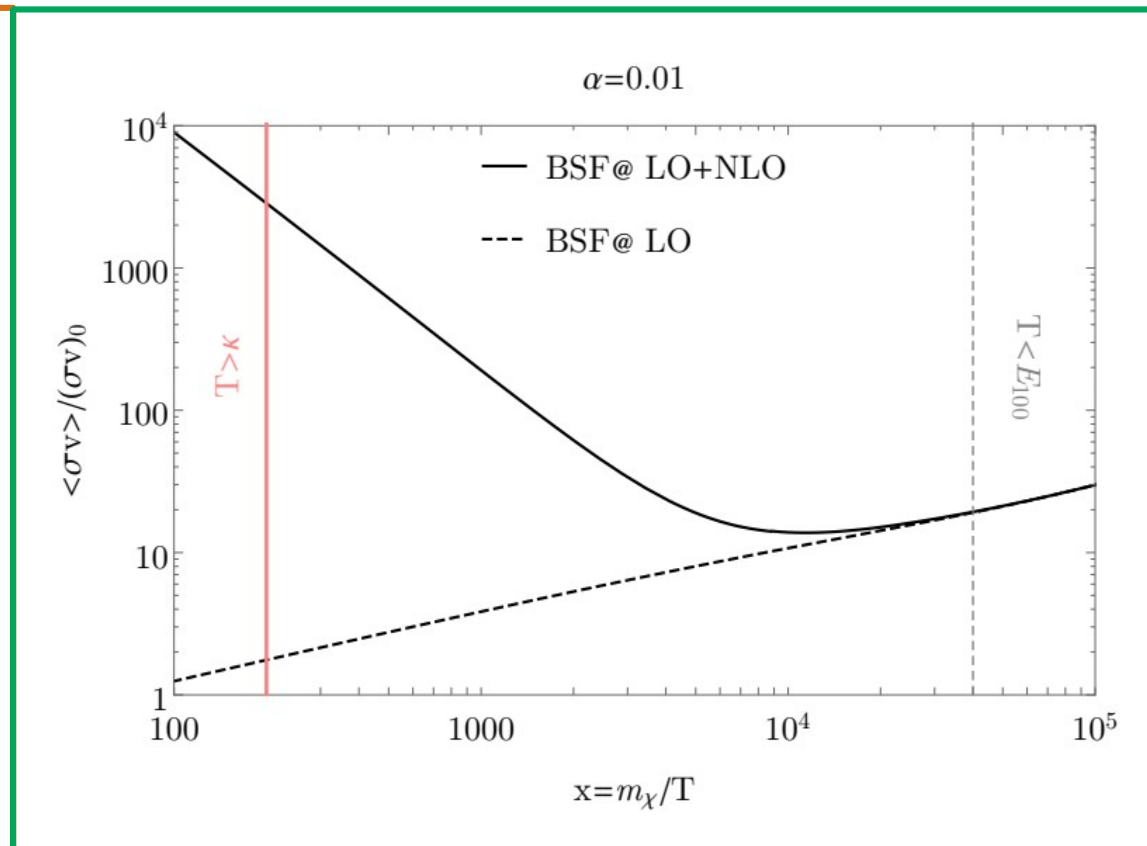
- **Thermal field theory approach required** for mediator masses smaller than or comparable to the binding energy.



More complete picture







Summary and conclusion

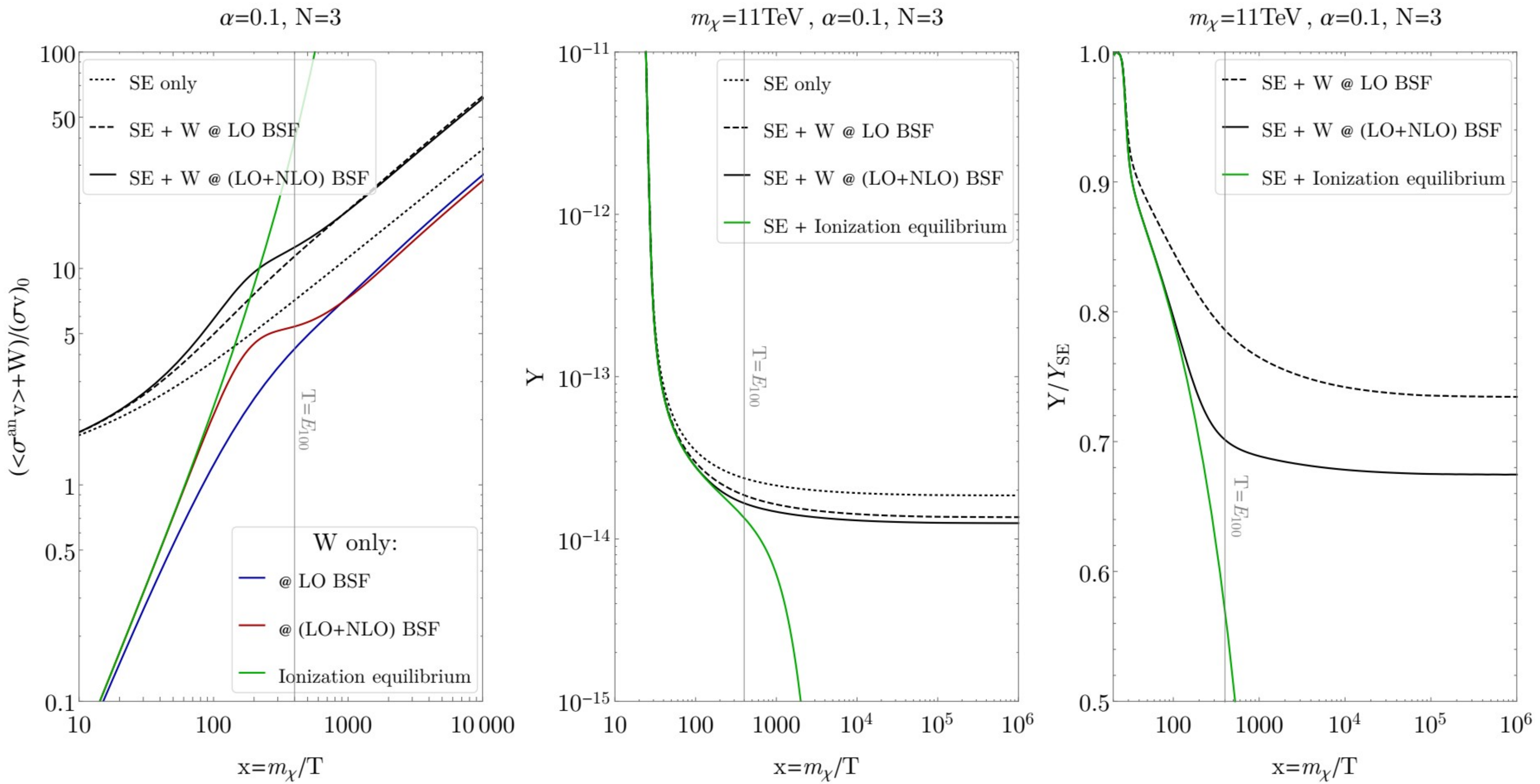
Formal achievements:

- Proof for **cancellation of collinear divergences**.
- More **complete description** of the DM freeze-out: from **melting of bound states** down to far below the **decoupling from ionization equilibrium**.

Phenomenological results and their implications:

- Real part corrections partially cancel each other.
- Impact of a large thermal width in co-annihilation scenarios unclear.
- T> binding energy: **dominant BSF channel** is **via bath-particle scattering**.
- Statement expected to be true also for non-abelian gauge or Yukawa theories.
- **Consequently, DM mass could be (a bit) heavier than previously expected.**

Backup



α=0.1, N=3
SE only
SE + W @ LO BSF
SE + W @ (LO+NLO) BSF
W only:
@ LO BSF
@ (LO+NLO) BSF
Ionization equilibrium
T=E_{100}
$(<\sigma^{an}v>+W)/(\sigma v)_0$
x=m_χ/T
m_χ=11TeV , α=0.1, N=3
SE + Ionization equilibrium
Y
m_χ=11TeV , α=0.1, N=3
Y/Y_{SE}